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BOOK ONE

1. Preface

Those who have been true philosophers, Syrus, seem to me to have very wisely separated the theoretical part of philosophy from the practical. For even if it happens the practical turns out to be theoretical prior to its being practical, nevertheless a great difference would be found in them; not only because some of the moral virtues can belong to the everyday ignorant man and it is impossible to come by the theory of whole sciences without learning, but also because in practical matters the greatest advantage is to be had from a continued and repeated operation upon the things themselves, while in theoretical knowledge it is to be had by a progress onward. We accordingly thought it up to us so to train our actions even in the application of the imagination as not to forget in whatever things we happen upon the consideration of their beautiful and well-ordered disposition, and to indulge in meditation mostly for the exposition of many beautiful theorems and especially of those specifically called mathematical.

For indeed Aristotle quite properly divides also the theoretical into three immediate genera: the physical, the mathematical, and the theological. For given that all beings have their existence from matter and form and motion, and that none of these can be seen, but only thought, in its subject separately from the others, if one should seek out in its simplicity the first cause of the first movement of the universe, he would find God invisible and unchanging. And the kind of science which seeks after Him is the theological; for such an act [ἐνέργεια] can only be thought as high above somewhere near the loftiest things of the universe and is absolutely apart from sensible things. But the kind of science which traces through the material and ever moving quality, and has to do with the white, the hot, the sweet, the soft, and such things, would be called physical; and such an essence [οὐσία], since it is only generally what it is, is to be found in corruptible things and below the lunar sphere. And the kind of science which shows up quality with respect to forms and local motions, seeking figure, number, and magnitude, and also place, time, and similar things, would be defined as mathematical. For such an essence falls, as it were, between the other two, not only because it can be conceived both through the senses and without the senses, but also because it is an accident in absolutely all beings both mortal and immortal, changing with those things that ever change, according to their inseparable form, and preserving unchangeable the changelessness of form in things eternal and of an ethereal nature.

And therefore meditating that the other two genera of the theoretical would be expounded in terms of conjecture rather than in terms of scientific understanding: the theological because it is in no way phenomenal and attainable, but the physical because its matter is unstable and obscure, so that for this reason philosophers could never hope to agree on them; and meditating that only the mathematical, if approached enquiringly, would give its practitioners certain

and trustworthy knowledge with demonstration both arithmetic and geometric resulting from indisputable procedures, we were led to cultivate most particularly as far as lay in our power this theoretical discipline [θεωρία]. And especially were we led to cultivate that discipline developed in respect to divine and heavenly things as being the only one concerned with the study of things which are always what they are, and therefore able itself to be always what it is—which is indeed the proper mark of a science—because of its own clear and ordered understanding, and yet to cooperate with the other disciplines no less than they themselves. For that special mathematical theory would most readily prepare the way to the theological, since it alone could take good aim at that unchangeable and separate act, so close to that act are the properties having to do with translations and arrangements of movements, belonging to those heavenly beings which are sensible and both moving and moved, but eternal and impassible. Again as concerns the physical there would not be just chance correspondances. For the general property of the material essence is pretty well evident from the peculiar fashion of its local motion—for example, the corruptible and incorruptible from straight and circular movements, and the heavy and light or passive and active from movement to the center and movement from the center. And indeed this same discipline would more than any other prepare understanding persons with respect to nobleness of actions and character by means of the sameness, good order, due proportion, and simple directness contemplated in divine things, making its followers lovers of that divine beauty, and making habitual in them, and as it were natural, a like condition of the soul.

And so we ourselves try to increase continuously our love of the discipline of things which are always what they are, by learning what has already been discovered in such sciences by those really applying themselves to them, and alsby making a small original contribution such as the period of time from them to us could well make possible. And therefore we shall try and set forth as briefly as possible as many theorems as we recognize to have come to light up to the present, and in such a way that those who have already been initiated somewhat may follow, arranging in proper order for the completeness of the treatise all matters useful to the theory of heavenly things. And in order not to make the treatise too long we shall only report what was rigorously proved by the ancients, perfecting as far as we can what was not fully proved or not proved as well as possible.

2. On the Order of the Theorems

A view, therefore, of the general relation of the whole earth to the whole of the heavens will begin this composition of ours. And next, of things in particular, there will first be an account of the ecliptic's position and of the places of that part of the earth inhabited by us, and again of the difference, in order, between each of them according to the inclinations of their horizons. For the theory of these, once understood, facilitates the examination of the rest. And, secondly, there will be an account of the solar and lunar movements and of their incidents. For without a prior understanding of these one could not profitably consider what concerns the stars. The last part, in view of this plan, will be an account of the stars. Those things having to do with the sphere of what are called the fixed stars would reasonably come first, and then those having to do with what are called the five planets. And we shall try and show each of these things using

as beginnings and foundations for what we wish to find, the evident and certain appearances from the observations of the ancients and our own, and applying the consequences of these conceptions by means of geometrical demonstrations.

And so, in general, we have to state that the heavens are spherical and move spherically; that the earth, in figure, is sensibly spherical also when taken as a whole; in position, lies right in the middle of the heavens, like a geometrical centre; in magnitude and distance, has the ratio of a point with respect to the sphere of the fixed stars, having itself no local motion at all. And we shall go through each of these points briefly to bring them to mind.

3. That the Heavens Move Spherically

It is probable the first notions of these things came to the ancients from some such observation as this. For they kept seeing the sun and moon and other stars always moving from rising to setting in parallel circles, beginning to move upward from below as if out of the earth itself, rising little by little to the top, and then coming around again and going down in the same way until at last they would disappear as if falling into the earth. And then again they would see them, after remaining some time invisible, rising and setting as if from another beginning; and they saw that the times and also the places of rising and setting generally corresponded in an ordered and regular way.

But most of all the observed circular orbit of those stars which are always visible, and their revolution about one and the same centre, led them to this spherical notion. For necessarily this point became the pole of the heavenly sphere; and the stars nearer to it were those that spun around in smaller circles, and those farther away made greater circles in their revolutions in proportion to the distance, until a sufficient distance brought one to the disappearing stars. And then they saw that those near the always-visible stars disappeared for a short time, and those farther away for a longer time proportionately. And for these reasons alone it was sufficient for them to assume this notion as a principle, and forthwith to think through also the other things consequent upon these same appearances, in accordance with the development of the science. For absolutely all the appearances contradict the other opinions.

If, for example, one should assume the movement of the stars to be in a straight line to infinity, as some have opined, how could it be explained that each star will be observed daily moving from the same starting point? For how could the stars turn back while rushing on to infinity? Or how could they turn back without appearing to do so? Or how is it they do not disappear with their size gradually diminishing, but on the contrary seem larger when they are about to disappear, being covered little by little as if cut off by the earth's surface? But certainly to suppose that they light up from the earth and then again go out in it would appear most absurd. For if anyone should agree that such an order in their magnitudes and number, and again in the distances, places, and times is accomplished in this way at random and by chance, and that one whole part of the earth has an incandescent nature and another a nature capable of extinguishing, or rather that the same part lights the stars up for some people and puts them out for others, and that the same stars happen to appear to some people either lit up or put out and to others not yet so—even if anyone, I say, should accept all such absurdities, what could we say about the always-visible stars which neither rise nor set? Or why don't the stars which light up and go out

rise and set for every part of the earth, and why aren't those which are not affected in this way always above the earth for every part of the earth? For in this hypothesis the same stars will not always light up and go out for some people, and never for others. But it is evident to everyone that the same stars rise and set for some parts, and do neither of these things for others.

In a word, whatever figure other than the spherical be assumed for the movement of the heavens, there must be unequal linear distances from the earth to parts of the heavens, wherever or however the earth be situated, so that the magnitudes and angular distances of the stars with respect to each other would appear unequal to the same people within each revolution, now larger now smaller. But this is not observed to happen. For it is not a shorter linear distance which makes them appear larger at the horizon, but the steaming up of the moisture surrounding the earth between them and our eyes, just as things put under water appear larger the farther down they are placed.

The following considerations also lead to the spherical notion: the fact that instruments for measuring time cannot agree with any hypothesis save the spherical one; that, since the movement of the heavenly bodies ought to be the least impeded and most facile, the circle among plane figures offers the easiest path of motion, and the sphere among solids; likewise that, since of different figures having equal perimeters those having the more angles are the greater, the circle is the greatest of plane figures and the sphere of solid figures, and the heavens are greater than any other body.

Moreover, certain physical considerations lead to such a conjecture. For example, the fact that of all bodies the ether has the finest and most homogeneous parts [ὁμοιομερέστερος]; but the surfaces of homogeneous parts must have homogeneous parts, and only the circle is such among plane figures and the sphere among solids. And since the ether is not plane but solid, it can only be spherical. Likewise the fact that nature has built all earthly and corruptible bodies wholly out of rounded figures but with heterogeneous parts, and all divine bodies in the ether out of spherical figures with homogeneous parts, since if they were plane or disc-like they would not appear circular to all those who see them from different parts of the earth at the same time. Therefore it would seem reasonable that the ether surrounding them and of a like nature be also spherical, and that because of the homogeneity of its parts it moves circularly and regularly.

4. That also the Earth, Taken as a Whole, Is Sensibly Spherical

Now, that also the earth taken as a whole is sensibly spherical, we could most likely think out in this way. For again it is possible to see that the sun and moon and the other stars do not rise and set at the same time for every observer on the earth, but always earlier for those living towards the orient and later for those living towards the occident. For we find that the phenomena of eclipses taking place at the same time, especially those of the moon, are not recorded at the same hours for everyone—that is, relatively to equal intervals of time from noon; but we always find later hours recorded for observers towards the orient than for those towards the occident. And since the differences in the hours is found to be proportional to the distances between the places, one would reasonably suppose the surface of the earth spherical, with the result that the general uniformity of curvature would assure every part's covering those following it

proportionately. But this would not happen if the figure were any other, as can be seen from the following considerations.

For, if it were concave, the rising stars would appear first to people towards the occident; and if it were flat, the stars would rise and set for all people together and at the same time; and if it were a pyramid, a cube, or any other polygonal figure, they would again appear at the same time for all observers on the same straight line. But none of these things appears to happen. It is further clear that it could not be cylindrical with the curved surface turned to the risings and settings and the plane bases to the poles of the universe, which some think more plausible. For then never would any of the stars be always visible to any of the inhabitants of the curved surface, but either all the stars would both rise and set for observers or the same stars for an equal distance from either of the poles would always be invisible to all observers. Yet the more we advance towards the north pole, the more the southern stars are hidden and the northern stars appear. So it is clear that here the curvature of the earth covering parts uniformly in oblique directions proves its spherical form on every side. Again, whenever we sail towards mountains or any high places from whatever angle and in whatever direction, we see their bulk little by little increasing as if they were arising from the sea, whereas before they seemed submerged because of the curvature of the water's surface.

5. That the Earth Is in the Middle of the Heavens

Now with this done, if one should next take up the question of the earth's position, the observed appearances with respect to it could only be understood if we put it in the middle of the heavens as the centre of the sphere. If this were not so, then the earth would either have to be off the axis but equidistant from the poles, or on the axis but farther advanced towards one of the poles, or neither on the axis nor equidistant from the poles.

The following considerations are opposed to the first of these three positions—namely, that if the earth were conceived as placed off the axis either above or below in respect to certain parts of the earth, those parts, in the right sphere, would never have any equinox since the section above the earth and the section below the earth would always be cut unequally by the horizon. Again, if the sphere were inclined with respect to these parts, either they would have no equinox or else the equinox would not take place midway between the summer and winter solstices. The distances would be unequal because the equator which is the greatest of those parallel circles described about the poles would not be cut in half by the horizon; but one of the circles parallel to it, either to the north or to the south, would be so cut in half. It is absolutely agreed by all, however, that these distances are everywhere equal because the increase from the equinox to the longest day at the summer tropic are equal to the decreases to the least days at the winter tropic. And if the deviation for certain parts of the earth were supposed either towards the orient or the occident, it would result that for these parts neither the sizes and angular distances of the stars would appear equal and the same at the eastern and western horizons, nor would the time from rising to the meridian be equal to the time from the meridian to setting. But these things evidently are altogether contrary to the appearances.

As to the second position where the earth would be on the axis but farther advanced towards one of the poles, one could again object that, if this were so,

the plane of the horizon in each latitude would always cut into uneven parts the sections of the heavens below the earth and above, different with respect to each other and to themselves for each different deviation. And the horizon could cut into two even parts only in the right sphere. But in the case of the inclined sphere with the nearer pole ever visible, the horizon would always make the part above the earth less and the part below the earth greater with the result that also the great circle through the centre of the signs of the zodiac [ecliptic] would be cut unequally by the plane of the horizon. But this has never been seen, for six of the twelve parts are always and everywhere visible above the earth, and the other six invisible; and again when all these last six are all at once visible, the others are at the same time invisible. And so—from the fact that the same semicircles are cut off entirely, now above the earth, now below—it is evident that the sections of the zodiac are cut in half by the horizon.

And, in general, if the earth did not have its position under the equator but lay either to the north or south nearer one of the poles, the result would be that, during the equinoxes, the shadows of the gnomons at sunrise would never perceptibly be on a straight line with those at sunset in planes parallel to the horizon. But the contrary is everywhere seen to occur. And it is immediately clear that it is not possible to advance the third position since each of the obstacles to the first two would be present here also.

In brief, all the observed order of the increases and decreases of day and night would be thrown into utter confusion if the earth were not in the middle. And there would be added the fact that the eclipses of the moon could not take place for all parts of the heavens by a diametrical opposition to the sun, for the earth would often not be interposed between them in their diametrical oppositions, but at distances less than a semicircle.

6. That the Earth Has the Ratio of a Point to the Heavens

Now, that the earth has sensibly the ratio of a point to its distance from the sphere of the so-called fixed stars gets great support from the fact that in all parts of the earth the sizes and angular distances of the stars at the same times appear everywhere equal and alike, for the observations of the same stars in the different latitudes are not found to differ in the least.

Moreover, this must be added: that sundials placed in any part of the earth and the centres of armillary spheres can play the role of the earth's true centre for the sightings and the rotations of the shadows, as much in conformity with the hypotheses of the appearances as if they were at the true midpoint of the earth.

And the earth is clearly a point also from this fact: that everywhere the planes drawn through the eye, which we call horizons, always exactly cut in half the whole sphere of the heavens. And this would not happen if the magnitude of the earth with respect to its distance from the heavens were perceptible; but only the plane drawn through the point at the earth's centre would exactly cut the sphere in half, and those drawn through any other part of the earth's surface would make the sections below the earth greater than those above.

7. That the Earth Does Not in any Way Move Locally

By the same arguments as the preceding it can be shown that the earth can neither move in any one of the aforesaid oblique directions, nor ever change at

all from its place at the centre. For the same things would result as if it had another position than at the centre. And so it also seems to me superfluous to look for the causes of the motion to the centre when it is once for all clear from the very appearances that the earth is in the middle of the world and all weights move towards it. And the easiest and only way to understand this is to see that, once the earth has been proved spherical considered as a whole and in the middle of the universe as we have said, then the tendencies and movements of heavy bodies (I mean their proper movements)[1] are everywhere and always at right angles to the tangent plane drawn through the falling body's point of contact with the earth's surface. For because of this it is clear that, if they were not stopped by the earth's surface, they too would go all the way to the centre itself, since the straight line drawn to the centre of a sphere is always perpendicular to the plane tangent to the sphere's surface at the intersection of that line.

All those who think it paradoxical that so great a weight as the earth should not waver or move anywhere seem to me to go astray by making their judgment with an eye to their own affects and not to the property of the whole. For it would not still appear so extraordinary to them, I believe, if they stopped to think that the earth's magnitude compared to the whole body surrounding it is in the ratio of a point to it. For thus it seems possible for that which is relatively least to be supported and pressed against from all sides equally and at the same angle by that which is absolutely greatest and homogeneous. For there is no "above" and "below" in the universe with respect to the earth, just as none could be conceived of in a sphere. And of the compound bodies in the universe, to the extent of their proper and natural motion, the light and subtle ones are scattered in flames to the outside and to the circumference, and they seem to rush in the upward direction relative to each one because we too call "up" from above our heads to the enveloping surface of the universe; but the heavy and coarse bodies move to the middle and centre and they seem to fall downwards because again we all call "down" the direction from our feet to the earth's centre. And they properly subside about the middle under the everywhere-equal and like resistance and impact against each other. Therefore the solid body of the earth is reasonably considered as being the largest relative to those moving against it and as remaining unmoved in any direction by the force of the very small weights, and as it were absorbing their fall. And if it had some one common movement, the same as that of the other weights, it would clearly leave them all behind because of its much greater magnitude. And the animals and other weights would be left hanging in the air, and the earth would very quickly fall out of the heavens. Merely to conceive such things makes them appear ridiculous.

[1]All local motions or movements according to place are divided by Aristotle into natural and violent local motions. In the case of compound bodies (that is, those bodies subject to generation and corruption and consisting of all those, and only those, bodies lying below the lunar sphere), the natural local motions are those of unimpeded and unpropelled fall; the violent local motions are any propelled or interrupted motions. In the case of simple bodies (that is, the heavenly bodies within and above the lunar sphere), there are only natural local motions: the regular or uniform circular motions. Ptolemy here calls the natural local motions of compound bodies their proper motions. This distinction between natural and violent motions is preserved by Galileo. For in his *Two New Sciences*, natural motion is treated in the "Third Day" and violent motion in the "Fourth Day." In the Newtonian system, the distinction is dissolved in a general mathematical treatment, a treatment more in line with the Platonic myth of the *Timaeus*, and so it loses all meaning.

Now some people, although they have nothing to oppose to these arguments, agree on something, as they think, more plausible. And it seems to them there is nothing against their supposing, for instance, the heavens immobile and the earth as turning on the same axis from west to east very nearly one revolution a day; or that they both should move to some extent, but only on the same axis as we said, and conformably to the overtaking of the one by the other.

But it has escaped their notice that, indeed, as far as the appearances of the stars are concerned, nothing would perhaps keep things from being in accordance with this simpler conjecture, but that in the light of what happens around us in the air such a notion would seem altogether absurd. For in order for us to grant them what is unnatural in itself, that the lightest and subtlest bodies either do not move at all or no differently from those of contrary nature, while those less light and less subtle bodies in the air are clearly more rapid than all the more terrestrial ones; and to grant that the heaviest and most compact bodies have their proper swift and regular motion, while again these terrestrial bodies are certainly at times not easily moved by anything else—for us to grant these things, they would have to admit that the earth's turning is the swiftest of absolutely all the movements about it because of its making so great a revolution in a short time, so that all those things that were not at rest on the earth would seem to have a movement contrary to it, and never would a cloud be seen to move toward the east nor anything else that flew or was thrown into the air. For the earth would always outstrip them in its eastward motion, so that all other bodies would seem to be left behind and to move towards the west.

For if they should say that the air is also carried around with the earth in the same direction and at the same speed, none the less the bodies contained in it would always seem to be outstripped by the movement of both. Or if they should be carried around as if one with the air, neither the one nor the other would appear as outstripping, or being outstripped by, the other. But these bodies would always remain in the same relative position and there would be no movement or change either in the case of flying bodies or projectiles. And yet we shall clearly see all such things taking place as if their slowness or swiftness did not follow at all from the earth's movement.

8. That There Are Two Different Prime Movements in the Heavens

It will be sufficient for these hypotheses, which have to be assumed for the detailed expositions following them, to have been outlined here in such a summary way since they will finally be established and confirmed by the agreement of the consequent proofs with the appearances. In addition to those already mentioned, this general assumption would also be rightly made that there are two different prime movements in the heavens. One is that by which everything moves from east to west, always in the same way and at the same speed with revolutions in circles parallel to each other and clearly described about the poles of the regularly revolving sphere. Of these circles the greatest is called the equator, because it alone is always cut exactly in half by the horizon which is a great circle of the sphere, and because everywhere the sun's revolution about it is sensibly equinoctial. The other movement is that according to which the spheres of the stars make certain local motions in the direction opposite to that of the movement just described and around other poles than those of that first revolu-

tion. And we assume that it is so because, while, from each day's observation, all the heavenly bodies are seen to move generally in paths sensibly similar and parallel to the equator and to rise, culminate, and set (for such is the property of the first movement), yet from subsequent and more continuous observation, even if all the other stars appear to preserve their angular distances with respect to each other and their properties as regards their places within the first movement, still the sun and moon and planets make certain complex movements unequal to each other, but all contrary to the general movement, towards the east opposite to the movement of the fixed stars which preserve their respective angular distances and are moved as if by one sphere.

If, then, this movement of the planets also took place in circles parallel to the equator—that is, around the same poles as those of the first revolution—it would be sufficient to assume for them all one and the same revolving movement in conformity with the first. For it would then be plausible to suppose that their movement was the result of a lag and not of a contrary movement. But they always seem, at the same time they move towards the east, to deviate towards the north and south poles without any uniform magnitude's being observed in this deviation, so that this seems to befall them through impulsions. But although this deviation is irregular on the hypothesis of one prime movement, it is regular when effected by a circle oblique to the equator. And so such a circle is conceived one and the same for, and proper to, the planets, quite exactly expressed and as it were described by the motion of the sun, but traveled also by the moon and planets which ever turn about it with every deviation from it on the part of any planet either way, a deviation within a prescribed distance and governed by rule. And since this is seen to be a great circle also because of the sun's equal oscillation to the north and south of the equator, and since the eastward movements of all the planets (as we said) take place on one and the same circle, it was necessary to suppose a second movement different from the general one, a movement about the poles of this oblique circle or ecliptic in the direction opposite to that of the first movement.

Then if we think of a great circle described through the poles of both the circles just mentioned, which necessarily cuts each of them—that is, the equator and the circle inclined to it—exactly in half and at right angles, there will be four points on the oblique circle or ecliptic: the two made by the equator diametrically opposite each other and called the equinoxes of which the one guarding the northern approach is called spring, and the opposite one autumn. And the two made by the circle drawn through both sets of poles, also clearly diametrically opposite each other, are called the tropics, of which the one to the south of the equator is called winter, and the one to the north summer.

The one first movement which contains all the others will be thought of then as described and as if defined by the great circle, through both sets of poles, which is carried around and carries with it all the rest from east to west about the poles of the equator. And these poles are as if they were on what is called the meridian, which differs from the circle through both sets of poles in this alone: that it is not always drawn through the poles of the ecliptic, but is conceived as continuously at right angles to the horizon and therefore called the meridian, since such a position cutting in half as it does each of the two hemispheres, that below the earth and that above, provides midday and midnight. But the second movement, consisting of many parts and contained by the first, and embracing

itself all the planetary spheres,[1] is carried by the first as we said, and revolves about the poles of the ecliptic in the opposite direction. And these poles of the ecliptic being on the circle effecting the first revolution—that is, on the circle drawn through all four poles together—are carried around with it as one would expect; and, moving therefore with a motion opposite to the second prime movement, in this way keep the position of the great circle which is the ecliptic ever the same with respect to the equator.

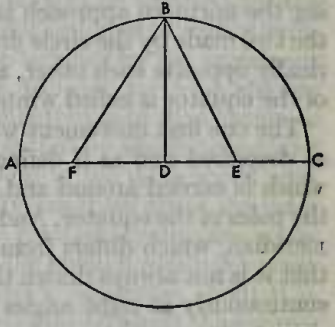
9. On the Particular Notions

A summary and general preliminary explanation would contain some such exposition as the foregoing of the things to be presupposed. But now we are going to begin the detailed proofs. And we think the first of these is that by means of which is calculated the length of the arc between the poles of the equator and the ecliptic, lying on the great circle drawn through these poles. To this end we must first see expounded the method of computing the size of chords inscribed in a circle, and we are now going to demonstrate this geometrically for each case, once for all.

10. On the Size of Chords in a Circle

With an eye to immediate use, we shall now make a tabular exposition of the size of these chords by dividing the circumference into 360 parts and setting side by side the chords as the arcs subtended by them increase by a half part. That is, the diameter of the circle will be cut into 120 parts for ease in calculation; [and we shall take the arcs, considering them with respect to the number they contain of the circumference's 360 parts, and compare them with the subtending chords by finding out the number the chords contain of the diameter's 120 parts.] But first we shall show how, with as few theorems as possible and the same ones, we make a methodical and rapid calculation of their sizes so that we may not only have the magnitudes of the chords set out without knowing the why and wherefore but so that we may also easily manage a proof by means of a systematic geometrical construction. In general we shall use the sexagesimal system because of the difficulty of fractions, and we shall follow out the multiplications and divisions, aiming always at such an approximation as will leave no error worth considering as far as the accuracy of the senses is concerned.

Then first let there be the semicircle ABC on the diameter ADC and around centre D, and let straight line DB be erected on AC at right angles. Let DC be bisected at E, and EB be joined; and let EF be laid out equal to EB, and let FB be joined.



I say that the straight line FD is the side of a regular inscribed decagon, and BF that of a pentagon.

For since the straight line DC is bisected at E and a straight line DF is added to it,

rect. CF, FD + sq. ED = sq. EF, [Eucl. II, 6]

since = sq. BE,

$BE = EF$.

[1] These are the two movements of the same and of the other described in Plato's myth of the *Timaeus*.

But

$$\text{sq. } ED + \text{sq. } DB = \text{sq. } BE. \qquad \text{[Eucl. I, 47]}$$

Therefore

$$\text{rect. } CF, FD + \text{sq. } ED = \text{sq. } ED + \text{sq. } DB.$$

And, subtracting the common square on ED,

$$\text{rect. } CF, FD = \text{sq. } DB$$
$$= \text{sq. } DC.$$

Therefore CF is cut at D in extreme and mean ratio [Eucl. VI, def. 3]. Since, then, the side of the hexagon and the side of the decagon which are inscribed in the same circle, when they are in the same straight line, cut that line in extreme and mean ratio [Eucl. XIII, 9], and since the radius DC is equal to the side of the hexagon [Eucl. IV, 15 coroll.], therefore FD is equal to the side of the decagon.

And likewise, since the square on the side of the pentagon is equal to the square on the side of the hexagon together with the square on the side of the decagon, all inscribed in the same circle [Eucl. XIII, 10], and since in the right triangle BDF

$$\text{sq. } BF = \text{sq. } DB + \text{sq. } FD$$

where DB is the side of the hexagon and FD the side of the decagon, the straight line BF is equal to the side of the pentagon.

Since, then, as I said, we suppose the diameter divided into 120 parts, therefore by what we have just established, being half the circle's radius,

$$ED = 30 \text{ such parts,}$$

and

$$\text{sq. } ED = 900;$$

and

$$\text{rad. } DB = 60 \text{ such parts,}$$

and

$$\text{sq. } DB = 3600;$$

and

$$\text{sq. } BE = \text{sq. } EF = 4500.$$

Then

$$EF = 67^{p}4'55''^{1}$$

in length, and by subtraction

$$FD = 37^{p}4'55''.$$

Therefore the side of the decagon, subtending an arc of 36° of the whole circumference's 360°, will have $37^{p}4'55''$ of the diameter's 120^{p}.

Since again

$$FD = 37^{p}4'55'',$$
$$\text{sq. } FD = 1375^{p}4'14'',$$
$$\text{sq. } DB = 3600^{p},$$

and

$$\text{sq. } FD + \text{sq. } DB = \text{sq. } BF,$$

therefore, in length,

[1]From now on, we shall indicate "parts such as the diameter's 120" by a p-superscript. Thus $67^{p}4'55''$ means "$67 + \frac{4}{60} + \frac{55}{3600}$ parts such as the diameter's 120." And we shall indicate "parts such as the circumference's 360" by the ordinary notation for angular degrees. For the measures of arcs and angles exactly correspond. Thus 47°42'40" means "$47 + \frac{42}{60} + \frac{40}{3600}$ parts such as the circumference's 360" or "parts such as 4 right angles' 360."

$$BF \fallingdotseq 70^{p}32'3''.$$

And therefore the side of the pentagon, subtending an arc of 72°, is $70^{p}32'3''$.

It is immediately clear that the side of the hexagon, subtending an arc of 60° and being equal to the radius, is itself 60^{p}. And likewise, since the side of the inscribed square, subtending an arc of 90°, is, when squared, double the square on the radius, and since the side of the inscribed equilateral triangle is, when squared, triple the square on the radius, and since the square on the radius is $3{,}600^{p}$, the square on the side of the square will add up to $7{,}200^{p}$, and the square on the side of the equilateral triangle to $10{,}800^{p}$. And so in length

$$\text{chord of arc } 90^\circ \fallingdotseq 84^{p}51'10'',$$

and

$$\text{chord of arc } 120^\circ \fallingdotseq 103^{p}55'23''.$$

And so these chords are easily gotten by themselves. Thence it is evident that, with these chords given, it will be easy to get the chords which subtend the supplements, since the squares on them added together are equal to the square on the diameter. For example, since it was shown

$$\text{chord of arc } 36^\circ = 37^{p}4'55'',$$

$$\text{sq. chord of arc } 36^\circ = 1375^{p}4'15'',$$

and

$$\text{sq. diameter} = 14{,}400^{p},$$

therefore, for the supplement,

$$\text{sq. chord of arc } 144^\circ = 13{,}024^{p}55'45'',$$

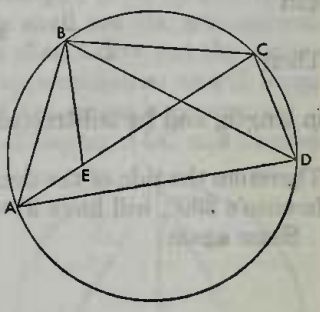
and, in length,

$$\text{chord of arc } 144^\circ \fallingdotseq 114^{p}7'37'';$$

and the others in like manner.

And we shall next show, by expounding a lemma very useful for this present business, how the rest of the chords can be derived successively from those we already have.

For let there be a circle with any sort of inscribed quadrilateral $ABCD$, and let AC and BD be joined.



It is to be proved that

$$\text{rect. } AC, BD = \text{rect. } AB, DC + \text{rect. } AD, BC.$$

For let it be laid out such that

$$\text{angle } ABE = \text{angle } DBC.$$

If then we add the common angle EBD,

$$\text{angle } ABD = \text{angle } EBC.$$

But also

$$\text{angle } BDA = \text{angle } BCE. \quad \text{[Eucl. III, 21]},$$

for they subtend the same arc. Then triangle ABD is equiangular with triangle BCE. Hence

$$BC : CE :: BD : AD \quad \text{[Eucl. VI, 4]}.$$

Therefore

$$\text{rect. } BC, AD = \text{rect. } BD, CE. \quad \text{[Eucl. VI, 16]}.$$

Again since

$$\text{angle } ABE = \text{angle } CBD$$

and also

$$\text{angle } BAE = \text{angle } BDC,$$

therefore triangle ABE is equiangular with triangle BCD. Hence

$$AB : AE :: BD : CD.$$

Therefore

rect. AB, CD = rect. BD, AE.

But it was also proved

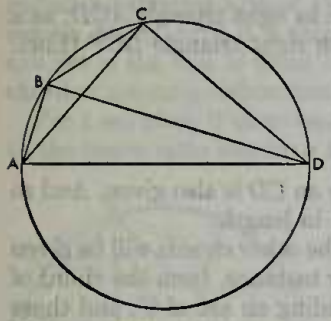
rect. BC, AD = rect. BD, CE.

Therefore also

rect. AC, BD = rect. AB, CD + rect. BC, AD. [Eucl. II, 1].

Which was to be proved.

Now that this has been expounded, let there be the semicircle $ABCD$ on diameter AD, and from the point A let there be drawn the two straight lines AB, AC, and let the length of each of them have been given in terms of such parts as the given diameter's 120; and let BC be joined.



I say that BC is also given.

For let BD and CD be joined. Then clearly they are also given because they subtend the supplements. Since, then, the quadrilateral $ABCD$ is inscribed in a circle, therefore

rect. AB, CD + rect. AD, BC = rect. AC, BD

[p. 16]

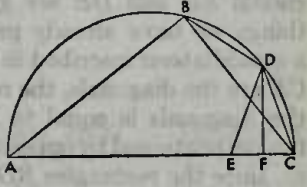
And rectangle AC, BD is given, and also rectangle AB, CD. Therefore the remaining rectangle AD, BC is also given. And AD is the diameter. Hence the straight line BC is also given. And it is now clear to us that, if two arcs are given and the two chords subtending them, then also the chord subtending the difference between the two arcs will be given. And it is evident that by means of this theorem we can inscribe many other chords in arcs which are the differences between arcs directly given; for instance, the chord subtending an arc of 12°, since we have the chords of 60° and 72°.

Again, given any chord in a circle, let it be proposed to find the chord of half the arc of the given chord.

And let there be the semicircle ABC on diameter AC, and let CB be the given chord. And let the arc be bisected at D, and let AB, AD, BD, and DC be joined. And let DF be drawn from D perpendicular to AC.

I say that

$$CF = \text{half } (AC - AB).$$



For let AE be laid out such that

$$AE = AB,$$

and let DB be joined. Since

$$AB = AE,$$

and AD is common, therefore the two sides AB and AD are equal to the two sides AE and AD respectively. And

angle BAD = angle EAD; [Eucl. III, 27]

therefore also

base BD = base DE.

But

chord BD = chord CD,

and therefore

chord CD = chord DE.

Since then, in the isosceles triangle DEC, DF has been dropped from the vertex perpendicular to the base, therefore

$$EF = CF.$$ [Eucl. I, 26]

But $$CE = AC - AB;$$

therefore

$$CF = \text{half } (AC - AB).$$

And so, since, given the chord of arc BC, chord AE of its supplement is also given [p. 16], therefore CF, which is half the difference between AC and AB, is given too. But when the perpendicular DF is drawn in right triangle ACD, as a consequence right triangle ACD is equiangular with right triangle DCF [Eucl. VI, 8], and

$$AC : CD :: CD : CF.$$

Therefore

$$\text{rect. } AC, CF = \text{sq. } CD.$$

But rectangle AC, CF is given, therefore the square on CD is also given. And so the chord CD of half the arc BC will also be given in length.

And so again, by means of this theorem, most of the other chords will be given as subtending the halves of arcs already found. For instance, from the chord of an arc of 12°, there can be gotten the chord subtending an arc of 6°, and those subtending arcs of 3°, 1½°, and ¾° respectively. And we shall find from calculation that

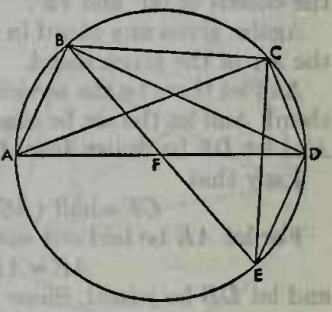
$$\text{chord of arc } 1\tfrac{1}{2}^\circ = 1^p 34' 15'',$$

and

$$\text{chord of arc } \tfrac{3}{4}^\circ = 0^p 47' 8''.$$

Again let there be the circle $ABCD$ on diameter AD with center at F. And from A let there be cut off consecutively two given arcs, AB and BC; and let the given chords subtending them, AB and BC, be joined.

I say that, if we join AC, then AC will be given also.

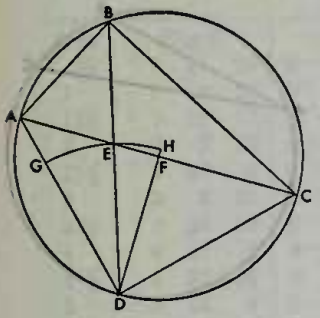


For let the circle's diameter BFE be drawn through B, and let BD, DC, CE, and DE be joined. Then from this it is clear that, by means of BC, chord CE is given; and, by means of AB, chords BD and DE are given [p. 16]. And by things we have already proved, since $BCDE$ is a quadrilateral inscribed in a circle, and BD and CE are the diagonals, the rectangle contained by the diagonals is equal to the sum of the rectangles contained by opposite sides [p. 16, 17]. And so, since the rectangles BD, CE and BC, DE are given, therefore the rectangle BE, CD is given also. But the diameter BE is given too, and the remaining side CD will be given. Therefore the chord AC of the supplement will be given also. And so, if two arcs and their chords are given, then by means of this theorem the chord of both these arcs together will be given.

And it is evident that, by continually combining the chord of an arc 1½° with those so far set out and calculating the sums, we shall inscribe all those chords which, when doubled, are divisible by three; and only those chords will still be

skipped which fall within $1\frac{1}{2}°$ intervals. For there will be two such chords skipped in each interval, since we are carrying out this inscribing of chords by successive additions of $\frac{1}{2}°$. And so if we could compute the chord subtending an arc of $\frac{1}{2}°$, then this chord, by addition to, and subtraction from, the chords which are separated by $1\frac{1}{2}°$ intervals and have already been given, will fill in all the rest of the intermediate chords. But since, given any chord such as that subtending an arc of $1\frac{1}{2}°$, the chord of a third of the arc is in no way geometrically given (and if it were possible, we could then compute the chord of an arc of $\frac{1}{2}°$), therefore we shall first look for the chord of an arc of 1° by means of chords subtending arcs of $1\frac{1}{2}°$ and $\frac{3}{4}°$. We shall do this by presenting a little lemma which, even if it may not suffice for determining their sizes in general, can yet in the case of these very small chords keep them indistinguishable from chords rigorously determined.

For I say that, if two unequal chords are inscribed in a circle, the greater has to the less a ratio less than the arc on the greater has to the arc on the less.



For let there be the circle $ABCD$; and let unequal chords be inscribed in it, AB the less and BC the greater.

I say that

chord BC : chord $AB <$ arc BC : arc AB.

For let angle ABC be bisected by BD, and let AEC, AD, and CD be joined. And since angle ABC has been bisected by the straight line DEB,

chord CD = chord AD, [Eucl. III, 26, 29]

and

$CE > AE$. [Eucl. VI. 3]

Then let DF be dropped from D perpendicular to AEC. Now since

$AD > DE$,
$DE > DF$,[1]

therefore the circle described with centre D and radius DE cuts AD and falls beyond DF. Then let the circle GEH be drawn and the straight line DFH be produced. And since

sect. $DEH >$ trgl. DEF,

and

trgl. $DEA >$ sect. DEG,

therefore

trgl. DEF : trgl. $DEA <$ sect. DEH : sect. DEG. [Eucl. V, 8]

But

trgl. DEF : trgl. DEA : : EF : AE, [Eucl. VI, 1]

and

sect. DEH : sect. DEG : : angle FDE : angle EDA.

Therefore

[1]For DF produced will bisect arc ABC [Eucl. III, 3, 26]; hence it will fall on the side of B towards C. Therefore

$DE > DF$, [Eucl. I, 21]
$AD > DE$.

$$EF : AE < \text{angle } FDE : \text{angle } EDA.$$

Then *componendo*

$$AF : AE < \text{angle } FDA : \text{angle } EDA.$$

And doubling the antecedents

$$CA : AE < \text{angle } CDA : \text{angle } EDA.$$

And *separando*

$$CE : AE < \text{angle } CDB : \text{angle } BDA.$$

But

$$CE : AE :: BC : AB, \qquad \text{[Eucl. VI, 3]}$$

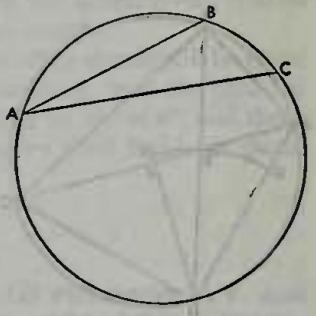
and

$$\text{angle } CDB : \text{angle } BDA :: \text{arc } BC : \text{arc } AB. \qquad \text{[Eucl. VI, 33]}$$

Therefore

$$\text{chord } BC : \text{chord } AB < \text{arc } BC : \text{arc } AB.$$

Now, then, with this laid down, let there be the circle ABC, and let the two chords AB and AC be inscribed in it. And first let AB be given as subtending an arc of $\frac{3}{4}°$, and AC an arc of 1°.



Since

$$\text{chord } AC : \text{chord } AB < \text{arc } AC : \text{arc } AB,$$

and

$$\text{arc } AC = 1\tfrac{1}{3} \text{ (arc } AB),$$

therefore

$$\text{chord } AC < 1\tfrac{1}{3} \text{ (chord } AB).$$

But it was proved [p. 18]

$$\text{chord } AB = 0^{\text{p}}47'8''.$$

Therefore

$$\text{chord } AC < 1^{\text{p}}2'50'',$$

for

$$1^{\text{p}}2'50'' \doteqdot 1\tfrac{1}{3} \ (0^{\text{p}}47'8'').$$

Again, with the same figure, let chord AB be given as subtending an arc of 1°, and chord AC an arc of $1\frac{1}{2}°$.

Likewise then, since

$$\text{arc } AC = 1\tfrac{1}{2} \text{ (arc } AB),$$
$$\text{chord } AC < 1\tfrac{1}{2} \text{ (chord } AB).$$

But we proved [p. 18]

$$\text{chord } AC = 1^{\text{p}}34'15''.$$

Therefore

$$\text{chord } AB > 1^{\text{p}}2'50'',$$

for

$$1^{\text{p}}34'15'' \doteqdot 1\tfrac{1}{2} \ (1^{\text{p}}2'50'').$$

And so, since it has been proved that the chord of an arc of 1° is both greater and less than the same number of parts, clearly we shall have

$$\text{chord of arc } 1° \doteqdot 1^{\text{p}}2'50'';$$

and by means of earlier proofs we saw

$$\text{chord of arc } \tfrac{1}{2}° \doteqdot 0^{\text{p}}31'25''.$$

And the remaining intervals will be filled in as we have just said. For example, in the first interval we find the chord subtending an arc of 2° by adding $\frac{1}{2}°$ and $1\frac{1}{2}°$, and the chord subtending an arc of $2\frac{1}{2}°$ by subtracting $\frac{1}{2}°$ from 3°, and so on for the rest.

So the business of chords in a circle can be easily handled in this way, I think.

And as I said, in order to have the magnitudes set out immediately to hand, we shall draw up tables of 45 rows each, for symmetry's sake. And the first column will contain the magnitudes of the arcs increasing by ½°, and the second column will contain the magnitudes of the chords subtending them in terms of the diameter's assumed 120 parts. The third column will contain the thirtieth of the increase of the chord as the corresponding arc increases by ½°, so that we may have a mean addition, accurate for the senses, for each increase of $\frac{1}{60}$° in the corresponding arcs, and so be able to calculate readily the chords falling within the ½° intervals. And it is to be remarked that by means of these same theorems, if we should suspect some typographical error in connection with any of the chords computed here, we can easily check and correct it either by means of the chord of an arc double the arc of the chord which is being examined, or by means of the difference of certain other given chords, or by means of the chord subtending the supplement. And here is the table:

11. Table of Chords

Arcs	*Chords*			*Sixtieths*[1]				*Arcs*	*Chords*			*Sixtieths*			
½	0	31	25	0	1	2	50	16½	17	13	9	0	1	2	10
1	1	2	50	0	1	2	50	17	17	44	14	0	1	2	7
1½	1	34	15	0	1	2	50	17½	18	15	17	0	1	2	5
2	2	5	40	0	1	2	50	18	18	46	19	0	1	2	2
2½	2	37	4	0	1	2	48	18½	19	17	21	0	1	2	0
3	3	8	28	0	1	2	48	19	19	48	21	0	1	1	57
3½	3	39	52	0	1	2	48	19½	20	19	19	0	1	1	54
4	4	11	16	0	1	2	47	20	20	50	16	0	1	1	51
4½	4	42	40	0	1	2	47	20½	21	21	11	0	1	1	48
5	5	14	4	0	1	2	46	21	21	52	6	0	1	1	45
5½	5	45	27	0	1	2	45	21½	22	22	58	0	1	1	42
6	6	16	49	0	1	2	44	22	22	53	49	0	1	1	39
6½	6	48	11	0	1	2	43	22½	23	24	39	0	1	1	36
7	7	19	33	0	1	2	42	23	23	55	27	0	1	1	33
7½	7	50	54	0	1	2	41	23½	24	26	13	0	1	1	30
8	8	22	15	0	1	2	40	24	24	56	58	0	1	1	26
8½	8	53	35	0	1	2	39	24½	25	27	41	0	1	1	22
9	9	24	54	0	1	2	38	25	25	58	22	0	1	1	19
9½	9	56	13	0	1	2	37	25½	26	29	1	0	1	1	15
10	10	27	32	0	1	2	35	26	26	59	38	0	1	1	11
10½	10	58	49	0	1	2	33	26½	27	30	14	0	1	1	8
11	11	30	5	0	1	2	32	27	28	0	48	0	1	1	4
11½	12	1	21	0	1	2	30	27½	28	31	20	0	1	1	0
12	12	32	36	0	1	2	28	28	29	1	50	0	1	0	56
12½	13	3	50	0	1	2	27	28½	29	32	18	0	1	0	52
13	13	35	4	0	1	2	25	29	30	2	44	0	1	0	48
13½	14	6	16	0	1	2	23	29½	30	33	8	0	1	0	44
14	14	37	27	0	1	2	21	30	31	3	30	0	1	0	40
14½	15	8	38	0	1	2	19	30½	31	33	50	0	1	0	35
15	15	39	47	0	1	2	17	31	32	4	7	0	1	0	31
15½	16	10	56	0	1	2	15	31½	32	34	22	0	1	0	27
16	16	42	3	0	1	2	13	32	33	4	35	0	1	0	22

[1]The sexagesimal system is carried out one place further in this column. Thus 0°1′2″50′ ″ represents $\frac{1}{60}+\frac{2}{3600}+\frac{50}{216000}$ which is $\frac{1}{30}$ of the increase of the second chord over the first.

Arcs	Chords			Sixtieths			
32½	33	34	46	0	1	0	17
33	34	4	55	0	1	0	12
33½	34	35	1	0	1	0	8
34	35	5	5	0	1	0	3
34½	35	35	6	0	0	59	57
35	36	5	5	0	0	59	52
35½	36	35	1	0	0	59	48
36	37	4	55	0	0	59	43
36½	37	34	47	0	0	59	38
37	38	4	36	0	0	59	32
37½	38	34	22	0	0	59	27
38	39	4	5	0	0	59	22
38½	39	33	46	0	0	59	16
39	40	3	25	0	0	59	11
39½	40	33	0	0	0	59	5
40	41	2	33	0	0	59	0
40½	41	32	3	0	0	58	54
41	42	1	30	0	0	58	48
41½	42	30	54	0	0	58	42
42	43	0	15	0	0	58	36
42½	43	29	33	0	0	58	31
43	43	58	49	0	0	58	25
43½	44	28	1	0	0	58	18
44	44	57	10	0	0	58	12
44½	45	26	16	0	0	58	6
45	45	55	19	0	0	58	0
45½	46	24	19	0	0	57	54
46	46	53	16	0	0	57	47
46½	47	22	9	0	0	57	41
47	47	51	0	0	0	57	34
47½	48	19	47	0	0	57	27
48	48	48	30	0	0	57	21
48½	49	17	11	0	0	57	14
49	49	45	48	0	0	57	7
49½	50	14	21	0	0	57	0
50	50	42	51	0	0	56	53
50½	51	11	18	0	0	56	46
51	51	39	41	0	0	56	39
51½	52	8	0	0	0	56	32
52	52	36	16	0	0	56	25
52½	53	4	29	0	0	56	18
53	53	32	38	0	0	56	10
53½	54	0	43	0	0	56	3
54	54	28	44	0	0	55	55
54½	54	56	42	0	0	55	48
55	55	24	36	0	0	55	40
55½	55	52	26	0	0	55	33
56	56	20	12	0	0	55	25
56½	56	47	54	0	0	55	17
57	57	15	33	0	0	55	9
57½	57	43	7	0	0	55	1
58	58	10	38	0	0	54	53
58½	58	38	5	0	0	54	45
59	59	5	27	0	0	54	37
59½	59	32	45	0	0	54	29
60	60	0	0	0	0	54	21
60½	60	27	11	0	0	54	12
61	60	54	17	0	0	54	4
61½	61	21	19	0	0	53	56
62	61	48	17	0	0	53	47
62½	62	15	10	0	0	53	39
63	62	42	0	0	0	53	30
63½	63	8	45	0	0	53	22
64	63	35	25	0	0	53	13
64½	64	2	2	0	0	53	4
65	64	28	34	0	0	52	55
65½	64	55	1	0	0	52	46
66	65	21	24	0	0	52	37
66½	65	47	43	0	0	52	28
67	66	13	57	0	0	52	19
67½	66	40	7	0	0	52	10
68	67	6	12	0	0	52	1
68½	67	32	12	0	0	51	52
69	67	58	8	0	0	51	43
69½	68	23	59	0	0	51	33
70	68	49	45	0	0	51	23
70½	69	15	27	0	0	51	14
71	69	41	4	0	0	51	4
71½	70	6	36	0	0	50	55
72	70	32	4	0	0	50	45
72½	70	57	26	0	0	50	35
73	71	22	44	0	0	50	26
73½	71	47	56	0	0	50	16
74	72	13	4	0	0	50	6
74½	72	38	7	0	0	49	56
75	73	3	5	0	0	49	46
75½	73	27	58	0	0	49	36
76	73	52	46	0	0	49	26
76½	74	17	29	0	0	49	16
77	74	42	7	0	0	49	6
77½	75	6	39	0	0	48	55
78	75	31	7	0	0	48	45
78½	75	55	29	0	0	48	34
79	76	19	46	0	0	48	24
79½	76	43	58	0	0	48	13
80	77	8	5	0	0	48	3
80½	77	32	6	0	0	47	52
81	77	56	2	0	0	47	41
81½	78	19	52	0	0	47	31
82	78	43	38	0	0	47	20
82½	79	7	18	0	0	47	9
83	79	30	52	0	0	46	58
83½	79	54	21	0	0	46	47
84	80	17	45	0	0	46	36

Arcs	*Chords*			*Sixtieths*			
84½	80	41	3	0	0	46	25
85	81	4	15	0	0	46	14
85½	81	27	22	0	0	46	3
86	81	50	24	0	0	45	52
86½	82	13	19	0	0	45	40
87	82	36	9	0	0	45	29
87½	82	58	54	0	0	45	18
88	83	21	33	0	0	45	6
88½	83	44	6	0	0	44	55
89	84	6	33	0	0	44	43
89½	84	28	54	0	0	44	31
90	84	51	10	0	0	44	20
90½	85	13	20	0	0	44	8
91	85	35	24	0	0	43	57
91½	85	57	23	0	0	43	45
92	86	19	15	0	0	43	33
92½	86	41	2	0	0	43	21
93	87	2	42	0	0	43	9
93½	87	24	17	0	0	42	57
94	87	45	45	0	0	42	45
94½	88	7	7	0	0	42	33
95	88	28	24	0	0	42	21
95½	88	49	34	0	0	42	9
96	89	10	39	0	0	41	57
96½	89	31	37	0	0	41	45
97	89	52	29	0	0	41	33
97½	90	13	15	0	0	41	21
98	90	33	55	0	0	41	8
98½	90	54	29	0	0	40	55
99	91	14	56	0	0	40	42
99½	91	35	17	0	0	40	30
100	91	55	32	0	0	40	17
100½	92	15	40	0	0	40	4
101	92	35	42	0	0	39	52
101½	92	55	38	0	0	39	39
102	93	15	27	0	0	39	26
102½	93	35	11	0	0	39	13
103	93	54	47	0	0	39	0
103½	94	14	17	0	0	38	47
104	94	33	41	0	0	38	34
104½	94	52	58	0	0	38	21
105	95	12	9	0	0	38	8
105½	95	31	13	0	0	37	55
106	95	50	11	0	0	37	42
106½	96	9	2	0	0	37	29
107	96	27	47	0	0	37	16
107½	96	46	24	0	0	37	3
108	97	4	56	0	0	36	50
108½	97	23	20	0	0	36	36
109	97	41	38	0	0	36	23
109½	97	59	49	0	0	36	9
110	98	17	54	0	0	35	56
110½	98	35	52	0	0	35	42
111	98	53	43	0	0	35	29
111½	99	11	27	0	0	35	15
112	99	29	5	0	0	35	1
112½	99	46	35	0	0	34	48
113	100	3	59	0	0	34	34
113½	100	21	16	0	0	34	20
114	100	38	26	0	0	34	6
114½	100	55	28	0	0	33	53
115	101	12	25	0	0	33	39
115½	101	29	15	0	0	33	25
116	101	45	57	0	0	33	11
116½	102	2	33	0	0	32	57
117	102	19	1	0	0	32	43
117½	102	35	22	0	0	32	29
118	102	51	37	0	0	32	15
118½	103	7	44	0	0	32	0
119	103	23	44	0	0	31	46
119½	103	39	37	0	0	31	32
120	103	55	23	0	0	31	18
120½	104	11	2	0	0	31	4
121	104	26	34	0	0	30	49
121½	104	41	59	0	0	30	35
122	104	57	16	0	0	30	21
122½	105	12	26	0	0	30	7
123	105	27	30	0	0	29	52
123½	105	42	26	0	0	29	37
124	105	57	14	0	0	29	23
124½	106	11	55	0	0	29	8
125	106	26	29	0	0	28	54
125½	106	40	56	0	0	28	39
126	106	55	15	0	0	28	24
126½	107	9	27	0	0	28	10
127	107	23	32	0	0	27	56
127½	107	37	30	0	0	27	40
128	107	51	20	0	0	27	25
128½	108	5	2	0	0	27	10
129	108	18	37	0	0	26	56
129½	108	32	5	0	0	26	41
130	108	45	25	0	0	26	26
130½	108	58	38	0	0	26	11
131	109	11	44	0	0	25	56
131½	109	24	42	0	0	25	41
132	109	37	32	0	0	25	26
132½	109	50	15	0	0	25	11
133	110	2	50	0	0	24	56
133½	110	15	18	0	0	24	41
134	110	27	39	0	0	24	26
134½	110	39	52	0	0	24	10
135	110	51	57	0	0	23	55
135½	111	3	54	0	0	23	40
136	111	15	44	0	0	23	25

Arcs	*Chords*			*Sixtieths*			
136½	111	27	26	0	0	23	9
137	111	39	1	0	0	22	54
137½	111	50	28	0	0	22	39
138	112	1	47	0	0	22	24
138½	112	12	59	0	0	22	8
139	112	24	3	0	0	21	53
139½	112	35	0	0	0	21	37
140	112	45	48	0	0	21	22
140½	112	56	29	0	0	21	7
141	113	7	2	0	0	20	51
141½	113	17	26	0	0	20	36
142	113	27	44	0	0	20	20
142½	113	37	54	0	0	20	4
143	113	47	56	0	0	19	49
143½	113	57	50	0	0	19	33
144	114	7	37	0	0	19	17
144½	114	17	15	0	0	19	2
145	114	26	46	0	0	18	46
145½	114	36	9	0	0	18	30
146	114	45	24	0	0	18	14
146½	114	54	31	0	0	17	59
147	115	3	30	0	0	17	43
147½	115	12	22	0	0	17	27
148	115	21	6	0	0	17	11
148½	115	29	41	0	0	16	55
149	115	38	9	0	0	16	40
149½	115	46	29	0	0	16	24
150	115	54	40	0	0	16	8
150½	116	2	44	0	0	15	52
151	116	10	40	0	0	15	36
151½	116	18	28	0	0	15	20
152	116	26	8	0	0	15	4
152½	116	33	40	0	0	14	48
153	116	41	4	0	0	14	32
153½	116	48	20	0	0	14	16
154	116	55	28	0	0	14	0
154½	117	2	28	0	0	13	44
155	117	9	20	0	0	13	28
155½	117	16	4	0	0	13	12
156	117	22	40	0	0	12	56
156½	117	29	8	0	0	12	40
157	117	35	28	0	0	12	24
157½	117	41	40	0	0	12	7
158	117	47	43	0	0	11	51
158½	117	53	39	0	0	11	35
159	117	59	27	0	0	11	19
159½	118	5	7	0	0	10	3
160	118	10	37	0	0	10	47
160½	118	16	1	0	0	10	31
161	118	21	16	0	0	10	14
161½	118	26	23	0	0	9	58
162	118	31	22	0	0	9	42
162½	118	36	13	0	0	9	25
163	118	40	55	0	0	9	9
163½	118	45	30	0	0	8	53
164	118	49	56	0	0	8	37
164½	118	54	15	0	0	8	20
165	118	58	25	0	0	8	4
165½	119	2	26	0	0	7	48
166	119	6	20	0	0	7	31
166½	119	10	6	0	0	7	15
167	119	13	44	0	0	6	59
167½	119	17	13	0	0	6	42
168	119	20	34	0	0	6	26
168½	119	23	47	0	0	6	10
169	119	26	52	0	0	5	53
169½	119	29	49	0	0	5	37
170	119	32	37	0	0	5	20
170½	119	35	17	0	0	5	4
171	119	37	49	0	0	4	48
171½	119	40	13	0	0	4	31
172	119	42	28	0	0	4	14
172½	119	44	35	0	0	3	58
173	119	46	35	0	0	3	42
173½	119	48	26	0	0	3	26
174	119	50	8	0	0	3	9
174½	119	51	43	0	0	2	53
175	119	53	10	0	0	2	36
175½	119	54	27	0	0	2	20
176	119	55	38	0	0	2	3
176½	119	56	39	0	0	1	47
177	119	57	32	0	0	1	30
177½	119	58	18	0	0	1	14
178	119	58	55	0	0	0	57
178½	119	59	24	0	0	0	41
179	119	59	44	0	0	0	25
179½	119	59	56	0	0	0	9
180	120	0	0	0	0	0	0

12. On the Arc Between the Tropics

Now that the question of the value of chords in a circle has been treated, our first task will be, as we have said, to show how much the oblique circle through the middle of the zodiac [the ecliptic] is inclined to the equator—that is, what ratio the great circle which passes through both sets of poles has to the arc intercepted on it by these poles. This arc is evidently equal to the arc from either of

the tropic points to the equator, along this same circle. And we can immediately get the measure of this arc instrumentally, with some such simple construction as the following.

We shall make a brass circle, suitable in size, accurately turned, with its surfaces standing square. And we shall use it as a meridian, dividing it into the 360 parts supposed for the great circle, and each of these into as many parts as there is room for. Then, fitting into this circle another little circle, thinner than it, in such a way that their sides remain in the same plane and the smaller circle can turn without hindrance within the greater circle to the north and south, always in the same plane, we shall place, on one of the smaller circle's two sides at two diametrically opposite points, small prisms of the same size pointing in exactly the same straight line with each other and with the centre of the circles. And in the middle of the prisms' width we shall place fine pointers touching the side of the greater circle and its divisions. And then we shall fit that circle securely on a small and convenient column for its several uses, and stand the base of the column on a pavement in the open air exactly parallel to the plane of the horizon. And we shall take care that the plane of the circles is perpendicular to that of the horizon and parallel to that of the meridian. The first condition is satisfied by means of a plumb line which is suspended from the highest point of the circles and which is watched until, by correction of the underpinning, it is directed to the diametrically opposite point below. And the second condition is satisfied by very visibly drawing a meridian line in the plane below the column and turning the circles from side to side until their plane is sighted parallel to the line. And after a placement of this kind had been made, we would observe the sun's advance to the north and south, moving the inside circle at middays until the lower prism was completely shadowed by the whole of the upper prism. And when this was done, the needle points would show us each time how many degrees from the zenith the sun's center stood on the meridian line.

And we used to make this observation still more readily by constructing, instead of circles, a steady square block of rock or wood having one of its faces regular and accurately shaped. And using as a centre the vertex of one of its angles, we drew a quadrant, drawing out from the centre point to the circumference those straight lines which contain the right angle of the quadrant, and likewise dividing the circumference into 90 parts, and dividing these again. And after this, on one of the straight lines, the one which was to be perpendicular to the plane of the horizon and to face south, we laid two small right cylinders in every way equal and similarly fashioned, one on the centre exactly at the midpoint of the indicated circle and the other at the lower extremity of the straight line. Then, standing the marked side of the block along the meridian line drawn in the supporting plane of the horizon so that it would also be parallel to the plane of the meridian, again making adjustments with fine splints in accord with a plumb line right across the cylinders perpendicular to the plane of the horizon, we would observe as before the midday shadow from the centre cylinder, putting something at the graduated circumference to show more clearly its position. And marking the middle of the shadow we would note down the division of the quadrant, and this would clearly indicate the latitude of the sun's path on the meridian.

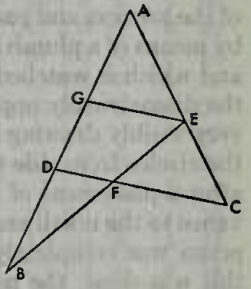
Now from such observations and especially from those made by us over several periods while the sun was near the tropics, seeing the marker count off from

the zenith always an equal number of divisions on the meridian and the same ones both for the winter tropics and for the summer tropics, we found the arc from the northernmost to the southernmost limit, which is the arc between the tropic points, to be always more than 47°40′ but less than 47°45′. And with this there results nearly the same ratio as that of Eratosthenes and as that which Hipparchus used. For the arc between the tropics turns out to be very nearly 11 out of the meridian's 83 parts.

And immediately from this observation it is easy to deduce the latitudes of the places where we make the observations, once we have taken the midpoint of the two limits, which is on the equator, and the arc between this point and the zenith. For the arc between the poles and the plane of the horizon is clearly equal to this arc.

13. Preliminaries to the Spherical Proofs

And since it is next in order to demonstrate the respective values of the arcs on great circles drawn through the poles of the equator and cut off between the equator and the ecliptic, we shall explain very short and easy lemmas by means of which we shall effect nearly all the proofs of theorems dealing with spheres.



Now let the two straight lines *BE* and *CD*, drawn to two straight lines *AB* and *AC*, cut each other at *F*.

I say that *AC* : *AE* comp. *CD* : *DF*, *BF* : *BE*.

For let *EG* be drawn through *E* parallel to *CD*. Since the straight lines *CD* and *EG* are parallel, *AC* : *AE* : : *CD* : *EG* [Eucl. VI, 4].

Then if we supply the straight line *DF*,

CD : *EG* comp. *CD* : *DF*, *DF* : *EG*.

And

DF : *EG* : : *BF* : *BE*, [Eucl. VI, 4].

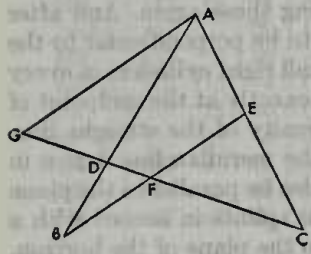
again because *EG* and *DF* are parallel. Therefore

AC : *AE* comp. *CD* : *DF*, *BF* : *BE*.

Which it was proposed to prove.

And in the same way it can be shown, by drawing a straight line through *A* parallel to *EF* and producing *CDG* to meet it, that

CE : *AE* comp. *CF* : *DF*, *BD* : *AB*.



For again since *AG* is parallel to *EF*

CF : *FG* : : *CE* : *AE* [Eucl. VI, 2].

But if we supply *DF*,

CF : *FG* comp. *CF* : *DF*, *DF* : *FG*.

But

DF : *FG* : : *BD* : *AB*

because *AB* and *FG* are drawn through to the parallels *AG* and *BF*. Therefore

CF : *FG* comp. *CF* : *DF*, *BD* : *AB*.

But

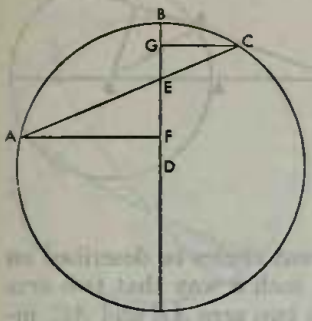
CE : *AE* : : *CF* : *FG*.

And therefore

CE : *AE* comp. *CF* : *DF*, *BD* : *AB*.

Which was to be proved.

Again, let there be the circle ABC with centre D; and let there be taken on its circumference three points A, B, and C in such a way that each of the arcs AB and BC is less than a semicircle (and let the same thing be understood of the arcs taken hereafter). And let the straight lines AC and DEB be joined.



I say that chord 2 arc AB : chord 2 arc BC : : $AE : CE$.

For let the straight lines AF and CG be drawn from the points A and C perpendicular to BD. Since AF is parallel to CG and AEC crosses them, therefore

$$AF : CG : : AE : CE \quad \text{[Eucl. VI, 4]}.$$

But

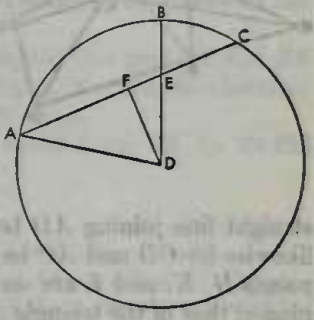
$AF : CG$: : chord 2 arc AB : chord 2 arc BC.

For the ones are the halves of the others respectively. And therefore

$AE : CE$: : chord 2 arc AB : chord 2 arc BC.

Which was to be proved.

And it follows from this that, if the whole arc AC is given and also the ratio of the chord of twice arc AB to the chord of twice arc BC, then each of the arcs AB and BC is given.

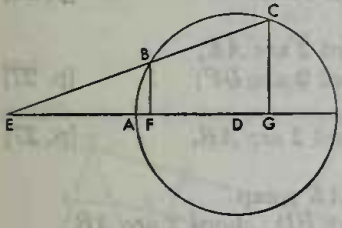


For with the same figure laid out, let AD be joined, and from D let DF be drawn perpendicular to AEC. It is clear then that, given arc AC, angle ADF which subtends the half of it is given, and also the whole triangle ADF. Now with the whole line AC given, since

$AE : CE$: : chord 2 arc AB : chord 2 arc BC,

therefore AE is given[1], and the remainder EF is given. And for this reason and since DF is given, both angle EDF of the right triangle EDF and the whole angle ADB are given. And so both arcs AB and BC are given. Which was to be proved.

Again, let there be the circle ABC with centre D, and on its circumference let there be taken three points A, B, and C in such a way that each of the arcs AB and AC is less than a semicircle (and let this same condition be understood for the arcs to be taken hereafter). And let AD and BC be joined and the resulting straight lines be produced to meet at E.



I say that

$CE : BE$: : chord 2 arc AC : chord 2 arc AB.

For, as in the previous lemma, if from the points B and C we drop BF and CG perpendicular to AD, then, since they are parallel,

$$CG : BF : : CE : BE.$$

And so also

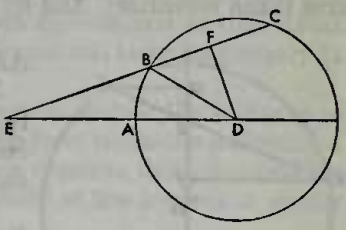
$$CE : BE : : \text{chord 2 arc } AC : \text{chord 2 arc } AB.$$

[1]For *componendo*

$AC : CE$: : chord 2 arc AB + chord 2 arc BC : chord 2 arc BC.

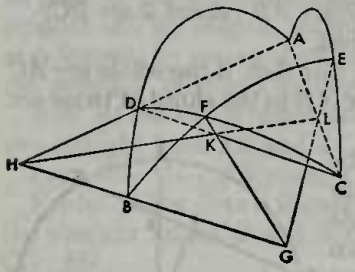
Which was to be proved.

And then from this it follows that, even if the arc BC alone be given along with the ratio of the chord of twice arc AC to the chord of twice arc AB, then arc AB is given.



For again in a similar figure, with BD joined and DF dropped perpendicular to BC, angle BDF subtending half arc BC is given, and also the whole right triangle BDF. And since also the ratio of CE to BE is given and chord BC, therefore BE is given and the whole line EBF. And so, since DF is given, angle EDF of the same right triangle is also given, and by subtraction angle EDB. Hence arc AB is also given.

And with these conclusions in mind, let arcs of great circles be described on a spherical surface in such a way that two arcs BE and CD, drawn to two arcs AB and AC, intersect each other at the point F. And let each of these arcs be less than a semicircle (and let the same condition be understood in all the figures).



Then I say that

chord 2 arc CE : chord 2 arc AE comp.
chord 2 arc CF : chord 2 arc DF,
chord 2 arc BD : chord 2 arc AB.

For let the centre of the sphere be taken, and let it be G; and from the point G let the straight lines BG, FG, and EG be drawn to the intersections of the circles at B, F, and E. And let the straight line joining AD be produced to meet BG produced at point H. And likewise let CD and AC be joined and cut FG and EG at K and L. Then the points H, K, and L are on one straight line, because they are at once in two planes: that of the triangle ACD, and that of the circle BFE. When these points are joined, we have two straight lines HL and CD intersecting at point K and cutting two other straight lines AH and AC. Therefore

$CL : AL$ comp. $CK : DK$, $DH : AH$. [p. 26]

But

$CL : AL$: : chord 2 arc CE : chord 2 arc AE,
$CK : KD$: : chord 2 arc CF : chord 2 arc DF; [p. 27]

and

$DH : AH$: : chord 2 arc BD : chord 2 arc AB. [p. 27]

And therefore

chord 2 arc CE : chord 2 arc AE comp.
chord 2 arc CF : chord 2 arc DF, chord 2 arc BD : chord 2 arc AB.

In the same way, by straight lines constructed in a plane, it is also proved that

chord 2 arc AC : chord 2 arc AE comp.
chord 2 arc CD : chord 2 arc DF, chord 2 arc BF : chord 2 arc BE.[1]

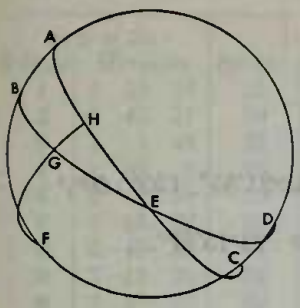
Which things it was required to prove.

[1]This analogous theorem is proved thus: With the same arcs as in the preceding figure, produce the straight lines CE, CF, and EF of the four intersecting great circles of a sphere whose

14. On the Arcs Between the Equator and the Ecliptic

Now that this theorem has been expounded, we shall make the first application of the arcs as worked out above, in the following way:

For let the circle *ABCD* be the circle through the poles of the equator and the poles of the ecliptic; and let *AEC* be the semicircle of the equator, and *BED* the semicircle of the ecliptic, and the point *E* their intersection at the spring equinox so that point *B* is the winter tropic and *D* the summer tropic. And let the pole of the equator *AEC* be taken on arc *ABC*, and let it be the point *F*. And let the arc *EG* be taken on the ecliptic, and suppose it to be 30°, and through the points *F* and *G* let the arc *FGH* of a great circle be drawn.



And let it be proposed to find the arc *GH*.

Let it be understood now and generally in all such proofs, so we shall not have to repeat the same thing for each one, that, whenever we say of how many parts or divisions the magnitudes of arcs or straight lines consist, we mean, in the case of arcs, of such divisions as the circumference of the great circle has 360; and, in the case of straight lines, of such as the same circle's diameter has 120.

Since, then, in the construction of the great circles, the two arcs *FH* and *BE* have been drawn to the arcs *AF* and *AE*, intersecting at the point *G*, therefore

chord 2 arc *AF* : chord 2 arc *AB* comp.
chord 2 arc *FH* : chord 2 arc *GH*, chord 2 arc *EG* : chord 2 arc *BE*. [p. 28 Fn]

But

2 arc *AF* = 180°,
chord 2 arc *AF* = 120^{p}.

---

centre is *G*. Let *CE* and *AG* meet at *H*, *CF* and *DG* at *L*, and *EF* and *BG* at *K*. Then the points *H*, *L*, and *K* lie in the same straight line since they all lie on the intersection of the same two planes: that of circle *BAG*, and that of triangle *FEC*.

Then

CH : *EH* comp. *CL* : *FL*, *FK* : *EK* [p. 26]

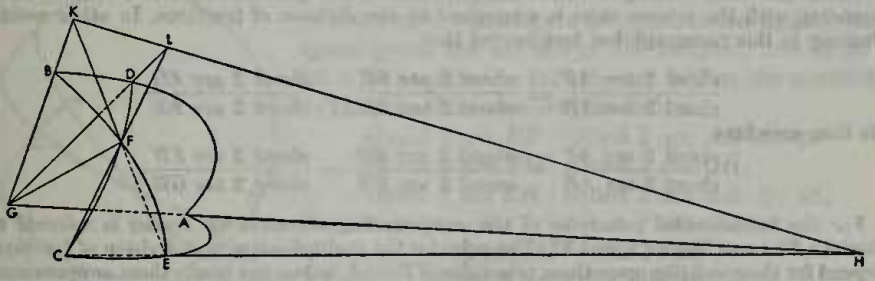
But

CH : *EH* : : chord 2 arc *AC* : chord 2 arc *AE*,
CL : *FL* : : chord 2 arc *CD* : chord 2 arc *DF*,

and

FK : *EK* : : chord 2 arc *BF* : chord 2 arc *BE*. [p. 27]

Substituting in the first expression, we have shown what was required.



And according to the ratio of 83 parts to 11, agreed to by us [p. 26],

$$2 \text{ arc } AB = 47°42'40'',$$

and therefore

$$\text{chord } 2 \text{ arc } AB = 48^{p}31'55''.$$

And again

$$2 \text{ arc } EG = 60°,$$
$$\text{chord } 2 \text{ arc } EG = 60^{p};$$

and

$$2 \text{ arc } BE = 180°,$$
$$\text{chord } 2 \text{ arc } BE = 120^{p}.$$

Therefore

chord 2 arc FH : chord 2 arc GH comp. $120^{p} : 48^{p}31'55''$, $120^{p} : 60^{p}$,[1]

or

$$\text{chord } 2 \text{ arc } FH : \text{chord } 2 \text{ arc } GH :: 120^{p} : 24^{p}15'57''.$$

And

$$2 \text{ arc } FH = 180°,$$
$$\text{chord } 2 \text{ arc } FH = 120^{p};$$

and therefore

$$\text{chord } 2 \text{ arc } GH = 24^{p}15'57''.$$

And so

$$2 \text{ arc } GH = 23°19'59'',$$
$$\text{arc } GH = 11°40'.$$

And again let

$$\text{arc } EG = 60°,$$

so that, the others remaining the same,

$$2 \text{ arc } EG = 120°$$
$$\text{chord } 2 \text{ arc } EG = 103^{p}55'23''.$$

Then again

$$\text{chord } 2 \text{ arc } FH : \text{chord } 2 \text{ arc } GH :: 120^{p} : 42^{p}1'48''.$$

And

$$\text{chord } 2 \text{ arc } FH = 120^{p}$$

And so also

$$\text{chord } 2 \text{ arc } GH = 42^{p}1'48''.$$

And therefore

$$2 \text{ arc } GH = 41°0'18''$$
$$\text{arc } GH = 20°30'9''.$$

Which things were to be proved.

[1]Since the compounding of ratios is equivalent to our multiplication of fractions, compounding with the inverse ratio is equivalent to our division of fractions. In other words, Ptolemy in this paragraph has first proved that

$$\frac{\text{chord } 2 \text{ arc } AF}{\text{chord } 2 \text{ arc } AB} = \frac{\text{chord } 2 \text{ arc } FH}{\text{chord } 2 \text{ arc } GH} \cdot \frac{\text{chord } 2 \text{ arc } EG}{\text{chord } 2 \text{ arc } BE}.$$

He then says here

$$\frac{\text{chord } 2 \text{ arc } AF}{\text{chord } 2 \text{ arc } AB} \div \frac{\text{chord } 2 \text{ arc } EG}{\text{chord } 2 \text{ arc } BE} = \frac{\text{chord } 2 \text{ arc } FH}{\text{chord } 2 \text{ arc } GH}.$$

For the fundamental principles of the compounding of ratios the reader is referred to Euclid's *Elements*, Books V and VI. The rules for the multiplication and division of fractions depend for their validity upon these principles of Euclid, unless one treats them as operations in a symbolic system, as in modern mathematical theory.

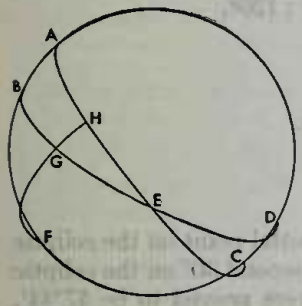
And now in the same way, by calculating the values for successive arcs, we shall lay out a table for the 90° of a quadrant, containing the values of arcs like those already proved. And here is the table:

15. Table of Obliquity

Arcs of the				*Arcs of the*				*Arcs of the*				*Arcs of the*			
Ecliptic	*Meridian*			*Ecliptic*	*Meridian*			*Ecliptic*	*Meridian*			*Ecliptic*	*Meridian*		
1	0	24	16	25	9	50	29	46	16	54	47	70	22	20	11
2	0	48	31	26	10	12	46	47	17	12	16	71	22	28	57
3	1	12	46	27	10	34	57	48	17	29	27	72	22	37	17
4	1	37	0	28	10	56	44	49	17	46	20	73	22	45	11
5	2	1	12	29	11	18	25	50	18	2	53	74	22	52	39
6	2	25	22	30	11	39	59	51	18	19	15	75	22	59	41
7	2	49	30	31	12	1	20	52	18	35	5	76	23	6	17
8	3	13	35	32	12	22	30	53	18	50	41	77	23	12	27
9	3	37	37	33	12	43	28	54	19	5	57	78	23	18	11
10	4	1	38	34	13	4	14	55	19	20	56	79	23	23	28
11	4	25	32	35	13	24	47	56	19	35	28	80	23	28	16
12	4	49	24	36	13	45	6	57	19	49	42	81	23	32	30
13	5	13	11	37	14	5	11	58	20	3	31	82	23	36	35
14	5	36	53	38	14	25	2	59	20	17	4	83	23	40	2
15	6	0	31	39	14	44	39	60	20	30	9	84	23	43	2
16	6	24	1	40	15	4	4	61	20	42	58	85	23	45	34
17	6	47	26	41	15	23	10	62	20	55	24	86	23	47	39
18	7	10	45	42	15	42	2	63	21	7	21	87	23	49	16
19	7	33	57	43	16	0	38	64	21	18	58	88	23	50	25
20	7	57	3	44	16	18	58	65	21	30	11	89	23	51	6
21	8	20	0	45	16	37	20	66	21	41	0	90	23	51	20
22	8	42	50					67	21	51	25				
23	9	5	32					68	22	1	25				
24	9	28	5					69	22	11	11				

16. On Ascensions in the Right Sphere

And next it would be proper to demonstrate all together the values of the arcs on the equator determined by circles drawn through the poles of the equator and given divisions of the ecliptic. For in this way we shall know corresponding to how many equatorial time-degrees the divisions of the ecliptic cross the meridian for any place and the horizon in the right sphere, because it is only in the right sphere that the horizon passes through the poles of the equator.



Then let the last figure be constructed again. And again given the arc EG of the ecliptic, first as 30°, let it be required to find the arc EH on the equator.

Then in the same way as before

chord 2 arc BF : chord 2 arc AB comp.
chord 2 arc FG : chord 2 arc GH,
chord 2 arc EH : chord 2 arc AE. [p. 28]

But

2 arc $BF = 132°17'20''$,
chord 2 arc $BF = 109^{p}44'53''$.

And

2 arc $AB = 47°42'40''$,
chord 2 arc $AB = 48^{p}31'55''$.

And again

2 arc $FG = 156°40'1''$,
chord 2 arc $FG = 117^{p}31'15''$;

and

2 arc $GH = 23°19'59''$ [table, §14]
chord 2 arc $GH = 24^{p}15'57''$.

Therefore

chord 2 arc EH : chord 2 arc AE comp.
$109^{p}44'53''$: $48^{p}31'55''$, $24^{p}15'57''$: $117^{p}31'15''$;

or

chord 2 arc EH : chord 2 arc AE : : $54^{p}52'26''$: $117^{p}31'15''$

And

$54^{p}52'26''$: $117^{p}31'15''$: : $56^{p}1'25''$: 120^{p}.

And

2 arc $AE = 180°$,
chord 2 arc $AE = 120^{p}$.

Therefore

chord 2 arc $EH = 56^{p}1'25''$;

and so

2 arc $EH \fallingdotseq 55°40'$,
arc $EH \fallingdotseq 27°50'$.

Again let

arc $EG = 60°$.

And so, the others remaining the same,

2 arc $FG = 138°59'42''$
chord 2 arc $FG = 112^{p}23'56''$;

and

2 arc $GH = 41°0'16''$,
chord 2 arc $GH = 42^{p}1'48''$.

Therefore

chord 2 arc EH : chord 2 arc AE comp.
$109^{p}44'53''$: $48^{p}31'55''$, $42^{p}1'48''$: $112^{p}23'56''$;

or

chord 2 arc EH : chord 2 arc AE : : $95^{p}2'40''$: $112^{p}23'56''$.

And

$95^{p}2'40''$: $112^{p}23'56''$: : $101^{p}28'20''$: 120^{p}.

But

chord 2 arc $AE = 120^{p}$;

and so

chord 2 arc $EH = 101^{p}28'20''$.

And therefore

2 arc $EH \fallingdotseq 115°28'$,
arc $EH \fallingdotseq 57°44'$.

It has been shown that the first 30°from the equinoctial point on the ecliptic corresponds to 27°50′ in time on the equator; and the second 30° on the ecliptic to 29°54′ on the equator, since they both together were proved to be 57°44′. And it is evident that the third 30° on the ecliptic will correspond in time to the

remainder of the equator's quadrant, that is to 32°16′, since the ecliptic's quadrant corresponds in time to the equator's quadrant, both being cut off by the same circles through the poles of the equator.

In the same way, following this same method, we calculated the arcs of the equator corresponding in time to each 10° of the ecliptic because arcs smaller than these have practically equal differences as they increase. We shall set them out also, in order to have at hand, as we said, the amount of time it takes each arc to cross the meridian for any place and the horizon in the right sphere. And we take as our starting place the point 10° from the equinox.

The first arc of 10° on the ecliptic takes 9°10′ equatorial time; the second, 9°15′; the third, 9°25′; so that the first 30° on the ecliptic totals 27°50′ equatorial time. The fourth arc takes 9°40′; the fifth, 9°58′; the sixth, 10°16′; so that the second 30° on the ecliptic totals 29°54′ equatorial time. The seventh arc takes 10°34′; the eighth, 10°47′; the ninth, 10°55′; so that the third 30° on the ecliptic totals 32°16′ equatorial time, and the whole quadrant 90°.

And from this it is evident that the order is the same for the other quadrants, everything being the same in each of them, because the sphere is given as right; that is, the equator is not oblique to the horizon.

BOOK TWO

1. On the General Position of the Inhabited World

Now that, in the first book of the Composition, we have gone through those things concerning the system of the universe which had first to be understood in summary form, and as many other things concerning the right sphere as would be considered useful to an understanding of questions treated here, we shall try in the following book to present again, in the handiest possible way, the more important of those things having to do with the oblique sphere.

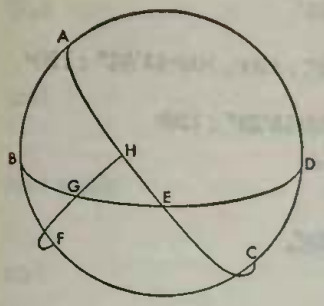
And now this we must first understand in a general way that, if the earth is supposed cut into four equal parts by the equator and one circle through its poles, the extent of the part inhabited by us is very nearly enclosed in one or the other of the northern quadrants. And this would certainly be evident in the case of latitude—that is, of the passage from south to north—because everywhere the noon shadows of gnomons at the equinox fall to the north and never to the south. And also in the case of longitude—that is, of the passage from east to west—because the same eclipses, and especially the lunar ones, observed at the same time by those inhabiting the extreme eastern sections of that part of the earth inhabited by us and by those inhabiting the extreme western sections, neither precede nor lag behind by more than twelve equatorial hours. But the quarter part of the sphere in longitude embraces a twelve-hour interval since it is bounded by one of the semicircular arcs of the equator.

Of the particular things which must be understood for the business at hand, one should especially consider the arrangement, one by one, of the circles north of the equator and parallel to it, according to the particularities of the places situated under them. That is, how far the poles of the equator are from the horizon; or how far the zenith is from the equator along the meridian circle; and in what places the sun reaches the zenith, and when and how many times that happens; what are the ratios of the equinoctial and tropical shadows cast at noon to their gnomons; and what are the differences of the longest and shortest days with respect to the equinoxes; and as many other things as are observed relative to the increase and decrease of solar days, with the correspondances in the risings and settings of the equator and ecliptic, and with the properties and magnitudes of the angles formed by the principal great circles.

2. Given the Magnitude of the Longest Day, How the Arcs on the Horizon Intercepted by the Equator and the Ecliptic are Given

As a general example, let the circle drawn through Rhodes parallel to the equator be considered, where the height of the pole is 36° and the longest day is 14½ equatorial hours.[1] And let the circle *ABCD* be the meridian; and *BED* the

[1]An equatorial hour is $\frac{1}{24}$ of the 360° of the equator or 15°. In other words an equatorial hour is $\frac{1}{24}$ of a stellar day, a stellar day being the time it takes a fixed star to pass from a meridian back again to that same meridan, that is one complete revolution of the equator. For the purposes of Book II, Ptolemy considers the sun such a fixed star, although it will be



eastern semicircle of the horizon; and likewise *AEC* the semicircle of the equator, and *F* its southern pole. And let the ecliptic's winter-tropic point rise through *G*; and let *FGH* be drawn as the quadrant of the great circle through *F* and *G*.

And first let the magnitude of the longest day be given; and let it be required to find the arc *EG* of the horizon.

Now, since the turning of the sphere takes place around the poles of the equator, it is evident that the points *G* and *H* will be at the meridian *ABCD* at the same time. And the time from the rising of *G* to its culmination at the upper meridian is contained by arc *HA* of the equator, and from its crossing the lower meridian to its rising by arc *CH*. It follows that the length of time of the day is double that contained by arc *HA*, and the length of time of the night is double that contained by arc *CH*. For the sections above and below the earth of the circles parallel to the equator are exactly cut in half by the meridian.

Therefore, since the arc *EH* is half the difference between the shortest or longest day and the equinoctial day, it is 1¼ hours on the given parallel or 18°45′ in time, and the remainder of the quadrant, arc *HA*, is 71°15′ in time. Since then, in the same way as in the previous proofs, the arcs *EB* and *FH* of great circles cutting each other at *G* have been drawn to the two arcs *AE* and *AF* of great circles,

chord 2 arc *HA* : chord 2 arc *AE* comp. chord 2 arc *FH* : chord 2 arc *FG*,
chord 2 arc *BG* : chord 2 arc *EB* [p. 28]

But

$$2 \text{ arc } HA = 142°30',$$
$$\text{chord } 2 \text{ arc } HA = 113^{p}37'54'';$$

and

$$2 \text{ arc } AE = 180°,$$
$$\text{chord } 2 \text{ arc } AE = 120^{p};$$

and

$$2 \text{ arc } FH = 180°,$$
$$\text{chord } 2 \text{ arc } FH = 120^{p};$$

and

$$2 \text{ arc } FG^{1} = 132°17'20''$$

[1]The arc *FG* is 66°8′40″, since the arc *GH* which marks the obliquity of the ecliptic is 23°51′20″.

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pointed out in detail in Book III that this is not the case, just as it has already been stated in general in Book I. Therefore all the calculations relative to the sun in Book II are in error by the small amount such neglect entails.

In Book III, there is introduced the notion of a solar day, a solar day being the time it takes the sun to pass from a meridian back to that same meridian. The solar day, it will be seen, is very nearly 59′ [of arc] longer than the stellar day. Now in Book III an hour is defined as $\frac{1}{24}$ of the solar day; and this hour is used interchangeably with the equatorial hour, although it is very nearly 2½′ longer than it. But the total error could never come to as much as 59′, which is less than 4 minutes in time; and Ptolemy declares that his instruments cannot be accurate within less than 15 minutes in time. Therefore this ambiguity will have no effect on the accuracy of the calculations.

The degrees on the equator are often referred to as "degrees in time" or "time-degrees" in referring to their time-measuring aspect.

chord 2 arc $FG = 109^{p}44'53''$

Then

chord 2 arc BG : chord 2 arc EB comp. $113^{p}37'54''$: 120^{p}, $109^{p}44'53''$: 120^{p}

or

chord 2 arc BG : chord 2 arc EB : : $103^{p}55'23''$: 120^{p}

And since it is a quadrant,

chord 2 arc $EB = 120^{p}$,

and therefore

chord 2 arc $BG = 103^{p}55'23''$.

And so

2 arc $BG = 120°$,

arc $BG = 60°$

And therefore as remainder

arc $EG = 30°$.

Which was to be shown.

3. How, with the Same Things Given, the Height of the Pole is Given, and Conversely

Now again, with this given, let it be required to find the height of the pole; that is, arc BF of the meridian. Then, in the same figure

chord 2 arc EH : chord 2 arc HA comp. chord 2 arc EG : chord 2 arc BG,

chord 2 arc BF : chord 2 arc AF. [p. 28]

But

2 arc $EH = 37°30'$,

chord 2 arc $EH = 38^{p}34'22''$;

and

2 arc $HA = 142°30'$,

chord 2 arc $HA = 113^{p}37'54''$;

and again

2 arc $EG = 60°$,

chord 2 arc $EG = 60^{p}$.

Then

chord 2 arc BF : chord 2 arc AF comp. $38^{p}34'22''$: $113^{p}37'54''$, $103^{p}55'23''$: 60^{p}

or very nearly

chord 2 arc BF : chord 2 arc AF : : $70^{p}33'$: 120^{p}.

And again

chord 2 arc $AF = 120^{p}$,

and therefore

chord 2 arc $BF = 70^{p}33'$.

And so

2 arc $BF = 72°1'$,

arc $BF = 36°$.

Again, conversely, let arc BF, the height of the pole, be given by observation as 36°; and let it be required to find the difference between the longest or shortest day and the equinoctial day, that is twice arc EH. Then

chord 2 arc BF : chord 2 arc AB comp. chord 2 arc FG : chord 2 arc GH,

chord 2 arc EH : chord 2 arc AE.

But

$$2 \text{ arc } BF = 72°,$$
$$\text{chord } 2 \text{ arc } BF = 70^{p}32'3'';$$

and

$$2 \text{ arc } AB = 108°,$$
$$\text{chord } 2 \text{ arc } AB = 97^{p}4'56'';$$

and again

$$2 \text{ arc } FG = 132°17'20''$$
$$\text{chord } 2 \text{ arc } FG = 109^{p}44'53'';$$

and

$$2 \text{ arc } GH = 47°42'40'',$$
$$\text{chord } 2 \text{ arc } GH = 48^{p}31'55''.$$

Then

$$\text{chord } 2 \text{ arc } EH : \text{chord } 2 \text{ arc } AE \text{ comp. } 70^{p}32'3'' : 97^{p}4'56'', \quad 48^{p}31'55'' : 109^{p}44'53''$$

or

$$\text{chord } 2 \text{ arc } EH : \text{chord } 2 \text{ arc } AE :: 31^{p}11'23'' : 97^{p}4'56''.$$

And since, very nearly,

$$31^{p}11'23'' : 97^{p}4'56'' :: 38^{p}34' : 120^{p},$$

and

$$\text{chord } 2 \text{ arc } AE = 120^{p},$$

therefore it is inferred

$$\text{chord } 2 \text{ arc } EH = 38^{p}34'.$$

And so

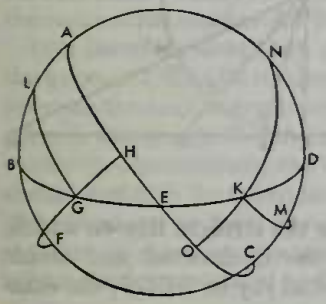
$$2 \text{ arc } EH = 37°30'$$

or $2\frac{1}{2}$ equatorial hours. Which was to be shown.

Likewise the arc EG of the horizon will be given because the given ratio of the chord of 2 arc AF to the chord of 2 arc AB is compounded of the given ratio of the chord of 2 arc FH to the chord of 2 arc GH and the ratio of the chord of 2 arc EG to the chord of 2 arc BE. And so, since BE is given, the magnitude of arc EG is left.

It is evident that—even if we did not suppose the point G to be the winter tropic, but some point at one of the other divisions of the ecliptic—each of the arcs EH and EG would still be given in the same way. For we have set up, by means of the Table of Obliquity, the arcs on the meridian cut off by each division of the ecliptic and by the equator—that is, the arcs like GH.

From this it follows that the divisions of the ecliptic made by the same parallels—that is, those equidistant from the same tropic point—make sections on the horizon the same and on the same side of the equator, and the magnitudes of the days and nights equal to each other, like to like.



At the same time it is proved that the points produced by equal parallels—that is, those equidistant from the same equinoctial point—make equal arcs on the horizon from either side of the equator, and make the magnitudes of the days and nights equal contrariwise, unlike to unlike. For if, in the figure already set out, we take also

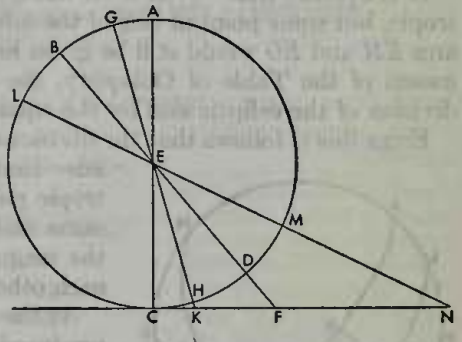
the point *K* as the point where the circle equal and parallel to that through *G* cuts the semicircle of the horizon *BED*, and if we fill in the sections *GL* and *KM* of the parallels, which are clearly contrariwise and equal, and if we draw the quadrant *NKO* through *K* and the north pole, then the arcs *AH* and *OC* will be equal. For *GL* and *KM* are equal as like to like; and the remainder *EH* will be equal to the remainder *EO*. And the two sides of the similar three-sided figures *EGH* and *EKO* will be equal to each other, *EH* to *EO* and *GH* to *KO*, and each of the angles at *H* and *O* are right, so that base *EG* is equal to base *EK*.

4. How One is to Calculate at What Places, When, and How Many Times, the Sun Comes to the Zenith

Now it is easy, once given these things, to calculate when and how many times the sun comes to the zenith. For since it is at once evident that the sun does not come to the zenith at all in those places under parallels farther away from the equator than the approximate 23°51′20″ of the distance of the summer tropic point, and that it does so once, at the summer tropic itself, for those places under parallels at an equal distance from the equator, therefore it is also clear it comes to the zenith twice in those places at a distance less than that just given. And the setup of the Table of Obliquity makes the answer to the question, "When?" an easy one. For taking to the second column the number of degrees by which the parallel in question is distant from the equator—that is, in the case of those parallels within the summer tropic—we shall have, in the first column, the corresponding number of degrees of the quarter of the ecliptic. And at the distance from either equinoctial point given by this number of degrees, the sun comes to the zenith for the places under this parallel on the side of the summer tropic.

5. How, from Things Explained, the Ratios of the Gnomons to their Equinoctial and Tropic Shadows at Noon are Found

Now it will be made clear in the following manner that also the proposed ratios of the shadows to the gnomons are obtained more simply, if the arc between the tropics and the arc between the horizon and the poles are given.



For let the circle *ABCD* be the meridian around center *E;* and with *A* as zenith let the diameter *AEC* be drawn; and let *CKFN*, in the plane of the meridian, be perpendicular to it. *CKFN* is clearly parallel to the intersection of the meridian-plane and the horizon-plane. And since the earth as a whole is sensibly in the ratio of a point and centre to the solar sphere so that the centre *E* does not differ from the top of the gnomon, therefore let *CE* be thought of as the gnomon, and *CKFN* as the straight line on which, at noon, the ends of the shadows fall. And let the equinoctial and tropic noontide rays be drawn through *E*. Let the equinoctial ray be *BEDF*, the sum-

mer tropic ray be *GEHK*, and the winter one *LEMN;* so that *CK* is the summer shadow, *CF* is the equinoctial shadow, and *CN* the winter shadow.

Therefore, since the arc *CD*, which is equal to the height of the pole above the horizon, is, in the given latitude, 36°, and the meridian *ABC* 360°, and each of the arcs *HD* and *DM* approximately 23°51′20″, it is evident that, as remainder,

arc $CH = 12°8′40″$,

and, by addition,

arc $CM = 59°51′20″$.

And so, of the angles subtending these arcs,

angle $KEC = 12°8′40″$,
angle $FEC = 36°$,
angle $NEC = 59°51′20″$.

And

angle $KEC = 24°17′20″$ to 2 rt.,
angle $FEC = 72°$ to 2 rt.,
angle $NEC = 119°42′40″$ to 2 rt.

And then, on the circles about the right triangles *KEC*, *FEC*, and *NEC*,

arc $CK = 24°17′20″$,
arc $CE = 155°42′40″$

as remainder of the semicircle; and

arc $CF = 72°$,
arc $CE = 108°$

likewise; and

arc $CN = 119°42′40″$,
arc $CE = 60°17′20″$

again as remainder of the semicircle. And so, of these chords,

chord $CE = 117^{p}18′51″$

where

chord $CK = 25^{p}14′43″$;

and

chord $CE = 97^{p}4′56″$

where

chord $CF = 70^{p}32′4″$;

and

chord $CE = 60^{p}15′42″$

where

chord $CN = 103^{p}46′16″$.

And therefore, where *CE* equals 60^{p}, *CK* the summer shadow equals $12^{p}55′$, *CF* the equinoctial shadow $43^{p}36′$, and *CN* the winter shadow very nearly $103^{p}20′$.

It is therefore clear that, conversely, if any two ratios alone of these three of the gnomon *CE* to its shadows are given, then the height of the pole and the arc between the tropics are given, since, if any two of the angles at *E* are given, the other is also given. For the arcs *HD* and *DM* are equal. But for the sake of accuracy in observations, the height of the pole and the arc between the tropics should certainly be taken in the way we have already shown. For the ratios of these shadows to their gnomons do not always agree, because the time of the equinoctial shadows is somehow in itself undetermined, and the ends of the winter shadows are hard to distinguish.

6. Exposition of the Properties of Each Pa

In this same way we shall consider the most important of
have expounded for the other parallels also. And taking para
inclination of ¼ equatorial hour, which is sufficient, we shall
position of their incidents before making a detailed one.

i. We shall begin with the parallel under the equator
bounds the southern part of the whole quarter of the earth in
which alone has all days and nights equal to each other. For
the sphere's parallels to the equator cut in half by the horizo
sections above the earth are similar to each other and equal
ing sections under the earth. And this is not the case with any
of the sphere. Again, only the equator is everywhere cut in
and makes the days along it sensibly equal to the nights, sin
circle. But the others are divided unequally; and, in the lati
us, the parallels south of the equator have their sections ab
their sections below earth and their days shorter than the
versely the parallels north of the equator have their sections a
and their days longer.

And this parallel is also amphiscian, since for those living
twice at the zenith, at the intersections of the equator and the
those times only are the gnomons shadowless at noon. A
passes through the northern hemisphere the gnomons' shadow
and when it passes through the southern, to the north. There
and winter shadows are very nearly 26½ to the gnomon's 60.

We say "the shadows at noon," speaking generally and
preciable difference, because the equinoxes and tropics are by
completed exactly at noon.

And all those stars which revolve along the equator pass
for those under this parallel. And they are all seen to rise and
of the sphere are on the horizon, none of the parallels mak
always visible or invisible, and no meridians are cut short. I
lieve there are human habitations under the equator; for, acc
very temperate because the sun does not tarry about the
speed of its passage through the divisions of the equator;
summer heat would be temperate. Nor is the sun in the tropic
zenith, so that the winter would not be severe. But we coul
conviction what sort of habitations they are; for up until
mained inaccessible to people from the part of the earth in
what is said about them should be considered conjecture r

and the stars contained by it are always visible. And the always-
is that drawn with the south pole as pole at the same distance,
within it are always invisible.

ii. The second parallel is that whose longest day is 12¼ eq
And it is 4¼° from the equator, and is drawn through the island
[Ceylon]. And it is one of the amphiscian parallels, since the sun
zenith for those living under it and twice makes the gnomons
noon when it is 79½° from the summer tropic on either side. An
its passage the shadows of the gnomons fall to the south, and for
to the north. Thereupon the equinoctial shadow is 4+⅓+1/1
shadow 21⅓, and the winter 32, to the gnomon's 60.

iii. The third parallel is that whose longest day is 12½ equato
it is 8°25′ from the equator and it is drawn through the Aualiti
Aden]. And it is amphiscian since the sun is twice at the zenith f
under it, and twice makes the gnomons shadowless at noon, whe
the summer tropic on either side. And so for 138° of its passage
the gnomons fall to the south, and for the other 222° to the no
equinoctial shadow is 8+½+⅓, the summer shadow 16+½
winter 37+½+⅓+1/15, to the gnomon's 60.

iv. The fourth parallel is that whose longest day is 12¾ equato
it is 12½° from the equator and is drawn through the Adulitic
Bay]. And it is amphiscian, since the sun is twice at the zenith
under it and twice makes the gnomons shadowless at noon, w
from the summer tropic on either side. And so for 115⅓° of i
shadows of the gnomons fall to the south, and for the other 244⅔
Thereupon the equinoctial shadow is 13⅓, the summer shado
winter 44⅙, to the gnomon's 60.

v. The fifth parallel is that whose longest day is 13 equatorial h
16°27′ from the equator and is drawn through the island of M
also amphiscian since the sun is twice at the zenith for those wh
and twice makes the gnomons shadowless at noon, when it is 45°
mer tropic on either side. And so for 90° of its passage the s
gnomons fall to the south, and for the other 270° to the north.
equinoctial shadow is 17+½+¼, the summer shadow 7+½
winter 51, to the gnomon's 60.

vi. The sixth parallel is that whose longest day is 13¼ equator
it is 20°14′ from the equator and is drawn through the country of t
And it is also amphiscian since the sun is twice at the zenith fo
under it and twice makes the gnomons shadowless at noon, when
the summer tropic on either side. And so for 62° of its passage t

point. And the rest of the time the shadows of the gnomons fall to the north. Thereupon the equinoctial shadow is 26½, the winter shadow 65+¼+⅓, to the gnomon's 60, and the summer shadowless. And all the parallels to the north of this one, as far as the one bounding our inhabited world, are heteroscian. For the gnomons at these parallels are never shadowless at noon and they never throw their shadows to the south but always to the north, because the sun is never at the zenith for these people.

viii. The eighth parallel is that whose longest day is 13¾ equatorial hours. And it is 27°12′ from the equator and is drawn through Ptolemais in the Thebais, also called Hermeias. Thereupon the summer shadow is 3½, the equinoctial shadow 36+½+⅛, and the winter 74⅙, to the gnomon's 60.

ix. The ninth parallel is that whose longest day is 14 equatorial hours. And it is 30°22′ from the equator and is drawn through the low countries of Egypt. Thereupon the summer shadow is 6+½+⅓, the equinoctial shadow 35 1/12, and the winter 83 1/12, to the gnomon's 60.

x. The tenth parallel is that whose longest day is 14¼ equatorial hours. And it is 33°18′ from the equator and is drawn through the middle of Phoenicia. Thereupon the summer shadow is 10, the equinoctial shadow 39½, and the winter 93 1/12, to the gnomon's 60.

xi. The eleventh parallel is that whose longest day is 14½ equatorial hours. And it is 36° from the equator and is drawn through Rhodes. Thereupon the summer shadow is 12+½+⅓+1/12, the equinoctial shadow 43+½+⅓, and the winter 103⅓.

xii. The twelfth parallel is that whose longest day is 14¾ equatorial hours. And it is 38°35′ from the equator and is drawn through Smyrna. Thereupon the summer shadow is 15⅔, the equinoctial shadow 47+½+⅓, and the winter 114+½+⅓+1/12, to the gnomon's 60.

xiii. The thirteenth parallel is that whose longest day is 15 equatorial hours. And it is 40°56′ from the equator and is drawn through the Hellespont. Thereupon the summer shadow is 18½, the equinoctial shadow 52⅙, and the winter 127+½+⅓, to the gnomon's 60.

xiv. The fourteenth parallel is that whose longest day is 15¼ equatorial hours. And it is 43°4′ from the equator and is drawn through Marseilles. Thereupon the summer shadow is 20+½+⅓, the equinoctial shadow 55+½+⅓+1/12, and the winter 144, to the gnomon's 60.

xv. The fifteenth parallel is that whose longest day is 15½ equatorial hours. And it is 45°1′ from the equator and is drawn through the middle of Pontus. Thereupon the summer shadow is 23¼, the equinoctial shadow 60, and the winter 155 1/12, to the gnomon's 60.

xvi. The sixteenth parallel is that whose longest day is 15¾ equatorial hours. And it is 46°51′ from the equator and is drawn through the source of the river Ister [Danube]. Thereupon the summer shadow is 25½, the equinoctial shadow 63+½+⅓+1/12, and the winter 171⅙, to the gnomon's 60.

xvii. The seventeenth parallel is that whose longest day is 16 equatorial hours. And it is 48°32′ from the equator and is drawn through the mouth of the Borysthenes [Dnieper]. Thereupon the summer shadow is 27½, the equinoctial shadow 67+½+⅓, and the winter 188+½+1/12.

xviii. The eighteenth parallel is that whose longest day is 16¼ equatorial hours. And it is 50°4′ from the equator and is drawn through the middle of

Palus Maeotis [the Sea of Azof]. Thereupon the summer shadow is $29+\frac{1}{2}+\frac{1}{3}+\frac{1}{12}$, the equinoctial shadow $71\frac{2}{3}$, and the winter $208\frac{1}{3}$, to the gnomon's 60.

xix. The nineteenth parallel is that whose longest day is $16\frac{1}{2}$ equatorial hours. And it is 51°40′ from the equator and is drawn through the southernmost parts of Britain. Thereupon the summer shadow is $31+\frac{1}{3}+\frac{1}{12}$, the equinoctial shadow $75+\frac{1}{3}+\frac{1}{12}$, and the winter $229\frac{1}{6}$, to the gnomon's 60.

xx. The twentieth parallel is that whose longest day is $16\frac{3}{4}$ equatorial hours. And it is 52°50′ from the equator and is drawn through the mouth of the Rhine. Thereupon the summer shadow is $33\frac{1}{3}$, the equinoctial shadow $79\frac{1}{12}$, and the winter $253\frac{1}{6}$, to the gnomon's 60.

xxi. The twenty-first parallel is that whose longest day is 17 equatorial hours. And it is 54°30′ from the equator and is drawn through the mouth of the Tanais [Don]. Thereupon the summer shadow is $34+\frac{1}{2}+\frac{1}{3}+\frac{1}{12}$, the equinoctial shadow $82+\frac{1}{2}+\frac{1}{12}$, and the winter $278+\frac{1}{2}+\frac{1}{4}$, to the gnomon's 60.

xxii. The twenty-second parallel is that whose longest day is $17\frac{1}{4}$ equatorial hours. And it is 55° from the equator and is drawn through Brigantium [York] of Great Britain. Thereupon the summer shadow is $36\frac{1}{4}$, the equinoctial shadow $85\frac{2}{3}$, and the winter $304\frac{1}{2}$.

xxiii. The twenty-third parallel is that whose longest day is $17\frac{1}{2}$ equatorial hours. And it is 56° from the equator and is drawn through the middle of Great Britain. Thereupon the summer shadow is $37\frac{2}{3}$, the equinoctial $88+\frac{1}{2}+\frac{1}{3}$, and the winter $335\frac{1}{4}$, to the gnomon's 60.

xxiv. The twenty-fourth parallel is that whose longest day is $17\frac{3}{4}$ equatorial hours. And it is 57° from the equator and is drawn through Caturactonium of Britain. Thereupon the summer shadow is $39\frac{1}{3}$, the equinoctial shadow $92+\frac{1}{3}+\frac{1}{12}$, and the winter $372\frac{1}{12}$, to the gnomon's 60.

xxv. The twenty-fifth parallel is that whose longest day is 18 equatorial hours. And it is 58° from the equator and is drawn through the southern parts of Little Britain. Thereupon the summer shadow is $40\frac{2}{3}$, the equinoctial shadow 96, and the winter $419\frac{1}{12}$, to the gnomon's 60.

xxvi. The twenty-sixth parallel is that whose longest day is $18\frac{1}{2}$ equatorial hours. And it is $59\frac{1}{2}$° from the equator and is drawn through the middle of Little Britain.

We have not here used an increase of a quarter hour, because the parallels are now very close and the difference between the heights of the pole does not come to a full degree, and because it does not seem called for to finish out completely, in like fashion, the still more northern parallels. And for this reason we have thought it superfluous also to set out the ratios of the shadows to the gnomons as in the case of those places already defined.

xxvii. And further where the longest day is 19 equatorial hours, there the parallel is 61° from the equator and is drawn through the northern parts of Little Britain.

xxviii. And where the longest day is $19\frac{1}{2}$ equatorial hours, there the parallel is 62° from the equator and is drawn through what are called the Ebrides Islands [Hebrides].

xxix. And where the longest day is 20 equatorial hours, there the parallel is 63° from the equator and is drawn through the Island of Thule.

xxx. And where the longest day is 21 equatorial hours, there the parallel is $64\frac{1}{2}$° from the equator and is drawn through unknown Scythian nations.

xxxi. And where the longest day is 22 equatorial hours, there the parallel is 65½° from the equator.

xxxii. And where the longest day is 23 equatorial hours, there the parallel is 66° from the equator.

xxxiii. And where the longest day is 24 equatorial hours, there the parallel is 66°8′40″ from the equator. And this is the first of the periscian parallels. For since the sun at the summer tropic does not set for that parallel, the shadows of the gnomons fall to every side of the horizon. And the parallel through the summer tropic is there always visible, and through the winter tropic always invisible, because they both touch the horizon from different directions. And the ecliptic coincides with the horizon whenever the point of the spring equinox is just rising from it.

xxxiv. And if anyone for the sake of knowledge generally should want some of the broader incidents of the still more northern latitudes, he would find, where the height of the north pole is very nearly 67°, there 15° along the ecliptic on either side of the summer tropic do not set; so that the longest day and the turning of the shadows to all parts of the horizon last for nearly a month. For these things can be easily understood from the Table of Obliquity, already given. For by as many degrees as we find the parallel distant from the equator—as for instance that parallel which cuts off 15° of the ecliptic on either side of the summer tropic and which is then either always invisible or always visible along the intercepted section of the ecliptic—by just so many degrees does the height of the north pole lack being a quadrant or 90°.

xxxv. And then where the height of the pole is 69½°, there it would be found that 30° on either side of the summer tropic do not set; so that the longest day and the periscian gnomons last nearly two months.

xxxvi. And where the height of the pole is 73⅓°, there it would be found that 45° on either side of the summer tropic do not set; so that the longest day and the periscian gnomons last nearly three months.

xxxvii. And where the height of the pole is 78⅓°, there it would be found that 60° on either side of the summer tropic do not set; so that the longest day and the wheeling of the shadows are accomplished in nearly 4 months.

xxxviii. And where the height of the pole is 84°, there it would be found that 75° on either side of the summer tropic do not set; so that again the longest day would last for nearly 5 months and the periscian gnomons the same length of time.

xxxix. And where the north pole is raised from the horizon the 90° of the whole quadrant, there the whole semicircle of the ecliptic north of the equator is never below the earth, and the whole southern semicircle is never above the earth; so that there is one day and one night in each year, each nearly six months long, and the gnomons are always periscian. And the properties of this latitude are that the north pole is always at the zenith, the equator occupies the position of the always visible and at the same time invisible circle and also of the horizon, putting the whole hemisphere north of itself above the earth and that south of it below the earth.

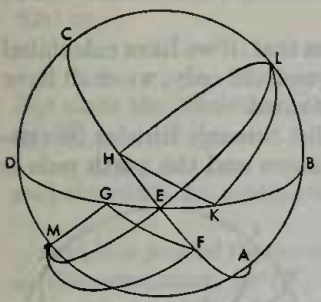
7. On the Co-ascensions of the Ecliptic and the Equator in the Oblique Sphere

Now that the general matters observed concerning the latitudes have been explained, the next thing would be to show how the co-ascending time-arcs of

the equator and the ecliptic can be gotten for each latitude. And from these all the other particularities will be deduced. We shall use the names of the zodiacal signs for the dodecatemories of the ecliptic as if they began from the equinoctial and tropic points, calling the first dodecatemory from the spring equinox in the direction opposite the movement of the universe the Ram, the second the Bull, and so on according to the order of the twelve signs handed down to us.

And first we shall show that arcs on the ecliptic equidistant from the same equinoctial point always ascend with equal arcs on the equator.

For let the circle $ABCD$ be the meridian, and BED the semicircle of the horizon, and AEC the semicircle of the equator. And let FG and HK be two sections of the ecliptic, so that each of the points F and H is assumed to be the point of the spring equinox, and so that the ends of the equal arcs FG and HK, taken on either side of it, ascend through the points K and G.



I say that the arcs of the equator ascending with either of them, that is EF and EH, are equal.

For let the points L and M be taken as the poles of the equator, and let the sections of great circles, LEM, HL, KL, FM, and GM be drawn through them. Then since

$$\text{arc } FG = \text{arc } HK,$$

and the parallels through K and G are equidistant from the equator on either side so that

$$\text{arc } LK = \text{arc } MG$$

and

$$\text{arc } EK = \text{arc } EG, \qquad \text{[p. 37, 38]}$$

therefore LKH and MGF have their sides equal respectively, and so also do LEK and MEG. Therefore

$$\text{angle } KLE = \text{angle } GME,$$
$$\text{angle } KLH = \text{angle } GMF,$$

so that, by subtraction,

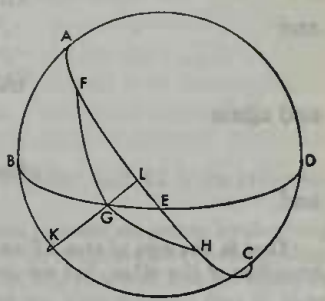
$$\text{angle } ELH = \text{angle } EMF.$$

And therefore

$$\text{base } EH = \text{base } EF.$$

Which was to be proved.

And again we shall prove that the arcs on the equator, ascending with equal arcs of the ecliptic equidistant from the same tropic point, are both together equal to the sum of the corresponding two in the right sphere.



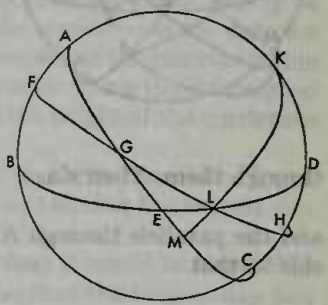
For let the meridian $ABCD$ be set out, and the semicircle of the horizon BED, and the semicircle of the equator AEC. And let two equal arcs on the ecliptic equidistant from the winter tropic point be drawn, the arc FG with F as the autumn

equinox and the arc *GH* with *H* as the spring equinox, so that the point *G* is the rising point of both on the horizon (because the arcs *FG* and *GH* are cut off by the same circle parallel to the equator), and so that clearly arcs *EH* and *GH* co-ascend and also *EF* and *FG*.

It is immediately evident, then, that the whole arc *HEF*[1] is equal to the ascensions in the right sphere of the arcs *FG* and *GH*. For if we suppose the south pole of the equator to be the point *K*, and draw through *K* and the point *G* the quadrant of a great circle *KGL* which represents the horizon in the right sphere, then arc *HL* ascends with arc *GH* in the right sphere, and likewise arc *LF* with arc *GF*. And so the two together, arcs *HL* and *LF*, are equal to the other two together, arcs *HE* and *EF*, and are contained by the same arc *HF*. Which was to be proved.

And for these reasons it has now become clear to us that, if we have calculated the particular co-ascensions in each latitude for a quadrant only, we shall have those of the other three quadrants already demonstrated.

Then with these things established, let the parallel through Rhodes be considered, where the longest day is 14½ equatorial hours and the north pole is raised 36° above the horizon. And let the circle *ABCD* be the meridian; and likewise *BED* the semicircle of the horizon, *AEC* the semicircle of the equator, and *FGH* the semicircle of the ecliptic, so that *G* is assumed to be the spring equinox. And with the north pole at the point *K*, let the quadrant of a great circle, arc *KLM*, be drawn through it and *L*, the intersection of the ecliptic and the horizon.



And given the arc *GL*, let it be required to find the arc on the equator ascending with it, that is *GE*. And let *GL* first embrace the dodecatemory corresponding to the Ram.

Now, since again in this construction the arcs *DE* and *KM* of great circles, cutting each other at *L*, have been drawn to the two arcs *CE* and *CK* of great circles, therefore

chord 2 arc *DK* : chord 2 arc *CD* comp. chord 2 arc *KL* : chord 2 arc *LM*, chord 2 arc *EM* : chord 2 arc *CE*.

But

$$2 \text{ arc } DK = 72°,$$
$$\text{chord } 2 \text{ arc } DK = 70^{p}32'4'';$$

and

$$2 \text{ arc } CD = 108°,$$
$$\text{chord } 2 \text{ arc } CD = 97^{p}4'56'';$$

and again

$$2 \text{ arc } KL = 156°41',$$
$$\text{chord } 2 \text{ arc } KL = 117^{p}31'15'';$$

and

[1]That is, the sum of arcs *EF* and *EH;* for they are not, of course, naturally the one the continuation of the other, but are considered so for the purposes of proof.

2 arc LM[1] = 23°19′59″,
chord 2 arc LM = 24ᵖ15′57″.

Then

chord 2 arc EM : chord 2 arc CE comp. 70ᵖ32′4″ : 97ᵖ4′56″,
24ᵖ15′57″ : 117ᵖ31′15″,

or

chord 2 arc EM : chord 2 arc CE : : 18ᵖ0′5″ : 120ᵖ.

And

chord 2 arc CE = 120ᵖ,

therefore

chord 2 arc EM = 18ᵖ0′5″.

And so

2 arc EM ≒ 17°16′,
arc EM = 8°38′

But since the whole arc GM ascends with arc GL in the right sphere, therefore [p. 32]

arc GM = 27°50′,

and therefore, by subtraction,

arc EG = 19°12′.

And it is proved therewith that the dodecatemory which is the Fishes ascends in the same time, 19°12′; and that of the Virgin and that of the Balance is, each, what is left when this is subtracted from double the ascension in the right sphere [twice arc GM], that is 36°28′ in time [p. 45].[2] Which was to be proved.

Again, let the arc GL embrace the 60° of the two dodecatemories of the ecliptic, the signs of the Ram and the Bull. Then, because of this assumption, the other things remaining the same,

2 arc KL = 138°59′42″,
chord 2 arc KL = 112ᵖ23′56″;

and

2 arc LM = 41°0′18″,
chord arc LM = 42ᵖ1′48″.

Then again

chord 2 arc EM : chord 2 arc CE comp. 70ᵖ32′4″ : 97ᵖ4′56″,
42ᵖ1′48″ : 112ᵖ23′56″,

or

chord 2 arc EM : chord 2 arc CE : : 32ᵖ36′4″ : 120ᵖ.

And

chord 2 arc CE = 120ᵖ,

therefore

chord 2 arc EM = 32ᵖ36′4″.

And so

2 arc EM ≒ 31°32′,

[1] GL is given as 30°; hence LM is given in the Table of Obliquity, and KL is the difference between LM and 90°.

[2] In other words, the ascensions on either side of the autumn equinox are gotten by doubling arc GM and subtracting arc EG. For the arc EG and the arc to be found are arcs on the equator ascending with equal arcs on the ecliptic equidistant from the winter tropic, and are therefore both together equal to the two equatorial arcs ascending in the right sphere—that is, twice arc GM.

arc $EM = 15°46'$.

But, for the same reasons [p. 32], it has already been proved

arc $GM = 57°44'$;

and therefore, by subtraction,

arc $EG = 41°58'$.

Therefore the Ram and the Bull together ascend in 41°58′ in time, with 19°12′ of which the Ram has just been proved to ascend. And therefore the dodecatemory which is the Bull ascends with 22°46′ in time.

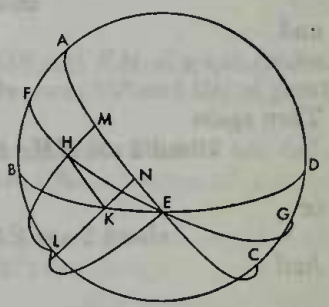
And again, for the same reasons as before, the dodecatemory which is the Water Bearer will ascend with an equal 22°46′ in time; and the Lion and the Scorpion, each, with what is left when this is subtracted from double the ascension in the right sphere; that is, 37°2′ in time.

And since the longest day is 14½ equatorial hours and the shortest day 9½ hours, it is evident that the semicircle from the Crab (Cancer) to the Archer will ascend with 217°30′ in equatorial time, and the semicircle from the Goat (Capricorn) to the Twins with 142°30′. And so each of the quadrants on either side of the spring equinox will ascend with 71°15′ in time, and each of the quadrants on either side of the autumn equinox with 108°45′ in time. And therefore there remain the dodecatemories, the Twins and Goat (Capricorn), each of which will ascend with the 29°17′ in time left over from the 71°15′ in time of the quadrant; and there remain the dodecatemories, the Crab (Cancer) and the Archer, each of which will ascend with the 35°15′ in time left over from the 108°45′ in time of the quadrant.

It is clear that in this same way we could also get the coascensions of the ecliptic for smaller divisions.

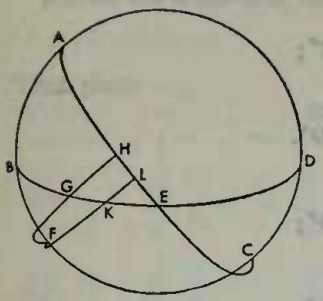
But we could also calculate them in an easier and more systematic way, thus:

For first let the circle *ABCD* be the meridian, *BED* the semicircle of the horizon, *AEC* that of the equator, and *FEG* that of the ecliptic, assuming the intersection *E* to be the spring equinox. And let the arc *EH* on the ecliptic be taken at random, and let the section *HK* of the parallel to the equator through *H* be drawn. And taking *L* as the pole of the equator, let there be drawn through it the quadrants of great circles *LHM*, *LKN*, and *LE*.



It is thus immediately evident that the section *EH* of the ecliptic ascends with arc *EM* of the equator in the right sphere, but in the oblique sphere with the arc equal to *MN*, since the parallel's arc *HK*, with which section *EH* ascends, is similar to arc *MN* of the equator and the similar arcs of parallels have everywhere their ascensions in equal times. Therefore the ascension of section *EH* in the oblique sphere is less than that in the right sphere by arc *EN*. And it has also now been proved in general that, if certain arcs of great circles like *LKN* are thus drawn, the section *EN* will contain the difference between the ascensions in the right and oblique spheres of arcs on the ecliptic cut off by *E* and the parallel drawn through *K*. Which was to be proved.

Now, with this first considered, let the figure be laid out with only the meridian and the semicircles of the horizon and the equator. And through *F*, the south



pole of the equator, let the quadrants of two great circles be drawn, *FGH* and *FKL*. And let *G* be assumed to be the intersection of the horizon and the parallel drawn through the winter tropic; and *K* the intersection of the horizon and the parallel drawn through the beginning of the Fishes, for example, or any other of the divisions of the given quadrant.

Now again the arcs of great circles, *FKL* and *EKG*, intersecting at *K*, have been drawn to the arcs of great circles *FH* and *EH*. And

chord 2 arc *GH* : chord 2 arc *FG* comp.
chord 2 arc *EH* : chord 2 arc *EL*,
chord 2 arc *KL* : chord 2 arc *FK*.

But 2 arc *GH* is given the same in all latitudes, for it is the distance between the tropics. And for this reason the remainder, 2 arc *FG*, is also given. And likewise, for the same sections of the ecliptic, 2 arc *KL* is the same for all latitudes and is given by the Table of Obliquity; and for this same reason 2 arc *FK* is given. And so the ratio

chord 2 arc *EH* : chord 2 arc *EL*

is left also as the same in all latitudes for the same divisions of the quadrant.

If, then, these things being true, we take the increasing values of arc *KL* for each increase of 10° on the quadrant from the spring equinox to the winter tropic (for a division into arcs of this length will be sufficient for our use), we always have

2 arc $GH = 47°42'40''$,
chord 2 arc $GH = 48^{p}31'55''$,

and

2 arc $FG = 132°17'20''$
chord 2 arc $FG = 109^{p}44'53''$.

And when *K* is 10° from the spring equinox in the direction of the winter tropic, we likewise also have

2 arc $KL = 8°3'16''$,
chord 2 arc $KL = 8^{p}25'39''$;

and

2 arc $FK = 171°56'44''$,
chord 2 arc $FK = 119^{p}42'14''$.

And likewise, when it is 20° away, we have

2 arc $KL = 15°54'6''$,
chord 2 arc $KL = 16^{p}35'56''$;

and

2 arc $FK = 164°5'54''$,
chord 2 arc $FK = 118^{p}50'47''$.

And when it is 30° away, we have

2 arc $KL = 23°19'58''$,
chord 2 arc $KL = 24^{p}15'56''$;

and

2 arc $FK = 156°40'2''$,
chord 2 arc $FK = 117^{p}31'15''$.

And when it is 40° away, we have

2 arc $KL = 30°8'8''$,

chord 2 arc $KL = 31^{p}11'43''$;

and

2 arc $FK = 149°51'52''$,

chord 2 arc $FK = 115^{p}52'19''$.

And when it is 50° away, we have

2 arc $KL = 36°5'46''$,

chord 2 arc $KL = 37^{p}10'39''$;

and

2 arc $FK = 143°54'14''$,

chord 2 arc $FK = 114^{p}5'44''$.

And when it is 60° away, we have

2 arc $KL = 41°0'18''$,

chord 2 arc $KL = 42^{p}1'48''$

and

2 arc $FK = 138°59'42''$,

chord 2 arc $FK = 112^{p}23'57''$.

And when it is 70° away, we have

2 arc $KL = 44°40'22''$

chord 2 arc $KL = 45^{p}36'18''$;

and

2 arc $FK = 135°19'38''$,

chord 2 arc $FK = 110^{p}59'47''$.

And when it is 80° away, we have

2 arc $KL = 46°56'32''$,

chord 2 arc $KL = 47^{p}47'40''$;

and

2 arc $FK = 133°3'28''$,

chord 2 arc $FK = 110^{p}4'16''$.

And therefore, if we compound the ratio

chord 2 arc GH : chord 2 arc FG

or

$48^{p}31'55'' : 109^{p}44'53''$

with the inverse of

chord 2 arc KL : chord 2 arc FK

for each 10° as just laid out, the result is the ratio

chord 2 arc EH : chord 2 arc EL.

And this is the same for all latitudes:—as 60^{p} to $9^{p}33'$, when K is 10° from the equinox; to $18^{p}57'$, when 20° away; to $28^{p}1'$, when 30° away; to $36^{p}33'$, when 40° away; to $44^{p}12'$, when 50° away; to $50^{p}44'$, when 60° away; to $55^{p}45'$, when 70° away; to $58^{p}55'$, when 80° away.

Therefore it is evident that, being given 2 arc EH for each latitude since it is equal to the difference between the equinoctial day and the shortest day in time, and being given its chord and the ratio of this chord to that of 2 arc EL, we also have given 2 arc EL. And if we subtract the half of it—that is, arc EL itself containing the difference we have spoken of before [p. 48]—from the ascensions in the right sphere of the given arc of the ecliptic, we shall find the ascension of the same arc in the given latitude.

For let the latitude of the parallel of Rhodes be taken for an example, where

2 arc $EH = 37°30$,

chord 2 arc $EH = 38^{p}34'$.

Then since

$$\begin{aligned} 60^{p} : 38^{p}34' &:: 9^{p}33' : 6^{p}8' \\ &:: 18^{p}57' : 12^{p}11' \\ &:: 28^{p}1' : 18^{p} \\ &:: 36^{p}33' : 23^{p}29' \\ &:: 44^{p}12' : 28^{p}25' \\ &:: 50^{p}44' : 32^{p}37' \\ &:: 55^{p}45' : 35^{p}52' \\ &:: 58^{p}55' : 37^{p}52', \end{aligned}$$

therefore the chord of 2 arc *EL* is also available for each of the sections as they increase in the usual manner by 10°. And the half of the arc subtending this chord (that is, *EL* itself) is 2°56′ at the first 10°; 5°50′ at the second; 8°38′ at the third; 11°17′ at the fourth; 13°42′ at the fifth; 15°46′ at the sixth; 17°24′ at the seventh; 18°24′ at the eighth; and clearly 18°45′ at the ninth.[1] And so, since in the right sphere the arc through the first 10° on the ecliptic ascends with 9°10′ in time; through the second, with 18°25′; through the third, with 27°50′; through the fourth, with 37°30′; through the fifth, with 47°28′; through the sixth, with 57°44′; through the seventh, with 68°18′; through the eighth, with 79°5′; and through the ninth, with the 90° in time of the whole quadrant; therefore it is evident that, if we subtract from each of the ascensions in the right sphere just mentioned the corresponding difference or arc *EL*, we shall have the ascensions of those same arcs in the given latitude. And so the arc through the first 10° of the ecliptic will ascend with 6°14′ in time; through the second 10° of the ecliptic, with 12°35′ in time; through the third 10°, with 19°12′; through the fourth, with 26°13′; through the fifth, with 33°46′; through the sixth, with 41°58′; through the seventh, with 50°54′; through the eighth, with 60°41′; and through the ninth or the whole quadrant, with the 71°15′ taken from the half of the day. And therefore the first 10° of the ecliptic with 6°14′ in time; the second with 6°21′; the third with 6°37′; the fourth with 7°1′; the fifth with 7°33′; the sixth with 8°12′; the seventh with 8°56′; the eighth with 9°47′; and the ninth with 10°34′.

And now with these things demonstrated, the ascensions of the other quadrants are in consequence also demonstrated immediately. And having calculated in the same way the ascensions of the other parallels for each 10°, which is as far as it is possible to anticipate their use in every case, we shall set them out in a table for use in the rest of this treatise, beginning with the parallel under the equator and going as far as that whose longest day is 17 hours, and advancing by half-hour increases because of the almost uniform increases of the periods within the half-hours. Then, placing first the 36 divisions of 10° each, we next set beside each one the time of its own ascension and then the successive additions of them, in the following manner:

[1]For when *K* is at 90° it coincides with *H*. Hence *EL* coincides with *EH*.

8. Table of Ascensions by 10°

Signs	Tens	Right Sphere: 12 hours time	Right Sphere: Lat. 0°0′ total time	Through the Atlantic Gulf: 12½ hours time	Through the Atlantic Gulf: Lat. 8°25′ total time	Through Meroë: 13 hours time	Through Meroë: Lat. 16°27′ total time
	10	9° 10′	9° 10′	8° 35′	8° 35′	7° 58′	7° 58′
Ram	20	9° 15′	18° 25′	8° 39′	17° 14′	8° 5′	16° 3′
	30	9° 25′	27° 50′	8° 52′	26° 6′	8° 17′	24° 20′
	10	9° 40′	37° 30′	9° 8′	35° 14′	8° 36′	32° 56′
Bull	20	9° 58′	47° 28′	9° 29′	44° 43′	9° 1′	41° 57′
	30	10° 16′	57° 44′	9° 51′	54° 34′	9° 27′	51° 24′
	10	10° 34′	68° 18′	10° 15′	64° 49′	9° 56′	61° 20′
Twins	20	10° 47′	79° 5′	10° 35′	75° 24′	10° 23′	71° 43′
	30	10° 55′	90° 0′	10° 51′	86° 15′	10° 47′	82° 30′
	10	10° 55′	100° 55′	10° 59′	97° 14′	11° 3′	93° 33′
Crab	20	10° 47′	111° 42′	10° 59′	108° 13′	11° 11′	104° 44′
(Cancer)	30	10° 34′	122° 16′	10° 53′	119° 6′	11° 12′	115° 56′
	10	10° 16′	132° 32′	10° 41′	129° 47′	11° 5′	127° 1′
Lion	20	9° 58′	142° 30′	10° 27′	140° 14′	10° 55′	137° 56′
	30	9° 40′	152° 10′	10° 12′	150° 26′	10° 44′	148° 40′
	10	9° 25′	161° 35′	9° 58′	160° 24′	10° 33′	159° 13′
Virgin	20	9° 15′	170° 50′	9° 51′	170° 15′	10° 25′	169° 38′
	30	9° 10′	180° 0′	9° 45′	180° 0′	10° 22′	180° 0′
	10	9° 10′	189° 10′	9° 45′	189° 45′	10° 22′	190° 22′
Balance	20	9° 15′	198° 25′	9° 51′	199° 36′	10° 25′	200° 47′
	30	9° 25′	207° 50′	9° 58′	209° 34′	10° 33′	211° 20′
	10	9° 40′	217° 30′	10° 12′	219° 46′	10° 44′	222° 4′
Scorpion	20	9° 58′	227° 28′	10° 27′	230° 13′	10° 55′	232° 59′
	30	10° 16′	237° 44′	10° 41′	240° 54′	11° 5′	244° 4′
	10	10° 34′	248° 18′	10° 53′	251° 47′	11° 12′	255° 16′
Archer	20	10° 47′	259° 5′	10° 59′	262° 46′	11° 11′	266° 27′
	30	10° 55′	270° 0′	10° 59′	273° 45′	11° 3′	277° 30′
	10	10° 55′	280° 55′	10° 51′	284° 36′	10° 47′	288° 17′
Goat	20	10° 47′	291° 42′	10° 35′	295° 11′	10° 23′	298° 40′
(Capricorn)	30	10° 34′	302° 16′	10° 15′	305° 26′	9° 56′	308° 36′
	10	10° 16′	312° 32′	9° 51′	315° 17′	9° 27′	318° 3′
Water Bearer	20	9° 58′	322° 30′	9° 29′	324° 46′	9° 1′	327° 4′
	30	9° 40′	332° 10′	9° 8′	333° 54′	8° 36′	335° 40′
	10	9° 25′	341° 35′	8° 52′	342° 46′	8° 17′	343° 57′
Fishes	20	9° 15′	350° 50′	8° 39′	351° 25′	8° 5′	352° 2′
	30	9° 10′	360° 0′	8° 35′	360° 0′	7° 58′	360° 0′

8. Table of Ascensions by 10°—Continued

Signs	Tens	Through Soëne 13½ hours time	Through Soëne Lat. 23°51′ total time	Through the low countries of Egypt 14 hours time	Through the low countries of Egypt Lat. 30°22′ total time	Through Rhodes 14½ hours time	Through Rhodes Lat. 36° total time
	10	7° 23′	7° 23′	6° 48′	6° 48′	6° 14′	6° 14′
Ram	20	7° 29′	14° 52′	6° 55′	13° 43′	6° 21′	12° 35′
	30	7° 45′	22° 37′	7° 10′	20° 53′	6° 37′	19° 12′
	10	8° 4′	30° 41′	7° 33′	28° 26′	7° 1′	26° 13′
Bull	20	8° 31′	39° 12′	8° 2′	36° 28′	7° 33′	33° 46′
	30	9° 3′	48° 15′	8° 37′	45° 5′	8 12′	41° 58′
	10	9° 36′	57° 51′	9° 17′	54° 22′	8° 56′	50° 54′
Twins	20	10° 11′	68° 2′	10° 0′	64° 22′	9° 47′	60° 41′
	30	10° 43′	78° 45′	10° 38′	75° 0′	10° 34′	71° 15′
	10	11° 7′	89° 52′	11° 12′	86° 12′	11° 16′	82° 31′
Crab	20	11° 23′	101° 15′	11° 34′	97° 46′	11° 47′	94° 18′
(Cancer)	30	11° 32′	112° 47′	11° 51′	109° 37′	12° 12′	106° 30′
	10	11° 29′	124° 16′	11° 55′	121° 32′	12° 20′	118° 50′
Lion	20	11° 25′	135° 41′	11° 54′	133° 26′	12° 23′	131° 13′
	30	11° 16′	146° 57′	11° 47′	145° 13′	12° 19′	143° 32′
	10	11° 5′	158° 2′	11° 40′	156° 53′	12° 13′	155° 45′
Virgin	20	11° 1′	169° 3′	11° 35′	168° 28′	12° 9′	167° 54′
	30	10° 57′	180° 0′	11° 32′	180° 0′	12° 6′	180° 0′
	10	10° 57′	190° 57′	11° 32′	191° 32′	12° 6′	192° 6′
Balance	20	11° 1′	201° 58′	11° 35′	203° 7′	12° 9′	204° 15′
	30	11° 5′	213° 3′	11° 40′	214° 47′	12° 13′	216° 28′
	10	11° 16′	224° 19′	11° 47′	226° 34′	12° 19′	228° 47′
Scorpion	20	11° 25′	235° 44′	11° 54′	238° 28′	12° 23′	241° 10′
	30	11° 29′	247° 13′	11° 55′	250° 23′	12° 20′	253° 30′
	10	11° 32′	258° 45′	11° 51′	262° 14′	12° 12′	265° 42′
Archer	20	11° 23′	270° 8′	11° 34′	273° 48′	11° 47′	277° 29′
	30	11° 7′	281° 15′	11° 12′	285° 0′	11° 16′	288° 45′
	10	10° 43′	291° 58′	10° 38′	295° 38′	10° 34′	299° 19′
Goat	20	10° 11′	302° 9′	10° 0′	305° 38′	9° 47′	309° 6′
(Capricorn)	30	9° 36′	311° 45′	9° 17′	314° 55′	8° 56′	318° 2′
	10	9° 3′	320° 48′	8° 37′	323° 32′	8° 12′	326° 14′
Water Bearer	20	8° 31′	329° 19′	8° 2′	331° 34′	7° 33′	333° 47′
	30	8° 4′	337° 23′	7° 33′	339° 7′	7° 1′	340° 48′
	10	7° 45′	345° 8′	7° 10′	346° 17′	6° 37′	347° 25′
Fishes	20	7° 29′	352° 37′	6° 55′	353° 12′	6° 21′	353° 46′
	30	7° 23′	360° 0′	6° 48′	360° 0′	6° 14′	360° 0′

8. TABLE OF ASCENSIONS BY 10°—Continued

Signs	Tens	*Through the Hellespont* 15 hours time	*Through the Hellespont* Lat. 40°56′ total time	*Through the middle of Pontus* 15½ hours time	*Through the middle of Pontus* Lat. 45°1′ total time	*Through the mouth of the Borysthenes* 16 hours time	*Through the mouth of the Borysthenes* Lat. 48° total time
	10	5° 40′	5° 40′	5° 8′	5° 8′	4° 36′	4° 36′
Ram	20	5° 47′	11° 27′	5° 14′	10° 22′	4° 43′	9° 19′
	30	6° 5′	17° 32′	5° 33′	15° 55′	5° 1′	14° 20′
	10	6° 29′	24° 1′	5° 58′	21° 53′	5° 26′	19° 46′
Bull	20	7° 4′	31° 5′	6° 34′	28° 27′	6° 5′	25° 51′
	30	7° 46′	38° 51′	7° 20′	35° 47′	6° 52′	32° 43′
	10	8° 38′	47° 29′	8° 15′	44° 2′	7° 53′	40° 36′
Twins	20	9° 32′	57° 1′	9° 19′	53° 21′	9° 5′	49° 41′
	30	10° 29′	67° 30′	10° 24′	63° 45′	10° 19′	60° 0′
	10	11° 21′	78° 51′	11° 26′	75° 11′	11° 31′	71° 31′
Crab	20	12° 2′	90° 53′	12° 15′	87° 26′	12° 29′	84° 0′
(Cancer)	30	12° 30′	103° 23′	12° 53′	100° 19′	13° 15′	97° 15′
	10	12° 46′	116° 9′	13° 12′	113° 31′	13° 40′	110° 55′
Lion	20	12° 52′	129° 1′	13° 22′	126° 53′	13° 51′	124° 46′
	30	12° 51′	141° 52′	13° 22′	140° 15′	13° 54′	138° 40′
	10	12° 45′	154° 37′	13° 17′	153° 32′	13° 49′	152° 29′
Virgin	20	12° 43′	167° 20′	13° 16′	166° 48′	13° 47′	166° 16′
	30	12° 40′	180° 0′	13° 12′	180° 0′	13° 44′	180° 0′
	10	12° 40′	192° 40′	13° 12′	193° 12′	13° 44′	193° 44′
Balance	20	12° 43′	205° 23′	13° 16′	206° 28′	13° 47′	207° 31′
	30	12° 45′	218° 8′	13° 17′	219° 45′	13° 49′	221° 20′
	10	12° 51′	230° 59′	13° 22′	233° 7′	13° 54′	235° 14′
Scorpion	20	12° 52′	243° 51′	13° 22′	246° 29′	13° 51′	249° 5′
	30	12° 46′	256° 37′	13° 12′	259° 41′	13° 40′	262° 45′
	10	12° 30′	269° 7′	12° 53′	272° 34′	13° 15′	276° 0′
Archer	20	12° 2′	281° 9′	12° 15′	284° 49′	12° 29′	288° 29′
	30	11° 21′	292° 30′	11° 26′	296° 15′	11° 31°	300° 0′
	10	10° 29′	302° 59′	10° 24′	306° 39′	10° 19′	310° 19′
Goat	20	9° 32′	312° 31′	9° 19′	315° 58′	9° 5′	319° 24′
(Capricorn)	30	8° 38′	321° 9′	8° 15′	324° 13′	7° 53′	327° 17′
	10	7° 46′	328° 55′	7° 20′	331° 33′	6° 52′	334° 9′
Water Bearer	20	7° 4′	335° 59′	6° 34′	338° 7′	6° 5′	340° 14′
	30	6° 29′	342° 28′	5° 58′	344° 5′	5° 26′	345° 40′
	10	6° 5′	348° 33′	5° 33′	349° 38′	5° 1′	350° 41′
Fishes	20	5° 47′	354° 20′	5° 14′	354° 52′	4° 43′	355° 24′
	30	5° 40′	360° 0′	5° 8′	360° 0′	4° 36′	360° 0′

8. TABLE OF ASCENSIONS BY 10°—Continued

Signs	Tens	Through the southernmost parts of Britain: 16½ hours time	Through the southernmost parts of Britain: Lat. 51°30′ total time	Through the mouth of the Tanais: 17 hours time	Through the mouth of the Tanais: Lat. 54°1′ total time
	10	4° 5′	4° 5′	3° 36′	3° 36′
Ram	20	4° 12′	8° 17′	3° 43′	7° 19′
	30	4° 31′	12° 48′	4° 0′	11° 19′
	10	4° 56′	17° 44′	4° 26′	15° 45′
Bull	20	5° 34′	23° 18′	5° 4′	20° 49′
	30	6° 25′	29° 43′	5° 56′	26° 45′
	10	7° 29′	37° 12′	7° 5′	33° 50′
Twins	20	8° 49′	46° 1′	8° 33′	42° 23′
	30	10° 14′	56° 15′	10° 7′	52° 30′
	10	11° 36′	67° 51′	11° 43′	64° 13′
Crab	20	12° 45′	80° 36′	13° 1′	77° 14′
(Cancer)	30	13° 39′	94° 15′	14° 3′	91° 17′
	10	14° 7′	108° 22′	14° 36′	105° 53′
Lion	20	14° 22′	122° 44′	14° 52′	120° 45′
	30	14° 24′	137° 8′	14° 54′	135° 39′
	10	14° 19′	151° 27′	14° 50′	150° 29′
Virgin	20	14° 18′	165° 45′	14° 47′	165° 16′
	30	14° 15′	180° 0′	14° 44′	180° 0′
	10	14° 15′	194° 15′	14° 44′	194° 44′
Balance	20	14° 18′	208° 33′	14° 47′	209° 31′
	30	14° 19′	222° 52′	14° 50′	224° 21′
	10	14° 24′	237° 16′	14° 54′	239° 15′
Scorpion	20	14° 22′	251° 38′	14° 52′	254° 7′
	30	14° 7′	265° 45′	14° 36′	268° 43′
	10	13° 39′	279° 24′	14° 3′	282° 46′
Archer	20	12° 45′	292° 9′	13° 1′	295° 47′
	30	11° 36′	303° 45′	11° 43′	307° 30′
	10	10° 14′	313° 59′	10° 7′	317° 37′
Goat	20	8° 49′	322° 48′	8° 33′	326° 10′
(Capricorn)	30	7° 29′	330° 17′	7° 5′	333° 15′
	10	6° 25′	336° 42′	5° 56′	339° 11′
Water Bearer	20	5° 34′	342° 16′	5° 4′	344° 15′
	30	4° 56′	347° 12′	4° 26′	348° 41′
	10	4° 31′	351° 43′	4° 0′	352° 41′
Fishes	20	4° 12′	355° 55′	3° 43′	356° 24′
	30	4° 5′	360° 0′	3° 36′	360° 0′

9. On Particular Consequences of the Ascensions

It will be evident through the systematic means given below that, once the times of the ascensions have been set out by us in the foregoing manner, all the rest of those things which concern the particular will be easily available, and we shall neither need proofs with figures for each of them, nor more tables.

For the first the length of the given day or night is gotten by calculating the degrees in time of the particular latitude: (1) in the case of the day by adding up the degrees in equatorial time from the point of the sun eastward to the point exactly opposite on the ecliptic, (2) in the case of the night, from the point opposite to the sun. For if we take a fifteenth of the total time, we shall get how many equatorial hours the given distance is; but if we take a twelfth, we shall get how many degrees in time the seasonal hour of the same distance is.

And the magnitude of the hour is also more easily found if, out of the foregoing table, the difference is taken laterally between the totals of successive additions of time in the parallel under the equator and that of the given latitude: (1) in the case of the day at the point of the sun; (2) in the case of the night, at the point directly opposite on the ecliptic. For taking $\frac{1}{6}$ of the difference so found and adding it to the 15° in time of the equatorial hour when the sun is in the northern hemisphere, and subtracting it when it is in the southern, we shall get the number of degrees in time of the given seasonal hour.[1]

And consequently we reduce the given seasonal hours to equatorial hours by multiplying the number of such hours in the day by the number of time-degrees in the hour for that day in that latitude, and the number in the night by the number of degrees in the hour for that night. For by taking $\frac{1}{15}$ of the total we shall have the number of equatorial hours. Conversely we reduce the given equatorial hours to seasonal hours by multiplying by 15 and dividing by the given number of time-degrees for the hour of that particular interval.

Again, given the time and any seasonal hour, we shall get the degree of the ecliptic which is rising at that time by multiplying the number of hours from the rising of the sun for the day, from the setting of the sun for the night, by the proper number of time-degrees for that hour. For we compare the resulting number with the ascensions of the given latitude: in the case of the day beginning with the sun and going eastward; in the case of the night, from the opposite point on the ecliptic. And to whatever degree the number corresponds we shall say that degree rises at that moment.

And if we wish to get the degree culminating, we multiply the seasonal hours, counted always from midday to the given hour, by the proper number of time-degrees for that hour. Beginning from the sun eastward, with the ascensions in the right sphere, we compare the resulting number, and on whatever degree the number falls, that degree will be culminating at that moment.

Likewise from the degree given as rising we shall get the degree culminating by locating the number resulting from the successive additions corresponding to the degree given as rising, in the table of the proper latitude. For by always subtracting from it the 90° in time of the quadrant, we shall find the degree culminating as that corresponding to the number resulting from this subtraction found in the column of successive additions of the right sphere. And con-

[1]For this difference, according to the construction of the Table [p. 52], is arc *EL* which is half the difference between the given and the equinoctial day [p. 50].

versely, from the degree given as culminating, we shall again get the degree which is rising by considering the number resulting from the successive additions corresponding to the degree given as rising, in the table of the right sphere. For after always adding the 90° in time, we shall look for the degree, corresponding to the resulting number, in the column of successive additions of the given latitude. And then we shall have the degree rising at that moment.

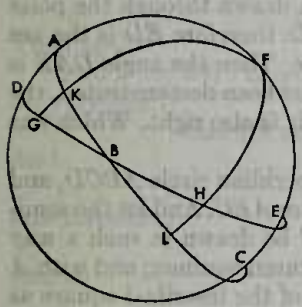
And it is also evident that for those living under the same meridian the sun is the same number of equatorial hours from noon or midnight; but for those not living under the same meridian, it will differ by as many equatorial hours as one meridian differs from the other.

10. On the Angles Formed by the Ecliptic and the Meridian

Since we still have to treat of the angles, to complete this theory—I mean those angles made by the ecliptic—it must first be understood that we say a right angle is contained by great circles if, when a circle is drawn with the intersection of the given great circles as pole and any distance, the arc intercepted on this circle by the sections of the great circles containing the angle is a quadrant. And in general we say that whatever ratio the intercepted arc has to the circle drawn in the manner we described, that same ratio does the angle contained by the inclination of the planes have to four right angles. And so, since we suppose the perimeter to be 360°, whatever number of degrees the intercepted arc is found to be, that same number will the angle subtending it be. For instance, one right angle is 90°.

Of the angles made by the ecliptic the most useful with respect to this theory are: (1) those contained by its intersection with the meridian, (2) those contained by its intersection with the horizon for each position, and likewise (3) those contained by its intersection with the great circle drawn through the horizon's poles. And along with the exposition of these last angles will be given also an exposition of the arcs intercepted on this great circle by the intersection and the horizon's pole—that is, the zenith. For every one of the aforementioned things, when demonstrated, has a very considerable place in the theory and contributes most especially to researches concerning the parallaxes of the moon, since such work cannot advance without the prior understanding of these.

But since there are four angles contained by the intersection of the two circles (that is, by the ecliptic and one of those intersecting it), and since we are going to speak of just one of them always similar in position, it is first necessary to determine that in general, of the two angles on either side of that arc of the ecliptic east of the intersection of the circles, the one on the north is to be considered, so that the incidents and quantities which will be demonstrated belong to such angles. And since the demonstration of the angles of the ecliptic considered with respect to the meridian is simpler, we shall begin with it, and we shall first prove that points on the ecliptic equidistant from the same equinoctial point make the resulting angles equal to each other.



For let *ABC* be the arc of the equator, and *DBE* the arc of the ecliptic, and point *F* the pole of the equator. And with arcs *BG* and *BH* cut off equal on

either side of the equinoctial point B, let there be drawn through the pole F and the points G and H the arcs of meridian circles FKG and FHL.

I say that

angle KGB = angle FHE.

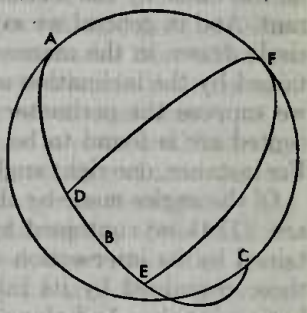
For the three-sided figure BGK is equiangular with the three-sided figure BHL, since it has its three sides equal to the three sides of the other respectively, GB to BH, GK to HL, and BK to BL. For all these things have been proved before [Book I, Chaps. 14, 15]. And therefore

angle KGB = angle BHL = angle FHE.

Which was to be proved.

Again it must be proved that, given two points on the ecliptic equidistant from the same tropic point, the angles there formed with the meridian are together equal to two right angles.

For let ABC be an arc of the ecliptic with B given as the tropic point. And with the equal arcs BD and BE intercepted on either side of it, let the arcs of meridian circles FD and FE be drawn through the points D and E and the pole F of the equator.



I say that

angle FDB + angle FEC = 2 rt. angles.

And this is immediately evident. For since the points D and E are equidistant from the same tropic point,

arc DF = arc EF.

And therefore

angle FDB = angle FEB

But

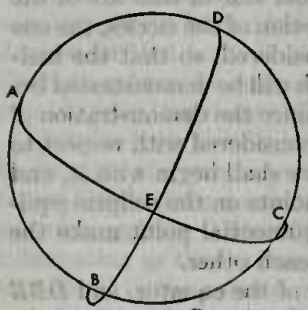
angle FEB + angle FEC = 2 rt. angles.

And therefore

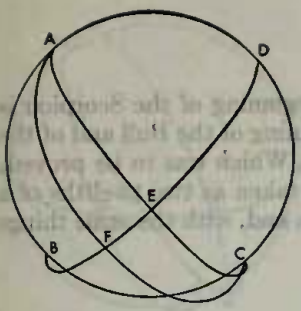
angle FDB + angle FEC = 2 rt. angles.

Which was to be proved.

And with these things understood first, let there be the meridian circle $ABCD$, and the semicircle of the ecliptic AEC with the point A taken as the winter tropic. And with A as pole and with a side of the inscribed square as distance let the semicircle BED be drawn. Since then the meridian $ABCD$ has been drawn through the poles of AEC and those of BED, therefore ED is the arc of the quadrant of a circle. Then the angle DAE is right. And, by what has just been demonstrated, the angle at the summer tropic is also right. Which was to be proved.



Again let there be the meridian circle $ABCD$, and the semicircle of the equator AEC. And let the semicircle of the ecliptic AFC be drawn in such a way that the point A is the autumn equinox; and with A as pole and with the side of the inscribed square as distance, let the semicircle $BFED$ be drawn.



For the same reasons, then, since $ABCD$ has been drawn through the poles of AEC and BED, therefore AF and ED are quadrants. And so the point F is the winter tropic, and as already proved

arc $EF = 23°51'$.

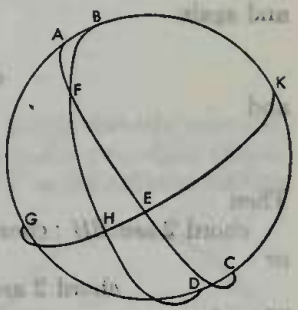
And therefore, by addition,

arc $FED = 113°51'$,

angle $DAF = 113°51'$.

And therefore, as already proved, the angle at the point of the spring equinox is what remains of two right angles or $66°9'$.

Again let there be the meridian circle $ABCD$, and the semicircle of the equator AEC, and that of the ecliptic BFD, so that the point F is taken as the autumn equinox, the arc BF first as one twelfth of the circle, that of the Virgin, and the point B clearly as the beginning of the Virgin. And again with B as pole and with the side of the inscribed square as distance let the semicircle $GHEK$ be drawn.



And let it be required to find angle KBH.

Since, then, the meridian $ABCD$ has been drawn through the poles of AEC and GEK, each of the arcs BG, BH, and EG is a quadrant. Because of the construction,

chord 2 arc AB : chord 2 arc AG comp.
chord 2 arc BF : chord 2 arc FH,
chord 2 arc EH : chord 2 arc EG.

But from things already proved

2 arc $AB = 23°20'$,
chord 2 arc $AB = 24^{p}16'$;

and

2 arc $AG = 156°40'$,
chord 2 arc $AG = 117^{p}31'$;

and again

2 arc $BF = 60°$,
chord 2 arc $BF = 60^{p}$;

and

2 arc $FH = 120°$,
chord 2 arc $FH = 103^{p}55'23''$.

Then again

chord 2 arc EH : chord 2 arc EG comp. $24^{p}16'$: $117^{p}31'$, $103^{p}55'23''$: 60^{p}

or, very nearly,

chord 2 arc EH : chord 2 arc EG : : $42^{p}58'$: 120^{p}

And

chord 2 arc $EG = 120^{p}$;

and therefore

chord 2 arc $EH = 42^{p}58'$.

And so

2 arc $EH = 42°$,
arc $EH = 21°$.

And therefore, by addition,

$$\text{arc } HEK = 111°,$$
$$\text{angle } KBH = 111°;$$

and, by previous proofs, the angle formed at the beginning of the Scorpion is also 111°, and each of the angles formed at the beginning of the Bull and of the Fishes is the 69° which remain from two right angles. Which was to be proved.

Again, with the same construction, let arc BF be taken as two twelfths of a circle, so that the point B is the beginning of the Lion and, with the same things being assumed, so that

$$2 \text{ arc } AB = 41°,$$
$$\text{chord } 2 \text{ arc } AB = 42^{p}2';$$

and

$$2 \text{ arc } AG = 139°,$$
$$\text{chord } 2 \text{ arc } AG = 112^{p}24';$$

and again

$$2 \text{ arc } BF = 120°,$$
$$\text{chord } 2 \text{ arc } BF = 103^{p}55'23'';$$

and

$$2 \text{ arc } FH = 60°$$
$$\text{chord } 2 \text{ arc } FH = 60^{p}.$$

Then

chord 2 arc EH : chord 2 arc EG comp. $42^{p}2'$: $112^{p}24'$, 60^{p} : $103^{p}55'23''$

or

chord 2 arc EH : chord 2 arc EG : : $25^{p}53'$: 120^{p}.

Therefore

$$\text{chord } 2 \text{ arc } EH = 25^{p}53',$$

And so

$$2 \text{ arc } EH = 25°,$$
$$\text{arc } EH = 12\tfrac{1}{2}°.$$

Therefore, by addition,

$$\text{arc } HEK = 102\tfrac{1}{2}°,$$
$$\text{angle } KBH = 102\tfrac{1}{2}°.$$

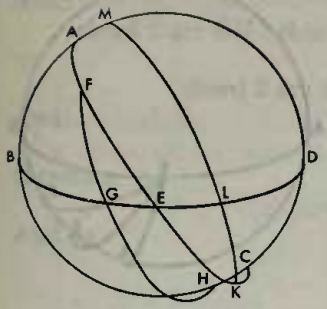
And for this reason the angle formed at the beginning of the Archer is also $102\frac{1}{2}°$, and each of the angles formed at the beginning of the Twins and of the Water Bearer is what remains of two right angles, or $77\frac{1}{2}°$.

And we have proved what was required; and the method will be the same for still smaller sections of the ecliptic. But the exposition of the twelve signs is sufficient for the needs of this treatise.

11. On the Angles Formed by the Ecliptic and the Horizon

And next we shall show how we can also get, for a given latitude, the angles formed by the ecliptic and the horizon; for they have a simpler system than the others that are left to do. Now it is clear that those formed with the meridian are the same as those formed with the horizon in the right sphere. But to get those in the oblique sphere it is first necessary to prove that points on the ecliptic equidistant from the same equinoctial point make with the horizon, angles equal to each other.

For let there be the meridian circle $ABCD$, and the semicircle of the equator



AEC, and that of the horizon *BED*. And let two sections of the ecliptic, *FGH* and *KLM*, be drawn in such a way that each of the points *F* and *K* is taken to be the autumn equinox, and

arc *FG* = arc *KL*.

I say that also

angle *EGH* = angle *DLK*.

And this is immediately evident. For the three-sided figure *EFG* is equiangular with the three-sided figure *EKL*, since, by previous proofs, it has its three sides equal respectively to the three sides of the other; *FG* to *KL*, the section of the horizon *GE* to *EL*, and the ascension arc *EF* to *EK*. Therefore also

angle *EGF* = angle *ELK*,

and the supplements

angle *EGH* = angle *DLK*.

Which was to be proved.

And I say also that the two angles formed at two points directly opposite, one at the east and the other at the west, together make two right angles. For if we draw circle *ABCD* as the horizon and *AECF* as the ecliptic, cutting each other at the points *A* and *C*,

angle *FAD* + angle *DAE* = 2 rt. angles.

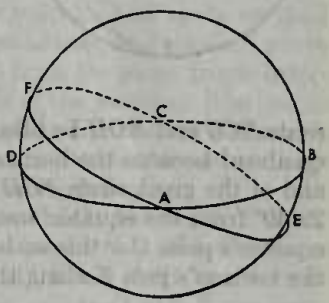
But

angle *FAD* = angle *FCD*

so that

angle *FCD* + angle *DAE* = 2 rt. angles.

Which was to be proved.



And now that these things are so—since angles formed by the ecliptic and the same horizon, at points on the ecliptic equidistant from the same equinoctial point, have been proved equal—it will also follow that the angle at the east and the angle at the west, for points equidistant from the same tropic and formed by the ecliptic and the horizon, will together equal two right angles. And so, for this reason, if we find the rising angles from the Ram to the Balance, the rising angles of the other semicircle will be made known at the same time, and also the setting angles of both. And we shall briefly expound how this is demonstrated, using again as an example the same parallel—that is, the one where the north pole is 36° above the horizon.

Now, the angles formed by the ecliptic and the horizon at the equinoctial points can be easily found. For if we draw the circle *ABCD* as the meridian and *AED* as the eastern semicircle of the horizon, and *EF* as a quadrant of the equator, and if we draw the two arcs of the ecliptic *BE* and *CE* in such a way that the point *E* is conceived as the fall equinox with respect to the quadrant *BE* and as the spring equinox with respect to *CE*, and *B* in the first instance becomes the winter tropic and *C* in the second becomes the summer tropic, then it is inferred that, since it is supposed

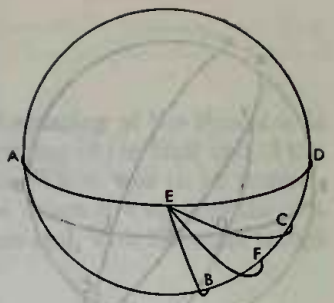
arc $DF = 54°$,

and

arc BF = arc $CG = 23°51'$,

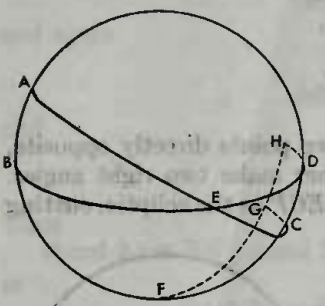
therefore

arc $CD = 30°9'$,
arc $BD = 77°51'$.



And so, since the point E is a pole of the meridian ABC, therefore the angle DEC formed at the beginning of the Ram will be $30°9'$, and angle DEB formed at the beginning of the Balance will be $77°51'$.

To show clearly the method for getting the others, let it be required, for example, to find the rising angle formed at the beginning of the Bull by the ecliptic and the horizon. And let there be the meridian circle $ABCD$ and the eastern semicircle of the given horizon BED. And let the semicircle of the ecliptic AEC be drawn so that the point E is the beginning of the Bull. And since in this same latitude, when the beginning of the Bull is just rising, the point $17°41'$ within the Crab is at the meridian under the earth (for we showed [p. 56] how to get such results easily from our Table of Ascensions). Therefore the arc CE is less than a quadrant. Then let the section HGF of a great circle be drawn with E as pole and the side of the inscribed square as distance; and let the quadrants ECG and EDH be filled out. But each of the arcs DCF and FGH is also a quadrant because the horizon BEH is through the poles of the meridian FCD and of the great circle FGH. Again, since the point $17°41'$ within the Crab is $22°40'$ from the equator towards the north along the great circle through the equator's poles (for this we have also proved), and since the equator is $36°$ from the horizon's pole F along the same arc FCD, therefore it is inferred



arc $CF = 58°40'$.

Now with this given, because of the construction,

chord 2 arc CD : chord 2 arc DF comp. chord 2 arc CE : chord 2 arc EG, chord 2 arc GH : chord 2 arc FH.

But, by what has already been laid down,

2 arc $CD = 62°40'$,
chord 2 arc $CD = 62^{p}24'$;

and

2 arc $DF = 180°$,
chord 2 arc $DF = 120^{p}$;

and again

2 arc $CE = 155°22'$,
chord 2 arc $CE = 117^{p}14'$;

and

2 arc $EG = 180°$,
chord 2 arc $EG = 120^{p}$.

Then

chord 2 arc GH : chord 2 arc FH comp. $62^p24'$: 120^p, 120^p : $117^p14'$

or

chord 2 arc GH : chord 2 arc FH : : $63^p52'$: 120^p.

And

chord 2 arc $FH = 120^p$;

therefore

chord 2 arc $GH = 63^p52'$.

And so

$$2 \text{ arc } GH = 64°20',$$
$$\text{arc } GH = 32°10',$$
$$\text{angle } GEH = 32°10'.$$

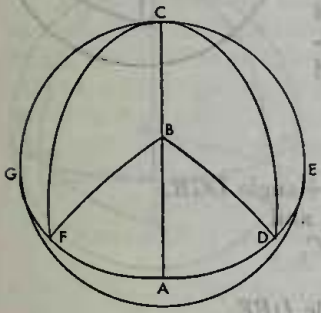
Which was to be proved.

Not to draw out this treatise by repeating for each case, the same method will be understood as used for the other signs and latitudes.

12. On the Angles and Arcs Made by the Ecliptic with, and on, the Circle through the Poles of the Horizon

Now there remains the method by which we can get the angles formed by the ecliptic and the circle through the poles of the horizon for each latitude and position. And, as we said, there will result, at the same time and in each case, a demonstration of the arc on the circle through the poles of the horizon intercepted by the zenith and the intersection with the ecliptic. And again we shall set out the fundamental things for this particular situation; and we shall prove first that, if two points on the ecliptic equidistant from the same tropic intercept an equal number of time degrees on either side of the meridian (the one to the east, the other to the west), then the arcs on great circles from the zenith to each of these points are equal to each other, and the angles formed at these points in the way we have stipulated are equal to two right angles.

For let there be a section of the meridian, ABC, and take the zenith B on it, and the pole of the equator C. And let two sections of the ecliptic, ADE and AFG, be drawn in such a way that the points D and F are equidistant from the same tropic and intercept equal arcs on the parallel through them from either side of the meridian ABC. And let the arcs of great circles, CD and CF, be drawn through the points D and F from the equator's pole C; and from the zenith B the arcs of great circles BD and BF.



I say that

arc BD = arc BF,

and

angle BDE + angle BFA = 2 rt. angles.

For since the points D and F are equidistant from the meridian ABC by equal arcs on the parallel through them,

angle BCD = angle BCF.

Then we have the two three-sided figures BCD and BCF having two sides equal to two sides respectively, CD to CF and BC common, and the angles

contained by the equal sides equal—that is, angle BCD to angle BCF. And therefore

$$\text{base } BD = \text{base } BF,$$
$$\text{angle } BFC = \text{angle } BCD.$$

But since it has just been proved that the angles at points equidistant from the same tropic and formed by the ecliptic and circles through the equator's poles are together equal to two right angles, therefore

$$\text{angle } CDE + \text{angle } CFA = 2 \text{ rt. angles.}$$

But it was proved also

$$\text{angle } BDC = \text{angle } BFC.$$

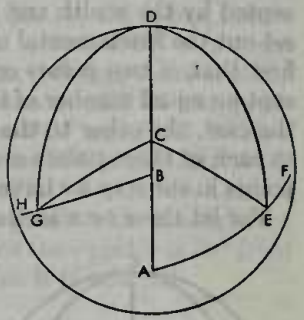
And therefore

$$\text{angle } BDE + \text{angle } BFA = 2 \text{ rt. angles.}$$

Which was to be proved.

Again, it must be proved that, when the same points on the ecliptic are equidistant in time from either side of the meridian, the arcs of great circles from the zenith to these points are equal to each other; and the angles formed by these arcs with the ecliptic (the one in the east and the other in the west) are together equal to the two angles at the same point formed by the ecliptic and the meridian at that point, whenever the culminations of the points in either position are either both to the north or both to the south of the zenith.

First let them be taken both to the south, and let there be the section of the meridian $ABCD$, and on it the zenith C, and the pole of the equator D. And let two sections of the ecliptic, AEF and BGH, be drawn so that points E and G, taken as the same, are equally distant from the meridian $ABCD$ on either side by an arc of the parallel through the point. And again let the sections of great circles, CE and CG, be drawn through these points from C; and DE and DG, from D.



By the same reasoning as before, since the points E and G, on the same parallel, make on that parallel equal arcs on either side from the meridian, therefore the three-sided figure CDE is equiangular and equilateral with the figure CDG, so that

$$\text{arc } CE = \text{arc } CG.$$

I say then that

$$\text{angle } CEF + \text{angle } CGB = \text{angle } DEF + \text{angle } DGB.$$

For since angle DEF is the same as angle DGB, and

$$\text{angle } CED = \text{angle } DGC,$$

therefore

$$\text{angle } CED + \text{angle } CGB = \text{angle } DEF.$$

And so also

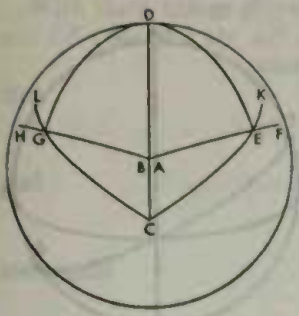
$$\text{angle } CEF + \text{angle } CGB = \text{angle } DEF + \text{angle } DGB.$$

Which was to be proved.

Let the same sections of the aforementioned circles be again constructed so that this time the points A and B are north of the point C.

I say that the same thing will follow, that is

$$\text{angle } KEF + \text{angle } LGB = \text{angle } DEF + \text{angle } DGB.$$



For since the angle DEF is the same as angle DGB and

angle DEK = angle DGL,

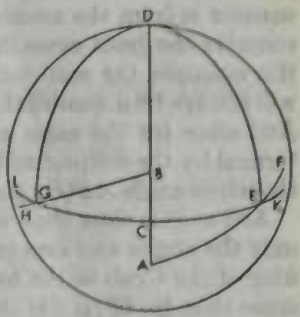
therefore, by addition,

angle LGB = angle DEF + angle DEK.

And so

angle KEF + angle LGB = angle DEF + angle DGB.

Now, again, let a similar figure be set out so that this time the point of the ecliptic's eastern section when culminating, that is A, is south of the zenith C, and the point of the western section when culminating, that is B, is north of it.



I say that

angle CEF + angle LGB = angle DEF + angle DGB + 2 rt. angles

For since

angle DGC = angle DEC,

and

angle DGC + angle DGL = 2 rt. angles,

therefore also

angle DEC + angle DGL = 2 rt. angles.

But angle DEF is the same as angle DBG. And so

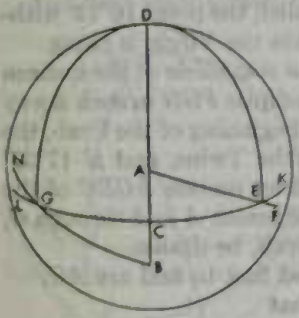
angle CEF + angle LGB = angle DEF + angle DGB + angle DEC + angle DGL

or

angle CEF + angle LGB = 2 angle DEF + 2 rt. angles.

Which was to be proved.

And let the final combination be set out in a similar figure: the point A of the ecliptic's eastern section culminating north of the point C, and the point B of the western section culminating south of it.



I say that

angle KEF + angle CGB + 2 rt. angles = 2 angle DEF.

For, by the same reasoning, again

angle KEF + angle CGB + angle DEK + angle DGC = angle DEF + angle DGB

or

angle KEF + angle CGB + angle DEK + angle DGC = 2 angle DEF.

But

angle DEK + angle DGC = 2 rt. angles,

because

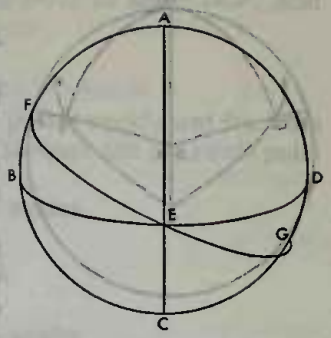
angle DEK + angle DEC = 2 rt. angles

and

angle DEC = angle DGC.

Which was to be proved.

And it will be immediately clear that, from the angles and arcs formed by the ecliptic with the great circle through the zenith in the way we have described,

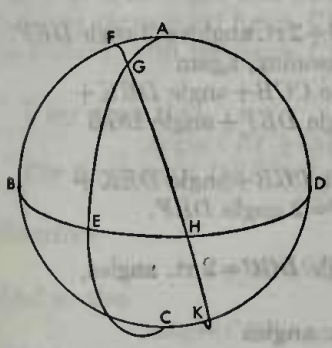


those formed on the meridian and the horizon can be easily gotten. For if we draw the meridian circle *ABCD*, and the semicircle of the horizon *BED*, and that of the ecliptic *FEG* in any manner whatsoever, then, whenever we conceive the great circle drawn through the zenith *A* as passing through the culminating point *F*, it will coincide with the meridian *ABCD*. And the angle *DFE* is immediately given us because the point *F* and the angle formed at it by the meridian [Book II, Chap. 10] are given. And the arc *AF* is given because we know how far on the meridian the point *F* is from the equator, and how far the equator is from the zenith *A*. And whenever we conceive the great circle drawn through *A* as passing through the rising point *E* (for example, the semicircle *AEC*) then it is immediately clear that the arc *AE* will always be a quadrant; because the point *A* is the pole of the horizon *BED*. And since for the same cause the angle *AED* is always right, and the angle formed by the ecliptic and the horizon (that is, angle *DEG*) is given. Therefore the whole angle *AEG* is also given. Which was to be proved.

And so it is clear that, since things are thus, if, in each latitude, we calculate only the angles and arcs preceding the meridian and only those from the beginning of the Crab to the beginning of the Goat, we shall have calculated at the same time [p. 63; p. 64] the angles and arcs which follow the meridian, and for the rest of the signs both those preceding and those following.

To make the method here clear for each position, again as an example, we shall set out what will be a general demonstration through one theorem, supposing that, in the same latitude (that is, where the north pole is 36° above the horizon), the beginning of the Crab for instance is one equatorial hour to the east of the meridian. And in this position, for this parallel, the point 16°12′ within the Twins is culminating and the point 17°37′ within the Virgin is rising.

Then let there be the meridian circle *ABCD;* and the semicircle of the horizon *BED;* and that of the ecliptic *FGH* in such a way that the point *G* is the beginning of the Crab, the point *F* 16°12′ within the Twins, and *H* 17°37′ within the Virgin. And let section *AGEC* of the great circle through the zenith *A* and through *G*, the beginning of the Crab, be drawn.



And let it be required first to find arc *AG*.

Then it is evident that

arc $FH = 91°25'$,

arc $GH = 77°37'$.

And likewise since the point 16°12′ within the Twins intercepts 23°7′ on the meridian from the equator northward and the equator is 36° from the zenith *A*, therefore

arc $AF = 12°53'$,

and, by subtraction from the quadrant,

arc $BF = 77°7'$.

With these things given, it again follows from the construction that chord 2 arc BF : chord 2 arc AB comp., chord 2 arc FH : chord 2 arc GH,
chord 2 arc EG : chord 2 arc AE.

But

$$2 \text{ arc } BF = 154°14',$$
$$\text{chord } 2 \text{ arc } BF = 116^p59';$$

and

$$2 \text{ arc } AB = 180°,$$
$$\text{chord } 2 \text{ arc } AB = 120^p;$$

and again

$$2 \text{ arc } FH = 182°50',$$
$$\text{chord } 2 \text{ arc } FH = 119^p58';$$

and

$$2 \text{ arc } GH = 155°14',$$
$$\text{chord } 2 \text{ arc } GH = 117^p12'.$$

Therefore

chord 2 arc EG : chord 2 arc AE comp. $116^p59'$: 120^p, $117^p12'$: $119^p58'$

or

chord 2 arc EG : chord 2 arc AE : : $114^p16'$: 120^p.

And

$$\text{chord } 2 \text{ arc } AE = 120^p;$$

and therefore

$$\text{chord } 2 \text{ arc } EG = 114^p16'.$$

And so

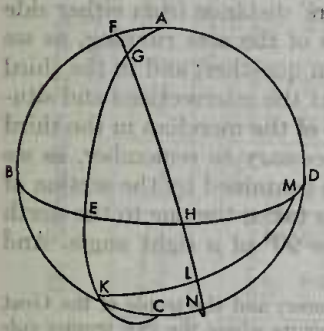
$$2 \text{ arc } EG = 114°26',$$
$$\text{arc } EG = 72°13'.$$

And therefore, by subtraction from the quadrant,

$$\text{arc } AG = 17°47'.$$

Which was to be proved.

And next we shall also find the angle AGH in this way: let the same figure be set out, and with G as pole and with the side of the inscribed square as distance let the section KLM of a great circle be drawn so that, since circle AGE has been drawn through the poles of EHM and KLM, each of the arcs EM and KM is a quadrant. Then again, because of the construction,



chord 2 arc EG : chord 2 arc EK comp.
chord 2 arc GH : chord 2 arc HL,
chord 2 arc LM : chord 2 arc KM.

But

$$2 \text{ arc } EG = 144°26',$$
$$\text{chord } 2 \text{ arc } EG = 114^p16';$$

and

$$2 \text{ arc } EK = 35°34',$$
$$\text{chord } 2 \text{ arc } EK = 36^p38';$$

and

$$2 \text{ arc } GH = 155°14',$$
$$\text{chord } 2 \text{ arc } GH = 117^p12';$$

and

$$2 \text{ arc } HL = 24°46'$$
$$\text{chord } 2 \text{ arc } HL = 25^{p}44'.$$

And so

chord 2 arc LM : chord 2 arc KM comp. $114^{p}16' : 36^{p}38'$, $25^{p}44' : 117^{p}12'$

or very nearly

$$\text{chord } 2 \text{ arc } LM : \text{chord } 2 \text{ arc } KM :: 82^{p}11' : 120^{p}.$$

And

$$\text{chord } 2 \text{ arc } KM = 120^{p};$$

therefore

$$\text{chord } 2 \text{ arc } LM = 82^{p}11'.$$

And so

$$2 \text{ arc } LM = 86°28',$$
$$\text{arc } LM = 43°14'.$$

And therefore, by subtraction,

$$\text{arc } KL = 46°46'$$
$$\text{angle } LGK = 46°46'.$$

And so, by subtraction from two right angles,

$$\text{angle } AGH = 133°14'.$$

Which was to be proved.

Now, the method of finding these arcs and angles is inferred to be the same also for the rest. But to have them set out, ready to hand, we have calculated as many of the other arcs and angles as seemed likely to be useful for the detailed researches. And we have begun geometrically with the parallel through Meroë where the longest day is 13 equatorial hours and have gone to the parallel drawn above Pontus through the mouth of the Borysthenes. And we have used again, as in the case of the ascensions, an increase each time of a half hour for the latitudes, of a twelfth of a circle for the sections of the ecliptic, and of one equatorial hour for the positions either east or west of the meridian. And we set them out in tables, one for each latitude and sign, putting in the first column for each position the number of its equatorial hours' distance from either side of the meridian; in the second column the lengths of the arcs running, as we said, from the zenith to the beginning of the sign in question; and in the third and fourth columns the sizes of the angles formed at the intersections and situated in the way we defined, with the positions east of the meridian in the third column and those west in the fourth. And it is necessary to remember, as we stipulated in the beginning, that, of the two angles contained by the section of the ecliptic east of the intersection, we have always taken the one to the north of it, comparing the size of each of them with the 90° of a right angle. And here are the tables.[1]

[1]Since, for latitudes between the tropic of the Crab (Cancer) and the tropic of the Goat (Capricorn), the zeniths lie on the meridian between the points where the two tropics culminate, therefore, as the tropic of the Crab, for instance, is considered at the meridian and then farther and farther eastward, the arc of the great circle through the zenith and the tropic, first south of the ecliptic, comes nearer and nearer to coinciding with the ecliptic. Finally it does coincide, and then passes to the northern side of the ecliptic. The angle in question starts as a right angle, becomes smaller and smaller, does not exist or is equal to two right angles, and then starting again as if from two right angles gets smaller and smaller. This is repeated, *mutatis mutandi*, for all the signs. The letter *N* or *S* placed after an angle in the tables indicates that the point of the ecliptic culminating is north or south of the zenith for that angle and all below it until a change is indicated.

13. Exposition of the Angles and Arcs by Parallels or Latitudes.

Of the Parallel through Meroë 13 hours lat. 16°27′.

Crab

Hours	Arcs	Eastern Angles	Western Angles
Noon	7° 24′	90°(N) 0′	
1°	15° 55′	25° 16′	154°(N)44′
2°	29° 3′	9° 15′	170° 45′
3°	42° 42′	1° 38′	178° 22′
4°	56° 25′	175°(S) 7′	4°(S) 53′
5°	70° 2′	170° 18′	9° 42′
6°	83° 27′	164° 41′	15° 19′
6° 30′	90° 0′	161° 57′	18° 3′

Lion

Hours	Arcs	Eastern Angles	Western Angles
Noon	4° 3′	102°(N)30′	
1°	14° 20′	26° 3′	178°(N)57′
2°	28° 42′	15° 28′	9°(S) 32′
3°	42° 43′	10° 5′	14° 55′
4°	56° 49′	6° 19′	18° 41′
5°	70° 38′	2° 33′	22° 27′
6°	84° 17′	177°(S) 0′	28° 0′
6° 25′	90° 0′	174° 51′	30° 9′

Virgin

Hours	Arcs	Eastern Angles	Western Angles
Noon	4° 47′	111°(S) 0′	0°(S) 0′
1°	15° 20′	0°(N) 0′	42° 0′
2°	29° 28′	8° 0′	34° 0′
3°	43° 40′	9° 15′	32° 45′
4°	58° 13′	8° 39′	33° 21′
5°	72° 36′	6° 53′	35° 7′
6°	86° 41′	5° 37′	36° 23′
6° 14′	90° 0′	4° 9′	37° 51′

Balance

Hours	Arcs	Eastern Angles	Western Angles
Noon	16° 27′	113°(S) 51′	
1°	22° 8′	154° 53′	72°(S) 49′
2°	33° 50′	173° 17′	54° 25′
3°	47° 20′	1°(N)23′	46° 19′
4°	61° 22′	5° 8′	42° 34′
5°	75° 39′	7° 9′	40° 33′
6°	90° 0′	7° 24′	40° 18′

Scorpion

Hours	Arcs	Eastern Angles	Western Angles
Noon	28° 7′	111°(S) 0′	
1°	31° 46′	139° 0′	83°(S) 0′
2°	40° 52′	157° 59′	64° 1′
3°	52° 30′	169° 23′	52° 37′
4°	65° 40′	176° 41′	45° 19′
5°	79° 18′	1°(N)41′	40° 19′
5° 46′	90° 0′	4° 9′	37° 51′

Archer

Hours	Arcs	Eastern Angles	Western Angles
Noon	36° 57′	102°(S) 30′	
1°	39° 46′	125° 12′	79°(S) 48′
2°	47° 15′	143° 5′	61° 55′
3°	57° 33′	156° 3′	48° 57′
4°	69° 30′	164° 48′	40° 12′
5°	82° 18′	171° 43′	33° 17′
5° 35′	90° 0′	174° 51′	30° 9′

Goat

Hours	Arcs	Eastern Angles	Western Angles
Noon	40° 18′	90°(S) 0′	
1°	42° 54′	111° 24′	68°(S) 36′
2°	49° 58′	128° 51′	51° 9′
3°	59° 35′	141° 49′	38° 11′
4°	71° 4′	151° 25′	28° 35′
5°	83° 31′	158° 48′	21° 12′
5° 30′	90° 0′	161° 57′	18° 3′

Water Bearer

Hours	Arcs	Eastern Angles	Western Angles
Noon	36° 57′	77°(S) 30′	
1°	39° 46′	100° 12′	54°(S) 48′
2°	47° 15′	118° 5′	36° 55′
3°	57° 33′	131° 3′	23° 57′
4°	69° 30′	139° 48′	15° 12′
5°	82° 18′	146° 43′	8° 17′
5° 35′	90° 0′	149° 51′	5° 9′

Fishes

Hours	Arcs	Eastern Angles	Western Angles
Noon	28° 7′	69°(S) 0′	
1°	31° 46′	97° 0′	41°(S) 0′
2°	40° 52′	115° 59′	22° 1′
3°	52° 30′	127° 23′	10° 37′
4°	65° 40′	134° 41′	3° 19′
5°	79° 18′	139° 41′	178°(N)19′
5° 46′	90° 0′	142° 9′	175° 51′

Ram

Hours	Arcs	Eastern Angles	Western Angles
Noon	16° 27′	66°(S) 27′	(S)
1°	22° 8′	107° 11′	25°(S) 7′
2°	33° 50′	125° 35′	6° 43′
3°	47° 20′	133° 41′	178°(N) 37′
4°	61° 22′	137° 26′	174° 52′
5°	75° 39′	139° 27′	172° 51′
6°	90° 0′	139° 42′	172° 36′

Bull

Hours	Arcs	Eastern Angles	Western Angles
Noon	4° 47′	69°(S) 0′	
1°	15° 20′	138° 0′	180°(N) 0′
2°	29° 28′	146° 0′	172° 0′
3°	43° 40′	147° 15′	170° 45′
4°	58° 13′	146° 39′	171° 21′
5°	72° 36′	144° 53′	173° 7′
6°	86° 41′	143° 37′	174° 23′
6° 14′	90° 0′	142° 9′	175° 51′

Twins

Hours	Arcs	Eastern Angles	Western Angles
Noon	4° 3′	77°(N)30′	
1°	14° 20′	1° 3′	153°(N) 57′
2°	28° 42′	170°(S) 28′	164° 32′
3°	42° 43′	165° 5′	169° 55′
4°	56° 49′	161° 19′	173° 41′
5°	70° 38′	157° 33′	177° 27′
6°	84° 17′	152° 0′	3°(S) 0′
6° 25′	90° 0′	149° 51′	5° 9′

13. Exposition of the Angles and Arcs by Parallels or Latitudes.—Continued

Of the Parallel through Soëne 13½ hours lat. 23°51′.

Hours	Arcs	Eastern Angles	Western Angles
	Crab		
Noon	0° 0′	90° 0′	0° 0′
1°	13° 43′	176° 15′	3° 45′
2°	27° 23′	173° 51′	6° 9′
3°	41° 20′	168° 15′	11° 45′
4°	54° 27′	166° 51′	13° 9′
5°	67° 42′	162° 42′	17° 18′
6°	80° 36′	157° 59′	22° 1′
6° 45′	90° 0′	153° 46′	26° 14′
	Lion		
Noon	3° 21′	102° 30′	
1°	14° 18′	176° 4′	28° 56′
2°	27° 56′	180° 0′	25° 0′
3°	41° 44′	179° 3′	25° 57′
4°	55° 14′	177° 18′	27° 42′
5°	68° 43′	173° 40′	31° 20′
6°	81° 52′	168° 56′	36° 4′
6° 38′	90° 0′	166° 53′	38° 7′
	Virgin		
Noon	12° 11′	111° 0′	
1°	18° 42′	158° 40′	63° 20′
2°	30° 57′	173° 44′	48° 16′
3°	44° 22′	178° 3′	43° 57′
4°	58° 1′	180° 0′	42° 0′
5°	71° 43′	179° 15′	42° 45′
6°	85° 20′	177° 39′	44° 21′
6° 21′	90° 0′	176° 41′	45° 19′

Hours	Arcs	Eastern Angles	Western Angles
	Balance		
Noon	23° 51′	113° 51′	
1°	27° 56′	144° 10′	83° 32′
2°	37° 36′	162° 13′	65° 29′
3°	49° 42′	171° 45′	55° 57′
4°	62° 47′	176° 59′	50° 43′
5°	76° 20′	179° 3′	48° 39′
6°	90° 0′	180° 0′	47° 42′
	Scorpion		
Noon	35° 31′	111° 0′	
1°	38° 25′	133° 15′	88° 45′
2°	46° 2′	150° 18′	71° 42′
3°	56° 30′	161° 41′	60° 19′
4°	68° 31′	169° 5′	52° 55′
5°	81° 22′	174° 30′	47° 30′
5° 39′	90° 0′	176° 41′	45° 19′
	Archer		
Noon	44° 21′	102° 30′	
1°	46° 40′	121° 30′	83° 30′
2°	53° 4′	137° 16′	67° 44′
3°	62° 18′	149° 25′	55° 35′
4°	73° 20′	157° 58′	47° 2′
5°	85° 23′	164° 46′	40° 14′
5° 22′	90° 0′	166° 53′	38° 7′

Hours	Arcs	Eastern Angles	Western Angles
	Goat		
Noon	47° 42′	90° 0′	
1°	49° 52′	108° 3′	71° 57′
2°	55° 52′	123° 31′	56° 29′
3°	64° 37′	135° 37′	44° 23′
4°	75° 12′	144° 57′	35° 3′
5°	86° 54′	152° 0′	28° 0′
5° 15′	90° 0′	153° 46′	26° 14′
	Water Bearer		
Noon	44° 21′	77° 30′	
1°	46° 40′	96° 30′	58° 30′
2°	53° 4′	112° 16′	42° 44′
3°	62° 18′	124° 25′	30° 25′
4°	73° 20′	132° 58′	22° 2′
5°	85° 23′	139° 46′	15° 14′
5° 22′	90° 0′	141° 53′	13° 7′
	Fishes		
Noon	35° 31′	69° 0′	
1°	38° 25′	91° 15′	46° 45′
2°	46° 2′	108° 18′	29° 42′
3°	56° 30′	119° 41′	18° 19′
4°	68° 31′	127° 5′	10° 55′
5°	81° 22′	132° 30′	5° 30′
5° 39′	90° 0′	134° 41′	3° 19′

Hours	Arcs	Eastern Angles	Western Angles
	Ram		
Noon	23° 51′	66° 9′	
1°	27° 56′	96° 28′	35° 50′
2°	37° 36′	114° 31′	17° 47′
3°	49° 42′	124° 3′	8° 15′
4°	62° 47′	129° 17′	3° 1′
5°	76° 20′	131° 21′	0° 57′
6°	90° 0′	132° 18′	0° 0′
	Bull		
Noon	12° 11′	69° 0′	
1°	18° 42′	116° 40′	21° 20′
2°	30° 57′	131° 44′	6° 16′
3°	44° 22′	136° 3′	1° 57′
4°	58° 1′	138° 0′	0° 0′
5°	71° 43′	137° 15′	0° 45′
6°	85° 20′	135° 39′	2° 21′
6° 21′	90° 0′	134° 41′	3° 19′
	Twins		
Noon	3° 21′	77° 30′	
1°	14° 18′	151° 4′	3° 56′
2°	27° 56′	155° 0′	0° 0′
3°	41° 44′	154° 3′	0° 57′
4°	55° 14′	152° 18′	2° 42′
5°	68° 43′	148° 40′	6° 20′
6°	81° 52′	143° 56′	11° 4′
6° 38′	90° 0′	141° 53′	13° 7′

13. Exposition of the Angles and Arcs by Parallels or Latitudes.—Continued

Of the Parallel through the Low Countries of Egypt 14 hours lat. 30°22′.

Hours	Arcs	Eastern Angles	Western Angles
	Crab		
Noon	6° 31′	90° 0′	
1°	14° 56′	150° 0′	30° 0′
2°	27° 23′	159° 38′	20° 22′
3°	40° 19′	160° 30′	19° 30′
4°	53° 14′	158° 51′	21° 9′
5°	65° 55′	156° 0′	24° 0′
6°	78° 15′	151° 49′	28° 11′
7°	90° 0′	146° 28′	33° 32′
	Lion		
Noon	9° 52′	102° 30′	
1°	16° 45′	153° 13′	51° 47′
2°	28° 44′	166° 22′	38° 38′
3°	41° 31′	169° 26′	35° 34′
4°	54° 27′	169° 8′	35° 52′
5°	67° 17′	167° 1′	37° 59′
6°	79° 48′	163° 46′	41° 14′
6° 51′	90° 0′	159° 49′	45° 11′
	Virgin		
Noon	18° 42′	111° 0′	
1°	23° 18′	145° 18′	76° 42′
2°	33° 30′	162° 25′	59° 35′
3°	45° 36′	169° 34′	52° 26′
4°	58° 21′	172° 10′	49° 50′
5°	71° 15′	172° 28′	49° 32′
6°	84° 7′	171° 5′	50° 55′
6° 28′	90° 0′	169° 55′	52° 5′

Hours	Arcs	Eastern Angles	Western Angles
	Balance		
Noon	30° 22′	113° 51′	
1°	33° 35′	137° 32′	90° 10′
2°	41° 39′	154° 19′	73° 23′
3°	52° 25′	164° 10′	63° 32′
4°	64° 28′	169° 47′	57° 55′
5°	77° 6′	172° 21′	55° 21′
6°	90° 0′	173° 29′	54° 13′
	Scorpion		
Noon	42° 2′	111° 0′	
1°	44° 26′	129° 32′	92° 28′
2°	50° 58′	144° 38′	77° 22′
3°	60° 19′	155° 33′	66° 27′
4°	71° 20′	162° 56′	59° 4′
5°	83° 19′	167° 54′	54° 6′
5° 32′	90° 0′	169° 55′	52° 5′
	Archer		
Noon	50° 52′	102° 30′	
1°	52° 53′	118° 39′	86° 21′
2°	58° 27′	132° 51′	72° 9′
3°	66° 44′	144° 1′	60° 59′
4°	76° 51′	152° 37′	52° 23′
5°	88° 9′	158° 43′	46° 17′
5° 9′	90° 0′	159° 49′	45° 11′

Hours	Arcs	Eastern Angles	Western Angles
	Goat		
Noon	54° 13′	90° 0′	
1°	56° 6′	105° 34′	74° 26′
2°	61° 22′	119° 23′	60° 37′
3°	69° 17′	130° 46′	49° 14′
4°	78° 59′	139° 30′	40° 30′
5°	90° 0′	146° 28′	33° 32′
	Water Bearer		
Noon	50° 52′	77° 30′	
1°	52° 53′	93° 39′	61° 21′
2°	58° 27′	107° 51′	47° 9′
3°	66° 44′	119° 1′	35° 59′
4°	76° 51′	127° 37′	27° 23′
5°	88° 9′	133° 43′	21° 17′
5° 9′	90° 0′	134° 49′	20° 11′
	Fishes		
Noon	42° 2′	69° 0′	
1°	44° 26′	87° 32′	50° 28′
2°	50° 58′	102° 38′	35° 22′
3°	60° 19′	113° 33′	24° 27′
4°	71° 20′	120° 56′	17° 4′
5°	83° 19′	125° 54′	12° 6′
5° 32′	90° 0′	127° 55′	10° 5′

Hours	Arcs	Eastern Angles	Western Angles
	Ram		
Noon	30° 22′	66° 9′	
1°	33° 35′	89° 50′	42° 28′
2°	41° 39′	106° 37′	25° 41′
3°	52° 25′	116° 28′	15° 50′
4°	64° 28′	122° 5′	10° 13′
5°	77° 6′	124° 39′	7° 39′
6°	90° 0′	125° 47′	6° 31′
	Bull		
Noon	18° 42′	69° 0′	
1°	23° 18′	103° 18′	34° 42′
2°	33° 30′	120° 25′	17° 35′
3°	45° 36′	127° 34′	10° 26′
4°	58° 21′	130° 10′	7° 50′
5°	71° 15′	130° 28′	7° 32′
6°	84° 7′	129° 5′	8° 55′
6° 28′	90° 0′	127° 55′	10° 5′
	Twins		
Noon	9° 52′	77° 30′	
1°	16° 45′	128° 13′	26° 47′
2°	28° 44′	141° 22′	13° 38′
3°	41° 31′	144° 26′	10° 34′
4°	54° 27′	144° 8′	10° 52′
5°	67° 17′	142° 1′	12° 59′
6°	79° 48′	138° 46′	16° 14′
6° 51′	90° 0′	134° 49′	20° 11′

13. Exposition of the Angles and Arcs by Parallels or Latitudes.—Continued

Of the Parallel through Rhodes 14½ hours lat. 36°.

Hours	Arcs	Eastern Angles	Western Angles
Crab			
Noon	12° 9′	90° 0′	
1°	17° 47′	133° 14′	46° 46′
2°	28° 22′	147° 45′	32° 15′
3°	40° 27′	151° 46′	28° 14′
4°	52° 36′	151° 52′	28° 8′
5°	64° 36′	149° 54′	30° 6′
6°	76° 16′	146° 25′	33° 35′
7°	87° 23′	141° 30′	38° 30′
7° 15′	90° 0′	140° 1′	39° 59′
Lion			
Noon	15° 30′	102° 30′	
1°	20° 20′	139° 32′	65° 28′
2°	30° 28′	155° 19′	49° 41′
3°	42° 6′	160° 37′	44° 23′
4°	54° 12′	162° 11′	42° 49′
5°	66° 17′	161° 5′	43° 55′
6°	78° 7′	158° 10′	46° 50′
7°	89° 27′	153° 39′	51° 21′
7° 4′	90° 0′	153° 36′	51° 24′
Virgin			
Noon	24° 20′	111° 0′	
1°	27° 51′	137° 38′	84° 22′
2°	36° 24′	153° 59′	68° 1′
3°	47° 14′	162° 10′	59° 50′
4°	59° 0′	165° 40′	56° 20′
5°	71° 5′	166° 34′	55° 26′
6°	83° 9′	165° 30′	56° 30′
6° 35′	90° 0′	164° 7′	57° 53′

Hours	Arcs	Eastern Angles	Western Angles
Balance			
Noon	36° 0′	113° 51′	
1°	38° 37′	133° 23′	94° 19′
2°	45° 31′	148° 23′	79° 19′
3°	55° 6′	158° 9′	69° 33′
4°	66° 9′	163° 58′	63° 44′
5°	77° 56′	166° 36′	61° 6′
6°	90° 0′	167° 51′	59° 51′
Scorpion			
Noon	47° 40′	111° 0′	
1°	49° 42′	126° 50′	95° 10′
2°	55° 26′	140° 20′	81° 40′
3°	63° 48′	150° 34′	71° 26′
4°	73° 55′	157° 51′	64° 9′
5°	85° 5′	162° 28′	59° 32′
5° 25′	90° 0′	164° 7′	57° 53′
Archer			
Noon	56° 30′	102° 30′	
1°	58° 14′	116° 39′	88° 21′
2°	63° 13′	129° 23′	75° 37′
3°	70° 41′	139° 47′	65° 13′
4°	80° 2′	147° 47′	57° 13′
4° 56′	90° 0′	153° 36′	51° 24′

Hours	Arcs	Eastern Angles	Western Angles
Goat			
Noon	59° 51′	90° 0′	
1°	61° 30′	103° 45′	76° 15′
2°	66° 12′	116° 10′	63° 50′
3°	73° 22′	126° 36′	53° 24′
4°	82° 24′	134° 56′	45° 4′
4° 45′	90° 0′	140° 1′	39° 59′
Water Bearer			
Noon	56° 30′	77° 30′	
1°	58° 14′	91° 39′	63° 21′
2°	63° 13′	104° 23′	50° 37′
3°	70° 41′	114° 47′	40° 13′
4°	80° 2′	122° 47′	32° 13′
4° 56′	90° 0′	128° 36′	26° 24′
Fishes			
Noon	47° 40′	69° 0′	
1°	49° 42′	84° 50′	53° 10′
2°	55° 26′	98° 20′	39° 40′
3°	63° 48′	108° 34′	29° 26′
4°	73° 55′	115° 51′	22° 9′
5°	85° 5′	120° 28′	17° 32′
5° 25′	90° 0′	122° 7′	15° 53′

Hours	Arcs	Eastern Angles	Western Angles
Ram			
Noon	36° 0′	66° 9′	
1°	38° 37′	85° 41′	46° 37′
2°	45° 31′	100° 47′	31° 31′
3°	55° 6′	110° 27′	21° 51′
4°	66° 9′	116° 16′	16° 2′
5°	77° 56′	118° 54′	13° 24′
6°	90° 0′	120° 9′	12° 9′
Bull			
Noon	24° 20′	69° 0′	
1°	27° 51′	95° 38′	42° 22′
2°	36° 24′	111° 59′	26° 1′
3°	47° 14′	120° 10′	17° 50′
4°	59° 0′	123° 40′	14° 20′
5°	71° 5′	124° 34′	13° 26′
6°	83° 9′	123° 30′	14° 30′
6° 35′	90° 0′	122° 7′	15° 53′
Twins			
Noon	15° 30′	77° 30′	
1°	20° 20′	114° 32′	40° 28′
2°	30° 28′	130° 19′	24° 41′
3°	42° 6′	135° 37′	19° 23′
4°	54° 12′	137° 11′	17° 49′
5°	66° 17′	136° 5′	18° 55′
6°	78° 7′	133° 10′	21° 50′
7°	89° 27′	128° 39′	26° 21′
7° 4′	90° 0′	128° 36′	26° 24′

13. Exposition of the Angles and Arcs by Parallels or Latitudes.—Continued

Of the Parallel through the Hellespont 15 hours lat. 40°56′.

Hours	Arcs	Eastern Angles	Western Angles
Crab			
Noon	17° 5′	90° 0′	
1°	21° 18′	122° 32′	57° 28′
2°	30° 17′	138° 29′	41° 31′
3°	41° 37′	144° 18′	35° 42′
4°	52° 25′	145° 38′	34° 22′
5°	63° 47′	144° 28′	35° 32′
6°	74° 48′	141° 30′	38° 30′
7°	85° 9′	137° 5′	42° 55′
7° 30′	90° 0′	134° 16′	45° 44′
Lion			
Noon	20° 26′	102° 30′	
1°	24° 5′	131° 6′	73° 54′
2°	32° 37′	147° 0′	58° 0′
3°	43° 8′	153° 50′	51° 10′
4°	54° 19′	156° 5′	48° 55′
5°	65° 36′	155° 8′	49° 52′
6°	76° 46′	153° 24′	51° 36′
7°	87° 24′	149° 6′	55° 54′
7° 16′	90° 0′	148° 6′	56° 54′
Virgin			
Noon	29° 16′	111° 0′	
1°	32° 5′	132° 30′	89° 30′
2°	39° 22′	147° 30′	74° 30′
3°	49° 3′	156° 0′	66° 0′
4°	59° 50′	160° 7′	61° 53′
5°	71° 5′	161° 24′	60° 36′
6°	82° 22′	160° 40′	61° 20′
6° 42′	90° 0′	158° 59′	63° 1′

Hours	Arcs	Eastern Angles	Western Angles
Balance			
Noon	40° 56′	113° 51′	
1°	43° 8′	129° 57′	97° 45′
2°	49° 7′	143° 38′	84° 4′
3°	57° 42′	153° 8′	74° 34′
4°	67° 50′	158° 47′	68° 55′
5°	78° 45′	161° 59′	65° 43′
6°	90° 0′	162° 55′	64° 47′
Scorpion			
Noon	52° 36′	111° 0′	
1°	54° 23′	124° 46′	97° 14′
2°	59° 25′	136° 55′	85° 5′
3°	66° 58′	146° 24′	75° 36′
4°	76° 15′	153° 10′	68° 50′
5°	86° 38′	157° 45′	64° 15′
5° 18′	90° 0′	158° 59′	63° 1′
Archer			
Noon	61° 26′	102° 30′	
1°	63° 0′	115° 5′	89° 55′
2°	67° 24′	126° 29′	78° 31′
3°	74° 13′	136° 10′	68° 50′
4°	82° 48′	143° 45′	61° 15′
4° 44′	90° 0′	148° 6′	56° 54′

Hours	Arcs	Eastern Angles	Western Angles
Goat			
Noon	64° 47′	90° 0′	
1°	66° 15′	102° 27′	77° 33′
2°	70° 30′	113° 35′	66° 25′
3°	77° 4′	122° 55′	57° 5′
4°	85° 18′	130° 58′	49° 2′
4° 30′	90° 0′	134° 16′	45° 44′
Water Bearer			
Noon	61° 26′	77° 30′	
1°	63° 0′	90° 5′	64° 55′
2°	67° 24′	101° 29′	53° 31′
3°	74° 13′	111° 10′	43° 50′
4°	82° 48′	118° 45′	36° 15′
4° 44′	90° 0′	123° 6′	31° 54′
Fishes			
Noon	52° 36′	69° 0′	
1°	54° 23′	82° 46′	55° 14′
2°	59° 25′	94° 55′	43° 5′
3°	66° 58′	104° 24′	33° 36′
4°	76° 15′	111° 10′	26° 50′
5°	86° 38′	115° 45′	22° 15′
5° 18′	90° 0′	116° 59′	21° 1′

Hours	Arcs	Eastern Angles	Western Angles
Ram			
Noon	40° 56′	66° 9′	
1°	43° 8′	82° 15′	50° 3′
2°	49° 7′	95° 56′	36° 22′
3°	57° 42′	105° 26′	26° 52′
4°	67° 50′	111° 5′	21° 13′
5°	78° 45′	114° 17′	18° 1′
6°	90° 0′	115° 13′	17° 5′
Bull			
Noon	29° 16′	69° 0′	
1°	32° 5′	90° 30′	47° 30′
2°	39° 22′	105° 30′	32°
3°	49° 3′	114° 0′	24° 0′
4°	59° 50′	118° 7′	19° 53′
5°	71° 5′	119° 24′	18° 36′
6°	82° 22′	118° 40′	19° 20′
6° 42′	90° 0′	116° 59′	21° 1′
Twins			
Noon	20° 26′	77° 30′	
1°	24° 5′	106° 6′	48° 54′
2°	32° 37′	122° 0′	33° 0′
3°	43° 8′	128° 50′	26° 10′
4°	54° 19′	131° 5′	23° 55′
5°	65° 36′	130° 8′	24° 52′
6°	76° 46′	128° 24′	26° 36′
7°	87° 24′	124° 6′	30° 54′
7° 16′	90° 0′	123° 6′	31° 54′

13. Exposition of the Angles and Arcs by Parallels or Latitudes.—Continued

Of the Parallel through the Middle of Pontus 15½ hours lat. 45°1′.

Hours	Arcs	Eastern Angles	Western Angles
Crab			
Noon	21° 10′	90° 0′	
1°	24° 32′	116° 5′	63° 55′
2°	32° 12′	131° 30′	48° 30′
3°	42° 1′	138° 17′	41° 43′
4°	52° 29′	140° 31′	39° 29′
5°	63 4′	140° 2′	39° 58′
6°	73° 24′	137° 32′	42° 28′
7°	83° 17′	133° 26′	46° 34′
7° 45′	90° 0′	129° 21′	50° 39′
Lion			
Noon	24° 31′	102° 30′	
1°	27° 29′	124° 49′	80° 11′
2°	34° 48′	140° 47′	64° 13′
3°	44° 20′	148° 5′	56° 55′
4°	54° 37′	151° 5′	53° 55′
5°	65° 15′	151° 7′	53° 53′
6°	75° 39′	149° 20′	55° 40′
7°	85° 39′	145° 39′	59° 21′
7° 28′	90° 0′	143° 25′	61° 35′
Virgin			
Noon	33° 21′	111° 0′	
1°	35° 43′	129° 15′	92° 45′
2°	42° 4′	142° 50′	79° 10′
3°	50° 46′	151° 9′	70° 51′
4°	60° 44′	155° 31′	66° 29′
5°	71° 12′	157° 3′	64° 57′
6°	81° 46′	156° 31′	65° 29′
6° 48′	90° 0′	154° 43′	67° 17′

Hours	Arcs	Eastern Angles	Western Angles
Balance			
Noon	45° 1′	113° 51′	
1°	46° 55′	128° 19′	99° 23′
2°	52° 17′	140° 26′	87° 16′
3°	60° 1′	149° 4′	78° 38′
4°	69° 19′	154° 48′	72° 54′
5°	79° 28′	157° 55′	69° 47′
6°	90° 0′	158° 50′	68° 59′
Scorpion			
Noon	56° 41′	111° 0′	
1°	58° 19′	123° 31′	98° 29′
2°	62° 49′	134° 16′	87° 44′
3°	69° 42′	143° 12′	78° 48′
4°	78° 16′	149° 31′	72° 29′
5°	87° 56′	154° 6′	67° 54′
5° 12′	90° 0′	154° 43′	67° 17′
Archer			
Noon	65° 31′	102° 30′	
1°	66° 55′	113° 50′	91° 10′
2°	70° 58′	124° 21′	80° 39′
3°	77° 14′	133° 19′	71° 41′
4°	85° 10′	140° 20′	64° 40′
4° 32′	90° 0′	143° 25′	61° 35′

Hours	Arcs	Eastern Angles	Western Angles
Goat			
Noon	68° 52′	90° 0′	
1°	70° 14′	101° 11′	78° 49′
2°	74° 5′	111° 30′	68° 30′
3°	80° 6′	120° 29′	59° 31′
4°	87° 42′	128° 13′	51° 47′
4° 15′	90° 0′	129° 21′	50° 39′
Water Bearer			
Noon	65° 31′	77° 30′	
1°	66° 55′	88° 50′	66° 10′
2°	70° 58′	99° 21′	55° 39′
3°	77° 14′	108° 19′	46° 41′
4°	85° 10′	115° 20′	39° 40′
4° 32′	90° 0′	118° 25′	36° 35′
Fishes			
Noon	56° 41′	69° 0′	
1°	58° 19′	81° 31′	56° 29′
2°	62° 49′	92° 16′	45° 44′
3°	69° 42′	101° 12′	36° 48′
4°	78° 16′	107° 31′	30° 29′
5°	87° 56′	112° 6′	25° 54′
5° 12′	90° 0′	112° 43′	25° 17′

Hours	Arcs	Eastern Angles	Western Angles
Ram			
Noon	45° 1′	66° 9′	
1°	46° 55′	80° 37′	51° 41′
2°	52° 17′	92° 44′	39° 34′
3°	60° 1′	101° 22′	30° 56′
4°	69° 19′	107° 6′	25° 12′
5°	79° 28′	110° 13′	22° 5′
6°	90° 0′	111° 8′	21° 10′
Bull			
Noon	33° 21′	69° 0′	
1°	35° 43′	87° 15′	50° 45′
2°	42° 4′	100° 50′	37° 10′
3°	50° 46′	109° 9′	28° 51′
4°	60° 44′	113° 31′	24° 29′
5°	71° 12′	115° 3′	22° 57′
6°	81° 46′	114° 31′	23° 29′
6° 48′	90° 0′	112° 43′	25° 17′
Twins			
Noon	24° 31′	77° 30′	
1°	27° 29′	99° 49′	55° 11′
2°	34° 48′	115° 47′	39° 13′
3°	44° 20′	123° 5′	31° 55′
4°	54° 37′	126° 5′	28° 55′
5°	65° 15′	126° 7′	28° 53′
6°	75° 39′	124° 20′	30° 40′
7°	85° 39′	120° 39′	34° 21′
7° 28′	90° 0′	118° 25′	36° 35′

13. Exposition of the Angles and Arcs by Parallels or Latitudes.—Continued

Of the Parallel through the Borysthenes 16 hours lat. 48°32′.

Hours	Arcs	Eastern Angles	Western Angles
Crab			
Noon	24° 41′	90° 0′	
1°	27° 30′	111° 44′	68° 16′
2°	34° 9′	126° 7′	53° 53′
3°	43° 2′	133° 18′	46° 42′
4°	52° 44′	136° 6′	43° 54′
5°	62° 40′	136° 4′	43° 56′
6°	72° 24′	134° 0′	46° 0′
7°	81° 38′	130° 16′	49° 44′
8°	90° 0′	124° 58′	55° 2′
Lion			
Noon	28° 2′	102° 30′	
1°	30° 32′	122° 9′	82° 51′
2°	36° 55′	135° 54′	69° 6′
3°	45° 30′	143° 28′	61° 32′
4°	55° 3′	146° 50′	58° 10′
5°	64° 59′	147° 19′	57° 41′
6°	74° 47′	145° 46′	59° 14′
7°	84° 10′	142° 27′	62° 33′
7° 40′	90° 0′	139° 20′	65° 40′
Virgin			
Noon	36° 52′	111° 0′	
1°	38° 56′	126° 45′	95° 15′
2°	44° 31′	139° 7′	82° 53′
3°	52° 25′	147° 9′	74° 51′
4°	61° 35′	151° 36′	70° 24′
5°	71° 22′	153° 23′	68° 37′
6°	81° 17′	152° 58′	69° 2′
6° 54′	90° 0′	151° 22′	70° 38′

Hours	Arcs	Eastern Angles	Western Angles
Balance			
Noon	48° 32′	113° 51′	
1°	50° 21′	126° 30′	101° 12′
2°	54° 59′	137° 40′	90° 2′
3°	62° 5′	145° 46′	81° 56′
4°	70° 41′	151° 18′	76° 24′
5°	80° 8′	154° 23′	73° 19′
6°	90° 0′	155° 19′	72° 23′
Scorpion			
Noon	60° 12′	111° 0′	
1°	61° 38′	122° 5′	99° 55′
2°	65° 36′	132° 10′	89° 50′
3°	72° 5′	140° 26′	81° 34′
4°	80° 3′	146° 28′	75° 32′
5°	89° 3′	151° 2′	70° 58′
5° 6′	90° 0′	151° 22′	70° 38′
Archer			
Noon	69° 2′	102° 30′	
1°	70° 20′	112° 49′	92° 11′
2°	74° 2′	122° 31′	82° 29′
3°	79° 48′	130° 49′	74° 11′
4°	87° 14′	137° 25′	67° 35′
4° 20′	90° 0′	139° 20′	65° 40′

Hours	Arcs	Eastern Angles	Western Angles
Goat			
Noon	72° 23′	90° 0′	
1°	73° 38′	100° 15′	79° 45′
2°	77° 10′	109° 47′	70° 13′
3°	82° 44′	118° 3′	61° 57′
4°	90° 0′	124° 58′	55° 2′
Water Bearer			
Noon	69° 2′	77° 30′	
1°	70° 20′	87° 49′	67° 11′
2°	74° 2′	97° 31′	57° 29′
3°	79° 48′	105° 49′	49° 11′
4°	87° 14′	112° 25′	42° 35′
4° 20′	90° 0′	114° 20′	40° 40′
Fishes			
Noon	60° 12′	69° 0′	
1°	61° 38′	80° 5′	57° 55′
2°	65° 36′	90° 16′	47° 44′
3°	72° 5′	98° 26′	39° 34′
4°	80° 3′	104° 28′	33° 32′
5°	89° 3′	109° 2′	28° 58′
5° 6′	90° 0′	109° 22′	28° 38′

Hours	Arcs	Eastern Angles	Western Angles
Ram			
Noon	48° 32′	66° 9′	
1°	50° 21′	78° 48′	53° 30′
2°	54° 59′	89° 58′	42° 20′
3°	62° 5′	98° 4′	34° 14′
4°	70° 41′	103° 36′	28° 42′
5°	80° 8′	106° 41′	25° 37′
6°	90° 0′	107° 37′	24° 41′
Bull			
Noon	36° 52′	69° 0′	
1°	38° 56′	84° 45′	53° 15′
2°	44° 31′	97° 7′	40° 53′
3°	52° 25′	105° 9′	32° 51′
4°	61° 35′	109° 36′	28° 24′
5°	71° 22′	111° 23′	26° 37′
6°	81° 17′	110° 58′	27° 2′
6° 54′	90° 0′	109° 22′	28° 38′
Twins			
Noon	28° 2′	77° 30′	
1°	30° 32′	97° 9′	57° 51′
2°	36° 55′	110° 54′	44° 6′
3°	45° 30′	118° 28′	36° 32′
4°	55° 3′	121° 50′	33° 10′
5°	64° 59′	122° 19′	32° 41′
6°	74° 47′	120° 46′	34° 14′
7°	84° 10′	117° 27′	37° 33′
7° 40′	90° 0′	114° 20′	40° 40′

And now, although the business of the angles has been worked out and there is left to add to these things now established a research into the positions of the principal cities of each province according to their longitudes and latitudes for the calculations of the appearance observed in them, we shall do this exposition as a special geographical treatise by itself; and we shall follow the writings of those who have especially worked in this kind of thing, finding out how many degrees each city is from the equator along the meridian drawn through that city, and how many degrees east or west of the meridian of Alexandria each city is. For we arrange the times of the other places with reference to this meridian.

Now we have thought it pertinent to add this much about these positions: whenever we wish to know, at a given hour in one of these places, what hour it is in some other if their meridians are different, we must take the number of degrees they differ from each other along the equator, and, according as the one sought is east or west of the one given, increase or decrease the given hour by that number of degrees to get the hour defined at the same time in the place sought.

BOOK THREE

Now that we have methodically gone through, in all that has been put together up until now, those things which have first to be completely grasped mathematically concerning the heavens and the earth, and also concerning the obliquity of the sun's path through the middle of the zodiac [along the ecliptic] and its particular incidents in the right sphere and the oblique sphere for each latitude, we consider it proper after all this to treat of the sun and moon and to take account of the incidents concerning their movements, since without a prior understanding of them none of the appearances having to do with the stars can be discovered. And we find the treatise on the sun's movement advanced first, for again, without this, matters concerning the moon could not be grasped in detail.

1. On the Year's Magnitude

Since finding the year's time-length is the first of all the things demonstrated concerning the sun, we shall first learn from the treatises of the ancients the disagreements and difficulties concerning their statement on this, and especially from that of Hipparchus, a diligent and truth-loving man. For he is brought to a difficulty of this kind especially by the fact that, for the apparent returns of the sun with respect to the tropics and equinoxes, the length of the year is found to be less than 365¼ days, but for its returns observed with respect to the fixed stars it is found to be more. And from that he conjectures that the sphere of the fixed stars also has a very slow movement, and like that of the planets is in the direction contrary to that of the prime movement which revolves the circle that passes through the poles of the equator and the ecliptic. And we shall show this is so and how it comes about, in the chapters on the fixed stars. For matters concerning them could not be seen in their entirety without a prior understanding of the sun and moon.

But for the present research we believe that we must consider the length of the year looking only to the sun's return with respect to itself—that is, with respect to the oblique circle made by it [the ecliptic]—and that we must define the length of the year as the time in which the sun proceeds continuously from some fixed point of this circle back to the same point, supposing as we do that the only proper principles of this return are the points of this circle determined by the tropics and equinoxes. For, if we tell the story mathematically, we shall not find a more proper return than that which carries the sun through the same configuration both in space and time, whether it be considered with respect to the horizons or the meridian or the magnitude of the solar days; and we shall find no other principles of the ecliptic except those accidentally defined by the tropic and equinoctial points. And if one examines the subject more physically, he will not find a more reasonable return than that which brings the sun from a like to a like weather-condition and from the same season to the same season; nor will

he find other principles than those by which the seasons are the most completely distinguished. And the return of the sun considered with respect to the fixed stars seems quite inept for this reason particularly that their sphere is observed to make an ordered movement contrary to that of the heavens. For, with things this way, nothing will keep one from saying that the length of the sun's year is the time it takes the sun to overtake Saturn, for instance, or some other star. And so there would be many different years.

And so we think it proper to consider such a period of time the sun's year which is found, by as many observations as possible taken over a rather long interval, from one tropic or equinox back to the same.

But since a suspected inequality in the periods of even this return, suspected through continuous and successive observations, more or less worried Hipparchus, we shall try briefly to show how this is not at all disturbing, since we are sure by the continuous instrumental observations we have made of the tropics and equinoxes that these periods are not unequal. For we find them differing by no appreciable amount from the additional quarter day, but at times by about as much as could be attributed to the error due to the construction or position of the instruments. For we guess from what Hipparchus reports that the error with regards to inequality belongs rather to the observations. For after he has first set out, in his treatise *On the Precession of the Tropic and Equinoctial Points*, the summer and winter tropics seeming to him to have been accurately observed and in order, he himself agrees that there is not such a difference between them as to recognize for this reason an inequality in the year. For he adds this: "Then it is clear from these observations that the differences in the years have been altogether small. But in the case of the tropics I do not despair of Archimedes' and my having made an error of as much as a quarter day both in observation and calculation. But the irregularity of the years can be accurately perceived from observations made on the bronze ring situated in what is called the Square Hall in Alexandria which is supposed to indicate the equinoctial day as that on which its concave surface begins to be lighted up from the opposite side."[1]

Then he lists, first, the dates of those autumn equinoxes which have been very accurately observed. One fell in the year 17 of the Third Callippic Period, Mesore 30, at the setting of the sun; and another three years after, year 20, on the first of the intercalated days in the morning, which should have been noon, so that there was a disagreement of a quarter day. And another a year after, year 21, at the sixth hour, which agreed with the preceding observation. And another

[1]The inequality of the year is here being judged in terms of the number of solar days. This might seem arbitrary, since the days might be unequal. And indeed it will be seen later on in this Book that the solar days are considered to be unequal. But the theory of their irregularity will be such that their inequality is exactly symmetrical within each year. But, of course, to judge that the inequality is symmetrical, it is necessary to fall back on something else supposed equal. The Greeks usually supposed it to be the stellar day, the time it takes a fixed star to go from a meridian back to the same meridian again. This is practically true for Ptolemy, but not absolutely so, because of the precession of the equinoxes. For, as we shall see in Book VII, the fixed stars move from west to east about the poles of the ecliptic nearly a half a degree in a hundred years. This introduces an irregularity in the length of the stellar day exactly parallel to one of the irregularities in the length of the solar day described in Chapter 9 of this Book, but one so small in magnitude that it could not be perceived from day to day, or even from year to year.

eleven years after, year 32, on the third of the intercalated days at midnight before the fourth. And it should have been in the morning, so that there was again a disagreement of a quarter day. And another a year after, year 33, on the fourth of the intercalated days in the morning, which agreed with the preceding observation. And another three years after, year 36, on the fourth of the intercalated days in the evening. And it should have been at midnight so that there was again a disagreement of a quarter day.

And next he lists those spring equinoxes likewise accurately observed. One fell in the year 32 of the Third Callippic Period, Mechir 27, in the morning. But he adds: "The ring in Alexandria was also lighted up equally on both sides at the fifth hour, so that the same equinox differently observed differed by near-nearly five hours." And he says the equinoxes following, up to the year 37, agreed with the addition of a quarter day. And eleven years after, year 43, Mechir 29-30, just after midnight, he says, there was a spring equinox, which also agreed with the observation in the year 32; and also, he says, with the observations in the following years up to the year 50. For in that year it fell on Phamenoth 1 at sunset, within very nearly $1\frac{3}{4}$ days of that of the year 43, which is also proportional to the 7 intervening years. And so in these observations there was no perceptible difference although it is possible for there to be an error of as much as a quarter day, not only as regards the tropic observations, but also the equinoctial. For even if the position or discrimination of the instruments is inaccurate by only $\frac{1}{3600}$ of the circle through the poles of the equator, at the equinoctial intersections the sun makes up for this advance in latitude by shifting $\frac{1}{4}°$ in longitude along the ecliptic, so that there could be an inconsistency of very nearly a quarter day. And there could be a greater error still in the case of instruments not placed permanently and not then corrected for each observation, but which have been attached for some time to the pavement with a view to keeping a steady position for a good while, where yet some long unnoticed shift has been made in them. And anyone can see an example of this in the bronze rings in the palestra of our city, which are supposed to be in the plane of the equator. For in making observations we find such a distortion in their placement, and especially in the case of the largest and oldest, that at times their concave surfaces twice suffer a shift in lighting at the same equinoxes.

But certainly from such things Hipparchus himself does not think there is anything solid to support a suspicion of inequality in the lengths of years. But he says he finds by calculating from certain eclipses of the moon that the irregularity of the years, on the average, does not embrace a difference greater than three quarters of a day. And this would merit some attention if it were so and not evidently belied by the reasons he offers. For he calculates, by the lunar eclipses observed near certain fixed stars, by how much in each case the star Spica precedes the autumn equinox. And in this way he thinks he finds that once it was at its greatest distance of $6\frac{1}{2}°$, for the time he observed, and once at its least distance of $5\frac{1}{4}°$. And he infers from this fact that, since it is not possible for Spica to move so far in such a short time, it is likely that the sun, from which Hipparchus examines the positions of the fixed stars, does not always make its return in an equal time.

But he has overlooked the point that, since this calculation cannot proceed without laying down the sun's position at the eclipse, he, taking for this purpose

in each case the tropics and equinoxes accurately observed by himself in those very years, thereby immediately makes clear that in comparing the years there is no difference beyond the addition of the quarter day.

For example, from the observation of the eclipse in the year 32 of the Third Callippic Period he thinks he finds Spica preceding the autumn equinox by $6\frac{1}{2}°$; but from the eclipse in the year 43 of the same period, by $5\frac{1}{4}°$. And likewise setting beside these calculations the spring equinoxes accurately observed in those same years—so that by means of these he may get the sun's positions at the middle of the eclipses, and from these the moon's positions, and from the moon's those of the stars—he says that the spring equinox of the year 32 fell on Mechir 27 in the morning, and that of the year 43 on Mechir 29-30 after midnight, nearly $2\frac{3}{4}$ days later than that of the year 32, which is just equal to the quarter day added for each of the 11 intervening years. If, then, the sun has made its return to these equinoxes in neither more nor less time than the additional quarter day, and if it is not possible for the star Spica to have moved $1\frac{1}{4}°$ in so few years, how could it be otherwise than absurd to take the results calculated from the principles assumed as an accusation against the very principles combined to produce them, as if one were unable to saddle anything else with the cause of this excessive movement of Spica except the equinoxes assumed at the same time to have been accurately and inaccurately observed, although there were many things which could have introduced such an error?[1] For it would seem much more possible that the distances of the moon at the eclipses with respect to the nearest fixed stars had been estimated rather roughly; or that the calculations either of the moon's parallaxes for the sighting of its apparent positions or of the sun's movement from the equinoxes to the middle of the eclipses had been effected neither truly nor accurately.

And I think Hipparchus himself recognized there is nothing convincing in such things as far as imposing a second anomaly on the sun, but I think he only wishes for the love of the truth not to keep back anything which could in anyway bring one to suspect. And so he himself used hypotheses concerning the sun and moon with just one anomaly belonging to the sun, an anomaly which is redeemed in the year considered with respect to the tropics and equinoxes. And in supposing these revolutions to be equal in time we do not observe the appearances at the eclipses differing in any perceptible way from the calculations based on these hypotheses. For it would be quite perceptible if a correction for the inequality of the year were not made at the same time, even if it were only a difference of one degree, or very nearly two standard hours.

From all these things, and from the times of the returns which we ourselves have gotten from the consecutive passages of the sun observed by us, we do not find the magnitude of the year unequal if it is considered with respect to some one thing and not one time with respect to the tropic and equinoctial points and another with respect to the fixed stars. Nor do we find any more proper period of return than that which carries the sun from one tropic or equinoctial point, or any other point on the ecliptic, back to that same point. And we do think it entirely proper to explain the appearances by the simplest hypotheses

[1] Hipparchus' logic, contrary to what Ptolemy says, seems impeccable insofar as he is saying that assumptions which lead to their contradictories are false. But the reasoning must be right, and Ptolemy is also suggesting that there were many steps in between which might have been false.

possible, so long as nothing perceptible appears contrary to this deduction.

And therefore it has become clear to us from what Hipparchus has shown that the length of the year observed with respect to the tropics and equinoxes is less than 365¼ days, but it would not be possible to find out with very great certainty by how much it is less since the increase of a quarter day remains perceptibly unchanged for many years because of the very small difference. And so the extra amount can be perceived only when it is found added up together over a longer period of time. And it must be divided among the intervening years of the interval and it must be observed for a greater and a smaller number of years than this same interval. And the period of return will be gotten the more accurately the longer the time between the observations compared. And this is the case not only with this period of return, but with all of them. For the error resulting from the weakness of the observations themselves, even if they are managed accurately, is small and very nearly the same as far as the senses are concerned both for appearances considered over a long time and for those considered over a short time. And this error of observation, when it is distributed over fewer years, makes the error in the length of the year greater and also in multiples of it over a longer period of time; and it makes the error in the length of the year smaller when distributed over a greater number of years.

And therefore it is properly thought sufficient if, when we consider how much the time between us and the old yet accurate observations can help in the approximation of the supposed periods of revolution, we try to introduce them with the others and do not willingly forego the proper verification, and if we suppose the establishing of dates for a whole long age or for some great multiple of time between observations is the work for another's love of wisdom and truth. Because of their age, then, the summer tropics observed by the pupils of Meton and Euctemon and those observed after them by the pupils of Aristarchus should be compared with those observed by us. But because the observations of the tropics are generally hard to determine and, moreover, because the observations handed down by these people were taken more or less in the rough, as Hipparchus also seems to have thought, we pass them over. We have used for this comparison the observation of the equinoxes, and, because of their accuracy, especially those given Hipparchus' approval as having been most certainly taken by him, and those most carefully observed by ourselves with the instruments for such purposes described at the beginning of this treatise. And from these we find that, in very nearly 300 years, the tropics and equinoxes fall one day sooner than the quarter-day addition to 365 days allows.

For in the year 32 of the Third Callippic Period Hipparchus singles out especially the autumn equinox as most accurately observed, and he says he calculates it to have fallen at midnight between the third and fourth of the intercalated days. And this is the year 178 after the death of Alexander. And 285 years after in the year 3 of Antonine (which is 463 years after the death of Alexander) we observed, again most correctly, the autumn equinox as having fallen on Athyr 9 about one hour after sunrise. Therefore the return added on in all the 285 Egyptian years, that is those of 365 days each, all told $70+\frac{1}{4}+\frac{1}{20}$ days instead of the 71¼ days due these years by the regular quarter-day addition. And so the return fell sooner by very nearly 1 day less $\frac{1}{20}$ than the regular quarter-day addition allowed.

And likewise Hipparchus again says the spring equinox in the same year 32 of the Third Callippic Period was very accurately observed to have fallen on Mechir 27 in the morning. And this is 178 years after the death of Alexander. And likewise 285 years later (463 years after the death of Alexander) we find the spring equinox has fallen on Pactom 7 very nearly one hour after noon, so that the period reached the aforesaid $70+\frac{1}{4}+\frac{1}{20}$ days very nearly, instead of the regular quarter-day addition to the 285 years of $71\frac{1}{4}$ days. Therefore the return of the spring equinox fell sooner by 1 day less $\frac{1}{20}$ than the regular quarter-day addition allowed. And so, since 300 years to 285 years, and 1 day to 1 day less $\frac{1}{20}$, are the same ratio, it is inferred that in very nearly 300 years the sun's return with respect to the equinoctial points is sooner by 1 day than the regular quarter-day addition allows.

Even if, because of its antiquity, we compare the summer tropic more or less roughly recorded by the pupils of Meton and Euctemon with that calculated by us, we shall find the same thing. For it is recorded to have taken place Athenianwise in the Magistracy of Apseudes, Egyptianwise Phamenoth 21 in the morning, and we, in the same 463rd year after the death of Alexander, very carefully calculated it to have fallen on Mesore 11-12 two hours after midnight. And from the summer tropic recorded under Apseudes to that observed by the pupils of Aristarchus in the year 50 of the First Callippic Period, as Hipparchus also says, is 152 years. And from this year 50 (which was the 44th year after the death of Alexander) to the 463rd year [after the death of Alexander], the year of our observation, is 419 years. Therefore, in the intervening 571 years of the whole interval, if the summer tropic observed by the pupils of Euctemon fell at the beginning of Phamenoth 21, very nearly $140+\frac{1}{2}+\frac{1}{3}$ days have been added to the complete Egyptian years instead of the regular quarter-day addition to the 571 years of $142+\frac{1}{2}+\frac{1}{4}$ days; so that this return fell sooner by 2 days less $\frac{1}{12}$ than the regular quarter-day addition allowed. It is therefore evident that in 600 complete Egyptian years, the lengths of the solar years anticipate the regular quarter-day addition by nearly 2 whole days.

And with many other observations we find this same thing happening, and we see Hipparchus several times agreeing to this. For in his treatise *On the Length of the Year*, when he compares the summer tropic observed by Aristarchus at the end of the year 50 of the First Callippic Period with the one taken again very accurately by himself at the end of the year 43 of the Third Callippic Period, he says as follows: "It is evident, therefore, that in 145 years the tropic has fallen sooner by half a day and night together than the regular quarter-day addition allows." And again in his treatise *On Intercalated Months and Days*, saying first that the year, for the pupils of Meton and Euctemon, contains $365+\frac{1}{4}+\frac{1}{76}$ days and according to Callippus only $365\frac{1}{4}$ days, he adds this: "And we have found as many whole months in the 19 years as they, but we have found that the year comes to $\frac{1}{300}$ day less than the regularly added quarter day, so that in 300 Egyptian years the sum of the solar years is 5 days less than Meton's and only 1 day less than Callippus'." And also in summarizing his views by citing his own works, he says as follows: "And I have also treated the question of the length of the year in a book in which I show that the solar year (that is, the time in which the sun goes from a tropic back to the same tropic or from an equinox back to the same equinox) contains 365 days and less than $\frac{1}{4}$ day by $\frac{1}{300}$ of a day and night, and not as the mathematicians think $365\frac{1}{4}$ days."

I think, then, it has been made clear that the appearances observed up to this time concerning the magnitude of the year agree with the size just assigned the return of the tropic and equinoctial points by a concurrence of present appearances with earlier ones. And since all this is so, if we distribute the one day over the 300 years, there falls to each year 12″ of a day. And if we subtract this from the 365 days 15′ where the quarter day has been added, we shall have the length of the year we are looking for—that is, 365 days 14′ 48″. And this number of days can be taken by us as the nearest approximation possible from the observations we have at present.

And as regards the scrutiny of the movements of the sun and the other planets in their particularities which is best furnished ready to hand and all set out by the orderly construction of tables, we believe it is the necessary purpose and aim of the mathematician to show forth all the appearances of the heavens as products of regular and circular motions. And it is incumbent upon him to construct such tables as, proper and consequent upon this purpose, separate out the particular regular[1] motions from the anomaly which seems to result from the hypotheses of circles, and show forth their apparent movements as a combination and union of all together. In order, then, that we may get this sort of thing in more serviceable form for the demonstration under consideration, we shall set out the regular movements of the sun in their particularities in this way.

For since a return has been proved to be 365 days 14′48″, if we divide these into the 360° of one circle, we shall have the sun's mean daily movement along the ecliptic as approximately $0°59^{i}8^{ii}17^{iii}13^{iv}12^{v}31^{vi}$;[2] for it will suffice to carry out the fractions to this power of sixtieths. And again, taking $\frac{1}{24}$ of the daily movement along the ecliptic, we shall have for the hourly movement approximately $0°2^{i}27^{ii}50^{iii}43^{iv}3^{v}1^{vi}$. Likewise multiplying the daily movement by the 30 days of a month, we shall have the mean monthly movement of $29°34^{i}8^{ii}36^{iii}36^{iv}15^{v}30^{vi}$; and multiplying by the 365 days of an Egyptian year we shall have the mean yearly movement of $359°45^{i}24^{ii}45^{iii}21^{iv}8^{v}35^{vi}$. Again multiplying the mean yearly movement by 18 years, because of the symmetry which will appear in the construction of the tables, and subtracting the whole circles, we shall have the surplus for the 18-year period, that is $355°37^{i}25^{ii}36^{iii}20^{iv}34^{v}30^{vi}$.

We have accordingly drawn up three tables of the regular movement of the sun, one in forty-five rows and the others in two parts. The first table contains the mean movements for the 18-year periods; the second table contains first the movements for the Egyptian years, and then for hours; the third, first the movements for months and, under that, for days. The numbers designating the time are set out in the first columns, and in the next columns the degrees, minutes, etc., are put beside them according to the proper combinations of each. And the tables are as follows:

[1]The word "regular" is here used as a translation of the Greek word ὁμαλός. On the other hand ἀνομαλία, its privative, is translated by the technical "anomaly" instead of by the more obvious "irregularity." There will be times, however, when "irregularity" is used. The Greek word ὁμαλός has three meanings, all significant in an astronomical context: (1) regular, (2) uniform, (3) mean or average. It is evident that "regular" and "uniform" are here synonymous. But also the regular movement of the sun is computed as the average or mean movement of the sun for the interval of a solar year.

[2]The superscripts indicate the powers of the sixtieths in the denominator. Thus in ordinary fractions this would be written $\frac{59}{60} + \frac{8}{60^2} + \frac{17}{60^3} + \frac{13}{60^4}$, etc.

2. Table of the Sun's Regular Movement

Distance from the apogee 265°15′; mean epoch 0°45′ within the Fishes

18-year period	*Degrees*	I	II	III	IV	V	VI	*18-year period*	*Degrees*	I	II	III	IV	V	VI
18	355	37	25	36	20	34	30	450	250	35	40	8	34	22	30
36	351	14	51	12	41	9	0	468	246	13	5	44	54	57	0
54	346	52	16	49	1	43	30	486	241	50	31	21	15	31	30
72	342	29	42	25	22	18	0	504	237	27	56	57	36	6	0
90	338	7	8	1	42	52	30	522	233	5	22	33	56	40	30
108	333	44	33	38	3	27	0	540	228	42	48	10	17	15	0
126	329	21	59	14	24	1	30	558	224	20	13	46	37	49	30
144	324	59	24	50	44	36	0	576	219	57	39	22	58	24	0
162	320	36	50	27	5	10	30	594	215	35	4	59	18	58	30
180	316	14	16	3	25	45	0	612	211	12	30	35	39	33	0
198	311	51	41	39	46	19	30	630	206	49	56	12	0	7	30
216	307	29	7	16	6	54	0	648	202	27	21	48	20	42	0
234	303	6	32	52	27	28	30	666	198	4	47	24	41	16	30
252	298	43	58	28	48	3	0	684	193	42	13	1	1	51	0
270	294	21	24	5	8	37	30	702	189	19	38	37	22	25	30
288	289	58	49	41	29	12	0	720	184	57	4	13	43	0	0
306	285	36	15	17	49	46	30	738	180	34	29	50	3	34	30
324	281	13	40	54	10	21	0	756	176	11	55	26	24	9	0
342	276	51	6	30	30	55	30	774	171	49	21	2	44	43	30
360	272	28	32	6	51	30	0	792	167	26	46	39	5	18	0
378	268	5	57	43	12	4	30	810	163	4	12	15	25	52	30
396	263	43	23	19	32	39	0								
414	259	20	48	55	53	13	30								
432	254	58	14	32	13	48	0								

2. Table of the Sun's Regular Movement—Continued

Distance from the apogee 265°15′; mean epoch 0°45′ within the Fishes

Single years	*Degrees*	I	II	III	IV	V	VI	*Single years*	*Degrees*	I	II	III	IV	V	VI
1	359	45	24	45	21	8	35	10	357	34	7	33	31	25	50
2	359	30	49	30	42	17	10	11	357	19	32	18	52	34	25
3	359	16	14	16	3	25	45	12	357	4	57	4	13	43	0
4	359	1	39	1	24	34	20	13	356	50	21	49	34	51	35
5	358	47	3	46	45	42	55	14	356	35	46	34	56	0	10
6	358	32	28	32	6	51	30	15	356	21	11	20	17	8	45
7	358	17	53	17	28	0	5	16	356	6	36	5	38	17	20
8	358	3	18	2	49	8	40	17	355	52	0	50	59	25	55
9	357	48	42	48	10	17	15	18	355	37	25	36	20	34	30

Hours	*Degrees*	I	II	III	IV	V	VI	*Hours*	*Degrees*	I	II	III	IV	V	VI
1	0	2	27	50	43	3	1	13	0	32	1	59	19	39	16
2	0	4	55	41	26	6	2	14	0	34	29	50	2	42	18
3	0	7	23	32	9	9	3	15	0	36	57	40	45	45	19
4	0	9	51	22	52	12	5	16	0	39	25	31	28	48	20
5	0	12	19	13	35	15	6	17	0	41	53	22	11	51	21
6	0	14	47	4	18	18	7	18	0	44	21	12	54	54	23
7	0	17	14	55	1	21	9	19	0	46	49	3	37	57	24
8	0	19	42	45	44	24	10	20	0	49	16	54	21	0	25
9	0	22	10	36	27	27	11	21	0	51	44	45	4	3	27
10	0	24	38	27	10	30	12	22	0	54	12	35	47	6	28
11	0	27	6	17	53	33	14	23	0	56	40	26	30	9	29
12	0	29	34	8	36	36	15	24	0	59	8	17	13	12	31

The surplus of the distance from the apogee of the sun 5°30′ within the Twins to its mean epoch in the year I of Nabonassar 0°45′ within the Fishes is 265°15′.

2. Table of the Sun's Regular Movement—Continued

Distance from the apogee 265°15′; mean epoch 0°45′ within the Fishes

Egyptian months	*Degrees*	I	II	III	IV	V	VI	*Egyptian months*	*Degrees*	I	II	III	IV	V	VI
30	29	34	8	36	36	15	30	210	206	59	0	16	13	48	30
60	59	8	17	13	12	31	0	240	236	33	8	52	50	4	0
90	88	42	25	49	48	46	30	270	266	7	17	29	26	19	30
120	118	16	34	26	25	2	0	300	295	41	26	6	2	35	0
150	147	50	43	3	1	17	30	330	325	15	34	42	38	50	30
180	177	24	51	39	37	33	0	360	354	49	43	19	15	6	0
Days	*Degrees*	I	II	III	IV	V	VI	*Days*	*Degrees*	I	II	III	IV	V	VI
1	0	59	8	17	13	12	31	16	15	46	12	35	31	20	16
2	1	58	16	34	26	25	2	17	16	45	20	52	44	32	47
3	2	57	24	51	39	37	33	18	17	44	29	9	57	45	18
4	3	56	33	8	52	50	4	19	18	43	37	27	10	57	49
5	4	55	41	26	6	2	35	20	19	42	45	44	24	10	20
6	5	54	49	43	19	15	6	21	20	41	54	1	37	22	51
7	6	53	58	0	32	27	37	22	21	41	2	18	50	35	22
8	7	53	6	17	45	40	8	23	22	40	10	35	3	47	53
9	8	52	14	34	58	52	39	24	23	39	18	53	17	0	24
10	9	51	22	52	12	5	10	25	24	38	27	10	30	12	55
11	10	50	31	9	25	17	41	26	25	37	35	27	43	25	26
12	11	49	39	26	38	30	12	27	26	36	43	44	56	37	57
13	12	48	47	43	51	42	43	28	27	35	52	2	9	50	28
14	13	47	56	1	4	55	14	29	28	35	0	19	23	2	59
15	14	47	4	18	18	7	45	30	29	34	8	36	36	15	30

3. On the Hypotheses concerning Regular and Circular Movement

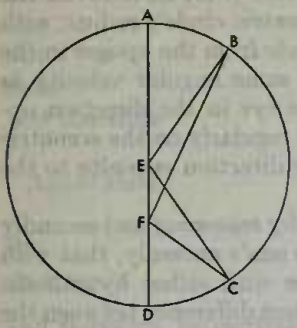
Since the next thing is to explain the apparent irregularity of the sun, it is first necessary to assume in general that the motions of the planets in the direction contrary to the movement of the heavens are all regular and circular by nature, like the movement of the universe in the other direction. That is, the straight lines, conceived as revolving the stars or their circles, cut off in equal times on absolutely all circumferences equal angles at the centres of each;[1] and their apparent irregularities result from the positions and arrangements of the circles on their spheres through which they produce these movements, but no departure from their unchangeableness has really occurred in their nature in regard to the supposed disorder of their appearances.

But the cause of this irregular appearance can be accounted for by as many as two primary simple hypotheses. For if their movement is considered with respect to a circle in the plane of the ecliptic concentric with the cosmos so that our eye is the centre, then it is necessary to suppose that they make their regular

[1]This principle of celestial mechanics will be considerably broadened in the case of the moon and the other five planets as treated in Books V, IX, and X.

movements either along circles not concentric with the cosmos, or along concentric circles; not with these simply, but with other circles borne upon them called epicycles. For according to either hypothesis it will appear possible for the planets seemingly to pass, in equal periods of time, through unequal arcs of the ecliptic circle which is concentric with the cosmos.

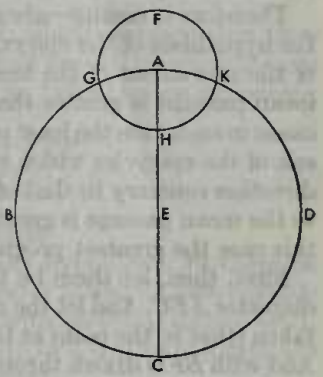
For if, in the case of the hypothesis of eccentricity, we conceive the eccentric circle *ABCD* on which the star moves regularly, with *E* as center and with diameter *AED*, and the point *F* on it as your eye so that the point *A* becomes the apogee and the point *D* the perigee; and if, cutting off equal arcs *AB* and *DC*, we join *BE*, *BF*, *CE*, and *CF*, then it will be evident that the star moving through each of the arcs *AB* and *CD* in an equal period of time will seem to have passed through unequal arcs on the circle described around *F* as a centre. For since



$$\text{angle } BEA = \text{angle } CED,$$

therefore angle *BFA* is less than either of them, and angle *CFD* greater [Eucl. I, 16].

And if in the hypothesis of the epicycle we conceive the circle *ABCD* concentric with the ecliptic with centre *E* and diameter *AEC*, and the epicycle *FGHK* borne on it on which the star moves, with its centre at *A*, then it will be immediately evident also that as the epicycle passes regularly along the circle *ABCD*, from *A* to *B* for example, and the star along the epicycle, the star will appear indifferently to be at *A* the centre of the epicycle when it is at *F* or *H;* but when it is at other points, it will not. But having come to *G*, for instance, it will seem to have produced a movement greater than the regular movement by the arc *AG;* and having come to *K*, likewise less by the arc *AK*.



Then with the hypothesis of eccentricity it is always the case that the least movement belongs to the apogee and the greatest movement to the perigee, since angle *AFB* is always less than angle *DFC*. But both cases can come about with the hypothesis of the epicycle. For when the epicycle moves contrary to the heavens [from west to east], for example from *A* to *B*, if the star so moves on the epicycle that it goes from the apogee again contrary to the heavens (that is, from *F* in the direction of *G*), there will result at the apogee the greatest advance, because the epicycle and the star are moving the same way. But if the movement of the star on the epicycle is in the direction of that of the heavens [from east to west], that is, from *F* towards *K*, conversely the least advance will be effected at the apogee because the star is then moving contrary to the movement of the epicycle.

With these things established, it must next be understood that, in the case of those planets which effect two anomalies, it is possible to combine both of these hypotheses, as we shall show in the chapters concerning them. But, in

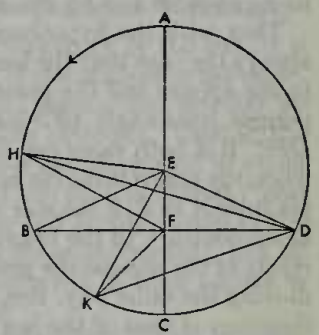
the case of those planets subject to only one anomaly, one of the hypotheses will suffice. And it must be understood that all the appearances can be cared for interchangeably according to either hypothesis, when the same ratios are involved in each. In other words, the hypotheses are interchangeable when, in the case of the hypothesis of the epicycle, the ratio of the epicycle's radius to the radius of the circle carrying it[1] is the same as, in the case of the hypothesis of eccentricity, the ratio of the line between the centres (that is, between the eye and the centre of the eccentric circle), to the eccentric circle's radius; with the added conditions that the star move on the epicycle from the apogee in the direction of the movement of the heavens with the same angular velocity as the epicycle moves on the circle concentric with the eye in the direction opposite to that of the heavens, and that the star move regularly on the eccentric circle with the same angular velocity also and in the direction opposite to the movement of the heavens.

And we shall briefly show in a systematic way, first by reasoning and secondly by the numbers discovered in the appearances of the sun's anomaly, that with the above assumptions the same appearances agree with either hypothesis.

I say first, then, that on either hypothesis the greatest difference between the regular movement and the apparent irregular movement (difference by which the mean passage of the stars is apprehended) occurs when the apparent angular distance cuts off a quadrant from the apogee; and that the time from the apogee to this mean passage is greater than from this mean passage to the perigee.

Therefore it results—always on the hypothesis of the eccentric circles, and on the hypothesis of the epicycle whenever their movements occur in the direction of the movement of the heavens—that the time from the least passage to the mean passage is greater than that from the mean to the greatest passage, because in each case the least progress is effected at the apogee. But on the hypothesis of the epicycles which supposes the revolutions of the stars on them in the direction contrary to that of the heavens, conversely the time from the greatest to the mean passage is greater than that from the mean to the least, because in this case the greatest progress is effected at the apogee.

First, then, let there be the star's eccentric circle $ABCD$ with centre E and diameter AEC. And let the centre of the ecliptic be taken (that is, the point at the eye), and let it be F. And with BFD drawn through F at right angles to AEC, let the star be supposed at points B and D, so that the apparent angular distance on either side from the apogee A is clearly a quadrant.



It must be proved that the greatest difference between the regular and irregular movements occurs at the points B and D.

For let EB and ED be joined. Then it is immediately evident that the arc of the anomalistic difference has to the whole circle the same ratio that angle EBF has to 4 right angles, since the angle AEB subtends the arc of the regular movement and angle AFB that of the apparent irregular movement, and angle EBF is the difference between them.

[1]The circle carrying the epicycle is often called the deferent.

I say, then, that no other angle can be constructed on the circumference of the circle $ABCD$ and on the straight line EF greater than these two at B and D.

For let the angles EHF and EKF be constructed at the points H and K, and let HD and KD be joined. Since then in every triangle the greater side subtends the greater angle, and

$$HF > FD, \qquad \text{[Eucl. III, 7,3]}$$

also angle $HDF >$ angle DHF

But angle $EDH =$ angle EHD

since $EH = ED$.

And therefore angle $EDF >$ angle EHF,

angle $EBD >$ angle EHF.

Again since $DF > KF$,

angle $FKD >$ angle FDK.

But angle $EKD =$ angle EDK,

since again $EK = ED$.

And therefore, by subtraction,

angle $EDF >$ angle EKF,

angle $EBF >$ angle EKF.

Therefore it is not possible to construct other angles in the way we have described greater than those at points B and D.[1]

And at the same time it is proved that arc AB, which embraces the time from the least to the mean movement, is greater than arc BC which embraces the time from the mean to the greatest movement, by twice the arc containing the anomalistic difference. For

angle $AEB =$ angle $EFB +$ angle EBF,

$=$ rt. angle $+$ angle EBF,

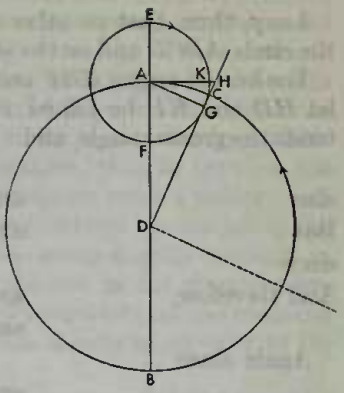
and

angle $BEC +$ angle $EBF =$ rt. angle.

Again, to prove the same thing occurs in the other hypothesis, let there be the circle ABC concentric with the cosmos, with center D and diameter ABD; and let there be in the same plane the epicycle EFG with centre A, carried on it. And let the star be supposed at G where it appears to be a quadrant's distance from the apogee point. And let AG and DGC be joined.

[1]The Greeks, in general, avoided the notion of a body's speed at a given point, and Ptolemy here handles the problem in the classic way, in terms of boundary points. Thus by proving that the greatest difference between the angle of the regular movement and that of the apparent irregular movement is at a point an apparent quadrant's distance from the apogee, it then follows that this point is a boundary point such that for all arcs between it and the apogee the star will appear to move more slowly than its regular or average movement, and for all arcs between it and the perigee the star will appear to move faster than its regular or average movement. Ptolemy therefore calls the point itself the point of the star's mean passage. It is not very different from saying in modern terms that the speed of the star at this point is its regular speed.

That the point of greatest anomalistic difference is such a boundary point is simply stated by Ptolemy and not explained. The explanation is this: From the apogee to the point of greatest difference the apparent angular speed of the star is always slower than its regular speed, for otherwise the difference between the two angles traveled would not be getting greater and greater. Likewise from the point of greatest difference to the perigee the apparent angular speed is greater than the regular speed, for otherwise the difference between the two angles traveled would not be getting less and less.



I say that the straight line *DGC* is tangent to the epicycle. For at that time there occurs the greatest difference between the regular and irregular movements.

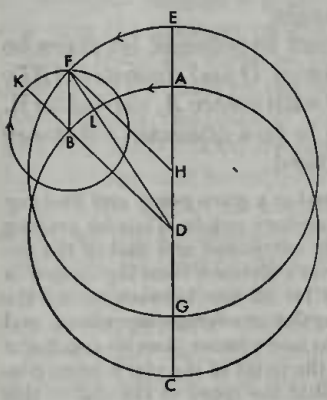
For since the regular movement from the apogee is contained by the angle *EAG* (for the star traverses the epicycle, and the epicycle the circle *ABC*, with the same angular velocity), and the difference between the regular and apparent movements is contained by the angle *ADG*, therefore it is evident that also the difference between angle *EAG* and angle *ADG* (that is, angle *AGD*) contains the apparent angular distance of the star from the apogee. And so, since it is assumed to be the angle of a quadrant, angle *AGD* will also be a right angle, and therefore the straight line *DGC* will be tangent to the epicycle *EFG* [Eucl. III, 16, Por.]. Therefore the arc *AC* between the centre *A* and the tangent is the greatest anomalistic difference.

And in the same way arc *EG*, which, according to the motion assumed for the epicycle, embraces the time from the least to the mean movement, is greater than arc *GF*, which embraces the time from the mean to the greatest movement. And it is greater by twice arc *AC*. For if we produce the straight line *DGH* and draw *AKH* at right angles to *EF*, then

angle *KAG* = angle *ADC*, [Eucl. VI, 8]

and arc *KG* is similar to arc *AC*. Therefore arc *EKG* is greater than a quadrant by arc *AC*, and arc *FG* is less than a quadrant by arc *AC*. Which it was required to prove.[1]

And next it will be clearly seen that, even in the other particular movements, in the case of both hypotheses, for equal times, all the same things will occur with respect to the regular and apparent movements and the differences between them—that is, the anomalistic difference.



For let there be the circle *ABC* with centre *D*, concentric with the ecliptic; and the eccentric *EFG* with center *H*, equal to the concentric circle *ABC;* and the diameter *EAHD* common to both, through the centres *D* and *H* and the apogee *E*. And with arc *AB* taken at random length on the concentric circle, let the epicycle *KF* with centre *B* and radius *DH* be described, and let *KBD* be joined.

I say that the star will be borne by either movement to *F*, the intersection of the eccentric

[1] In the case where the star moves on the epicycle in the same direction that the epicycle moves on the concentric circle, the mean passage and greatest anomalistic difference do not occur an apparent quadrant's distance from the apogee—that is, if the angular velocity of the star on the epicycle is the same as that of the epicycle's centre on the concentric circle. But it is greater than a quadrant's distance from the apogee. This is immediately evident if we refer to the previous figure and suppose *DGH* drawn tangent on the opposite side of the epicycle.

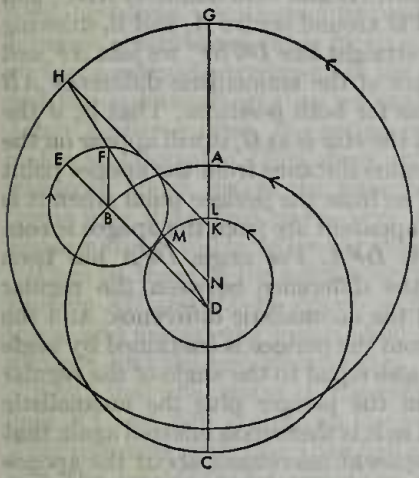
circle and the epicycle, in the same amount of time. That is, the three arcs, *EF* on the eccentric, *AB* on the concentric, and *KF* on the epicycle are similar to each other; and the difference between the regular and irregular movements, and the apparent passage of the star, will be similar and the same under either hypothesis.

For let *FH*, *BF*, and *DF* be joined. Since the opposite sides of the quadrilateral *BDHF* are equal to each other, *FH* to *BD*, and *BF* to *DH*, the quadrilateral *BDFH* is a parallelogram. Therefore the three angles *EHF*, *ADB*, and *FBK* are equal. And so, since they are angles at the centres, the arcs subtended by them—*EF* on the eccentric, *AB* on the concentric, and *KF* on the epicycle—are similar to one another. Therefore by either motion, the star will be brought to the same point *F* in an equal period of time, and will appear to have passed from the apogee along the same arc of the ecliptic, *AL*. And accordingly the anomalistic difference will be the same according to either hypothesis, since we have already proved that the difference contained by angle *DFH* on the hypothesis of eccentricity is of the same kind as that contained by angle *BDF* on the hypothesis of the epicycle, and since these angles are here also alternate and equal, with *FH* proved parallel to *BD*.

And it is clear that for all distances these same results will follow, *HDFB* being always a parallelogram and the eccentric circle being described by the movement of the star on the epicycle whenever the relations under either hypothesis are both similar and equal.

But it will also become clear in the following way that, even if they are only similar but unequal in magnitude, the same appearances will again result. For in the same way, let there be the circle *ABC* with centre *D*, concentric with the cosmos; and its diameter *ADC* passing through the star's apogee and perigee; and the epicycle about *B* at the random distance of arc *AB* from the apogee *A*. And let the star have moved through arc *EF* similar to *AB* since the returns of the circles take place in the same time. And let the straight lines *DBE*, *BF*, and *DF* be joined.

It is immediately clear that, on this hypothesis, angle *ADE* and angle *FBE* are always equal, and that the star will appear on the straight line *DF*.



I say also that, on the hypothesis of eccentricity, both if the eccentric circle is greater than the concentric circle and if it is less, with only the similarity of the relations and the isochronism of the returns assumed, the star will again appear on the same straight line *DF*.

For let the eccentric circle *GH* be drawn greater, as we said, with its centre at *K* on *AC;* and likewise *LM* less, with centre *N*. And producing *DMFH* and *DLAG*, let *HK* and *MN* be joined. Since

$$DB : BF :: HK : KD :: MN : ND,$$

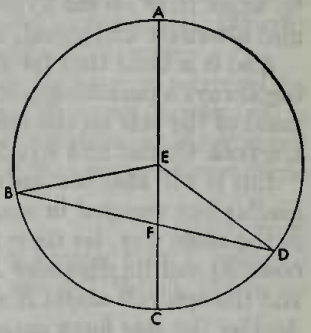
and since

$$\text{angle } BFD = \text{angle } MDN$$

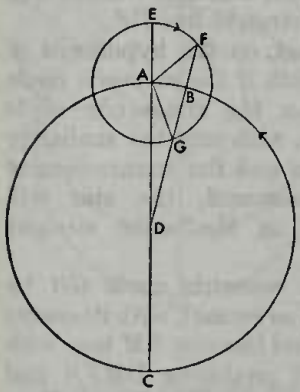
because of the parallels *DA* and *BF*, therefore the three triangles are equiangular [Eucl. VI, 7], and the angles *BDF*, *DHK*, and *DMN*, subtending the corresponding sides, are equal. Therefore the straight lines *BD*, *HK*, and *MN* are parallel, so that also angles *ADB*, *AKH*, and *ANM* are equal. And since they are angles at the centres of the circles, therefore the arcs subtended by them, *AB*, *GH*, and *LM*, will be similar. Therefore, in the same length of time, not only has the epicycle traversed arc *AB*, and the star arc *EF*, but also on the eccentric circles the star will have traversed arcs *GH* and *LM;* and in each case, therefore, it will be observed on the same straight line *DMFH*, being at the point *F* in the case of the epicycle, at *H* for the greater eccentric, at *M* for the smaller eccentric, and likewise for all positions.

And furthermore it results that, when the star appears to have traversed equal arcs both from the apogee and the perigee, the anomalistic difference in either position will be equal.

For, on the hypothesis of eccentricity, if we describe the eccentric circle *ABCD* about centre *E* with diameter *AEC* through the apogee *A*, with the eye supposed on it at *F*, and if, drawing through *F* the straight line *BFD* at random, we join *EB* and *ED*, then the apparent courses will be equal and opposite; that is, angle *AFB* of the course from the apogee and angle *CFD* of that from the perigee. And the anomalistic difference will be the same because *BE* is equal to *DE* and angle *EBF* to angle *EDF*. And so the arc from the apogee *A* and the arc from the perigee *C* (that is, the arcs contained by angles *AFB* and *CFD*, respectively) are, the one greater and the other less, than the regular movement by the same difference of the apparent arc; because *AEB* is greater than angle *AFB* and angle *CED* less than angle *CFD*.



And, on the hypothesis of the epicycle, if we describe likewise the concentric circle around centre *D* and with diameter *ADC*, and the epicycle *EFG* around centre *A*, and if, drawing at random the straight line *DGBF*, we join *AF* and *AG*, then the arc of the anomalistic difference *AB* will be the same for both positions. That is, if the star is at *F* or if the star is at *G*, it will appear on the ecliptic at the same distance from the apogee point when it is at *F* as from the perigee point when it is at *G*, since the apparent arc from the apogee is contained by angle *DFA*. For angle *DFA* has been shown to be the difference between the regular movement and the anomalistic difference. And the apparent arc from the perigee is contained by angle *FGA*. For it is also equal to the angle of the regular movement from the perigee plus the anomalistic difference. And so it is thereupon inferred again that the mean movement is greater than the apparent movement about the apogee (that is, angle *EAF* than angle *AFD*) and the mean movement is less than the



apparent about the perigee (that is, angle *GAD* than angle *AGF*), both by the same difference, by angle *ADG*. Which was to be proved.

4. On the Apparent Irregularity or Anomaly of the Sun

With these things explained, it is now necessary to take up the apparent irregularity or anomaly of the sun; because there is one only, and it is such that the time from the least movement to the mean is greater than the time from the mean to the greatest movement. For we find this agrees with the appearances. And this can be accomplished by either hypothesis:—(1) by that of the epicycle when the movement of the sun is in the direction of the movement of the heavens on its arc at the apogee. But (2) it would be more reasonable to stick to the hypothesis of eccentricity which is simpler and completely effected by one and not two movements.

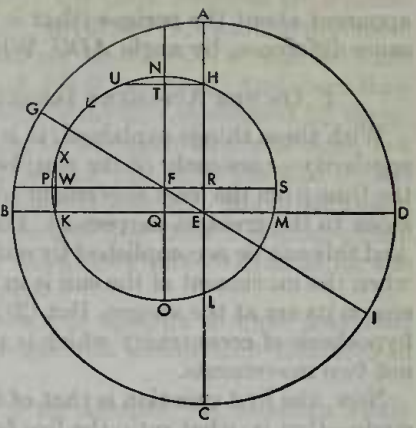
Now, the first question is that of finding the ratio of eccentricity of the sun's circle—that is, what ratio the line between the eccentric circle's centre and the ecliptic's centre at the eye has to the radius of the eccentric circle; and next at what section of the ecliptic the apogee of the eccentric circle is to be found. And these things have been shown in a serious way by Hipparchus. For having supposed the time from the spring equinox to the summer tropic to be 94½ days, and the time from the summer tropic to the autumn equinox to be 92½ days, he proves from these appearances alone that the straight line between the aforesaid centres is very nearly 1/24 the radius of the eccentric circle; and that its apogee precedes the summer tropic by very nearly 24½° of the ecliptic's 360°.

And we too find that the time-periods of these quarters and these ratios are very nearly the same even now, so that in this way it is clear to us that the sun's eccentric circle always preserves the same position with respect to the tropic and equinoctial points. And not to establish this position on hearsay only, but to expound the theory systematically with our own numbers, we shall prove these things ourselves, using these same appearances as regards the eccentric circle—that is, as we said, 94½ days from the spring equinox to the summer tropic and 92½ days from the summer tropic to the autumn equinox. For with the very accurate observations made by us in the year 463 after the death of Alexander we find a complete agreement in the number of days between the summer tropic and the equinoxes. For as we said [pp. 81-82], the autumn equinox fell on Athyr 9 after sunrise, the spring equinox on Pachom 7 after midday, which makes an interval of 178¼ days, and the summer tropic on Mesore 11-12 after midnight, which makes the interval from the spring equinox to the summer tropic 94½ days, and leaves very nearly 92½ days for the interval from the summer tropic to the following autumn equinox.

Then let there be the ecliptic circle *ABCD* with centre *E*, and let the two diameters *AC* and *BD* be drawn in it perpendicular to each other through the tropic and equinoctial points. And let *A* be supposed the spring point, *B* the summer, and the rest accordingly.

Now, that the centre of the eccentric circle will fall between the straight lines *EA* and *EB*, is clear on the one hand from the fact that the semicircle *ABC* embraces more time than half a year and therefore cuts off a section of the eccentric greater than a semicircle, and on the other hand from the fact that the quadrant *AB* itself also embraces more time and cuts off a greater arc of

the eccentric than quadrant *BC*.



This being so, let the point *F* be supposed the centre of the eccentric circle, and let the diameter *EFG* be drawn through both centres and the apogee. And with centre *F* and any radius, let the eccentric circle of the sun *HKLM* be drawn, and through *F* the line *NQO* parallel to *AC* and the line *PRS* parallel to *BD*. And again let *HTU*, the perpendicular from *H* to *NQO*, and *KWX*, the perpendicular from *K* to *PRS*, be drawn.

Since, then, the sun, moving regularly on the circle *HKLM*, traverses arc *HK* in 94½ days and arc *KL* in 92½ days, and since it covers regularly in 94½ days very nearly 93°9′, and in 92½ days 91°11′ [Chap. II, Table of Sun's Regular Movement], therefore

$$\text{arc } HKL = 184°20',$$

and

$$\text{arc } NH + \text{arc } LO = 4°20'$$

by subtraction of the semicircle *NPO*. And

$$\text{arc } HNU = 2 \text{ arc } HN = 4°20'.$$

And so

$$\text{chord } HU = 4^{p}32'$$

where

$$\text{ecc. diam.} = 120^{p}$$

And, the half of chord *HU*,

$$HT = EQ = 2^{p}16'.$$

Again, since

$$\text{arc } HNPK = 93°9'$$

and

$$\text{arc } HN = 2°10'$$

and

$$\text{quadrant } NP = 90°,$$

therefore, by subtraction,

$$\text{arc } PK = 0°59'$$

and

$$\text{arc } KPX = 2 \text{ arc } PK = 1°58'.$$

And so

$$\text{chord } KWX = 2^{p}4'$$

and, the half of it,

$$KW = FQ = 1^{p}2'.$$

But, it was proved

$$EQ = 2^{p}16'.$$

And since

$$\text{sq. } FQ + \text{sq. } EQ = \text{sq. } EF,$$

therefore, in length,

$$EF \fallingdotseq 2^{p}29'30''$$

where

$$\text{rad. ecc.} = 60^{p}.$$

Therefore the radius of the eccentric circle is very nearly twenty-four times the line between its centre and that of the ecliptic.

Again since

$$FQ = 1^{p}2'$$

where it was proved

$$EF = 2^{p}29'30'',$$

therefore

$$FQ \fallingdotseq 49^{p}46'$$

where

$$\text{hypt. } EF = 120^{p},$$

and, on the circle about right triangle EFQ,

$$\text{arc } FQ = 49°.$$

And therefore

$$\text{angle } FEQ = 49° \text{ to 2 rt.}$$
$$= 24°30'.$$

And so, since the angle is at the centre of the ecliptic, arc BG by which the apogee G precedes the summer tropic point B, is also 24°30′.

Finally, since the quadrants OS and SN are each 90°, and

$$\text{arc } OL = \text{arc } HN = 2°10',$$

and

$$\text{arc } MS = 0°59',$$

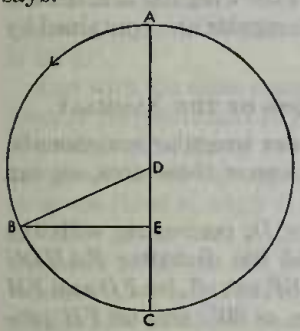
therefore

$$\text{arc } LM = 86°51'$$

and

$$\text{arc } MH = 88°49'.$$

But the sun moves regularly through 86°51′ in 88⅛ days, and through 88° 49′ in very nearly 90⅛ days [Chap. 2, Table of Sun's Regular Movement]. And so the sun will appear to traverse arc CD, which is the arc from the autumn equinox to the winter tropic in 88⅛ days; and arc DA, which is the arc from the winter tropic to the spring equinox, in very nearly 90⅛ days. And these things have been found by us in accord with what Hipparchus says.



Now with these quantities, let us find out first how much is the greatest difference between the regular and irregular movements, and at what point this occurs.

Then let there be the eccentric circle ABC with centre D and diameter ADC through the apogee A; and on it let there be the centre of the ecliptic E. And let EB be drawn perpendicular to AC, and DB be joined. Since

$$\text{1. betw. c. } DE = 2^{p}30'$$

where

$$\text{rad. } BD = 60^{p}$$

according to the ratio of 1 to 24, therefore

$$DE = 5^{p}$$

where

$$\text{hypt. } BD = 120^{p},$$

and, on the circle about right triangle BDE,

$$\text{arc } DE = 4°46'.$$

And so angle DBE which contains the greatest anomalistic difference will be 4°46′ to 2 right angles' and 2°23′ to 4 right angles' 360°. And

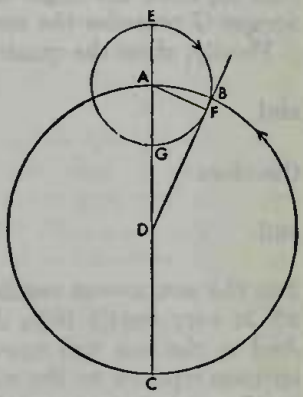
$$\text{rt. angle } BED = 90°$$

and

$$\text{angle } BDA = \text{angle } BED + \text{angle } DBE = 92°23'.$$

And since angle BDA is at the centre of the eccentric circle and angle BED of the ecliptic, we shall have the greatest anomalistic difference as 2°23′. And as for the arcs at which this occurs, that of the eccentric which is regular is 92°23′ from the apogee, and that of the ecliptic which is apparent and irregular is a quadrant or 90°, as we have already proved. And it is clear from what has been set out that in the opposite section the apparent mean passage and the greatest anomalistic difference will be at 270°, and the regular mean passage at 267°37′ on the eccentric.

In order to show with numbers also that the same quantities can be inferred on the hypothesis of the epicycle when there are the same ratios in the way we described, let there be the circle ABC with centre D and diameter ADC, concentric with the ecliptic, and the epicycle EFG with centre A. And let the straight line DFB be drawn from D tangent to the epicycle, and let AF be joined. Then likewise



$$AD = 24\ AF,$$

so that again also

$$AF = 5^{p}$$

where

$$\text{hypt. } AD = 120^{p},$$

and, on the circle about right triangle ADF,

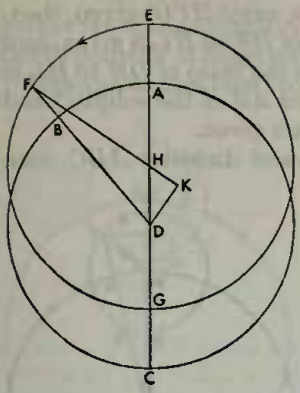
$$\text{arc } AF = 4°46' \text{ to 2 rt.}$$
$$= 2°23'$$

Therefore, the greatest anomalistic difference (that is, arc AB) is thereupon found rightly to be 2°23′; and the irregular arc, since it is contained by the right angle AFD, to be 90° and the regular arc, contained by angle EAF, again to be 92°23′.

5. On the Examination of Particular Sections of the Anomaly

And to be able to distinguish at any time particular irregular movements, we shall again show for either hypothesis how, given one of these arcs, we can get the others also.

Then first let there be the circle ABC with centre D, concentric with the ecliptic; and the eccentric circle with centre H; and the diameter $EAHDG$ through both centres and the apogee E. And with arc EF cut off, let FD and FH be joined. And first let arc EF be given, for instance, as 30°; and on FH produced let fall the perpendicular DK from D.



Since then it is assumed

$$\text{arc } EF = 30°,$$

therefore

$$\text{angle } EHF = \text{angle } DHK = 30° = 60° \text{ to 2 rt.}$$

And therefore, on the circle about right triangle DHK,

$$\text{arc } DK = 60°,$$

and, the rest of the semicircle,

$$\text{arc } HK = 120°. \qquad \text{[Eucl. III, 31]}$$

And therefore

$$DK = 60^p$$

and

$$KH = 103^p55'$$

where

$$\text{hypt. } DH = 120^p.$$

And so

$$DK = 1^p15',$$
$$HK = 2^p10',$$
$$KHF = 62^p10'$$

where

$$DH = 2^p30',$$
$$\text{rad. } FH = 60^p.$$

And since

$$\text{sq. } DK + \text{sq. } FHK = \text{sq. } FD,$$
$$\text{hypt. } FD = 62^p11'.$$

And therefore

$$DK = 2^p25'$$

where

$$FD = 120^p,$$

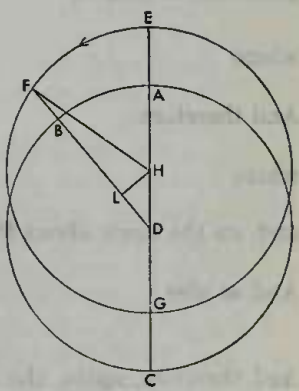
and, on the circle about right triangle FDK,

$$\text{arc } DK = 2°18'.$$

And so

$$\text{angle } DFK = 2°18' \text{ to 2 rt.} = 1°9'.$$

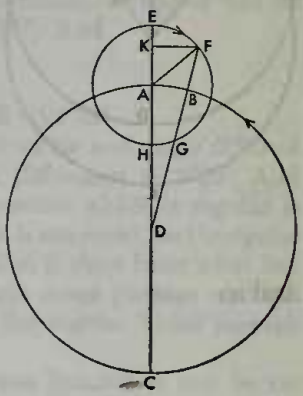
Therefore the anomalistic difference at that time is 1°9′. But angle EHF was 30°, and therefore the remaining angle ADB (that is, arc AB on the ecliptic) is 28°51′.



And with the same construction, if HL is dropped from H perpendicular to FD, it will be immediately clear that also, if any other angle is given, the rest are given. For if we suppose the arc AB on the ecliptic given (that is, angle HDL), then the ratio of DH to HL is given. And if the ratio DH to HF is given, the ratio of HF to HL is also given; and therefore we shall have angle HFL given (that is, the anomalistic difference) and angle EHF (that is, arc EF of the eccentric circle).

And if we suppose the anomalistic difference (that is, angle HFD) given, then, conversely, the same things will occur. For the ratio of HF to HL is given, and given from the first is the ratio of HF to HD. And so the ratio of DH to HL is also given; and therefore also angle HDL (that is, arc AB of the ecliptic) and angle EHF (that is, arc EF on the eccentric circle) are given.

Again let there be the circle ABC with centre D and diameter ADC, concentric with the ecliptic, and the epicycle $EFGH$ with centre A, in the same ratio. And let arc EF be cut off, and FBD and FA joined. Again let arc EF be supposed 30°, and let the line KF be drawn from F perpendicular to AE.



Since

$$\text{arc } EF = 30°,$$
$$\text{angle } EAF = 30°$$
$$= 60° \text{ to 2 rt.}$$

And so, on the circle about right triangle AFK,

$$\text{arc } FK = 60°,$$

and, the remainder of the semicircle,

$$\text{arc } AK = 120°.$$

And therefore

$$\text{chord } FK = 60^{\text{p}},$$
$$\text{chord } AK = 103^{\text{p}}55'$$

where

$$\text{diam. } AF = 120^{\text{p}}.$$

And so

$$FK = 1^{\text{p}}15',$$
$$AK = 2^{\text{p}}10',$$
$$KAD = 62^{\text{p}}10'$$

where

$$\text{hypt. } AF = 2^{\text{p}}30',$$
$$\text{rad. } AD = 60^{\text{p}}.$$

And since

$$\text{sq. } FK + \text{sq. } KAD = \text{sq. } FBD,$$

therefore, in length,

$$FD = 62^{\text{p}}11'$$

where

$$FK = 1^{\text{p}}15'.$$

And therefore

$$FK = 2^{\text{p}}25'$$

where

$$\text{hypt. } DF = 120^{\text{p}},$$

and, on the circle about the right triangle DFK,

$$\text{arc } FK = 2°18'.$$

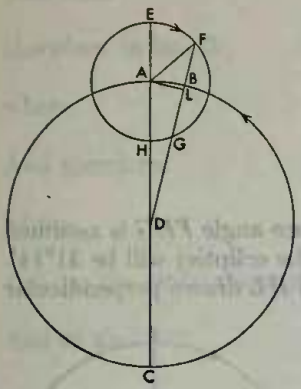
And so also

$$\text{angle } FDK = 2°18' \text{ to 2 rt.}$$
$$= 1°9'.$$

And therefore, again, the anomalistic difference, which is arc AB, is also 1°9'. But angle EAF was 30°. Therefore the remaining angle AFD (that is, the ap-

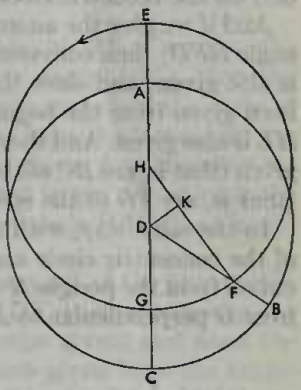
parent arc of the ecliptic) is 28°51′, which agrees with the magnitudes proved on the hypothesis of eccentricity.

Likewise in this case too, even if any other angle is given, the others will also be given if, with the same construction, AL is drawn from A perpendicular to FD. For if, conversely, we give the apparent arc of the ecliptic (that is, angle AFD) then the ratio of AF to AL is also given. And since the ratio of AF to AD was given from the beginning, the ratio of AD to AL is also given. And therefore angle ADB (that is, arc AB which is the anomalistic difference) is given; and also angle EAF (that is, arc EF) of the epicycle.



And if we suppose the anomalistic difference given (that is, angle ADB) then, conversely, the ratio of AD to AL will likewise be given. But since the ratio of AD to AF was given from the beginning, the ratio of AF to AL is also given, and therefore angle AFD is given (that is, the apparent arc of the ecliptic) and angle EAF (that is, arc EF of the epicycle).

Again, with the previous construction of the eccentric circle, let the arc FG be cut off from the perigee G on the eccentric circle and be assumed to be 30°. And let DFB and FH be joined, and let DK be dropped from D perpendicular to HF.



Since

$$\text{arc } FG = 30°,$$
$$\text{angle } FHG = 30°$$
$$= 60° \text{ to 2 rt.}$$

And so, on the circle about right triangle DHK,

$$\text{arc } DK = 60°$$

and, the remainder of the semicircle,

$$\text{arc } KH = 120°.$$

And therefore

$$\text{chord } DK = 60^{\text{p}},$$
$$\text{chord } KH = 103^{\text{p}}55'$$

where

$$\text{diam. } DH = 120^{\text{p}}.$$

And therefore

$$DK = 1^{\text{p}}15',$$
$$HK = 2^{\text{p}}10',$$
$$KF = 57^{\text{p}}50',$$

where

$$\text{hypt. } DH = 2^{\text{p}}30',$$
$$\text{rad. } HF = 60^{\text{p}}.$$

And since

$$\text{sq. } DK + \text{sq. } KF = \text{sq. } DF,$$

therefore, in length,

$$DF = 57^{\text{p}}51'$$

where

$$DK = 1^{p}15'.$$

And therefore

$$DK = 2^{p}34'36''$$

where

$$\text{hypt. } DF = 120^{p},$$

and, on the circle about right triangle DFK,

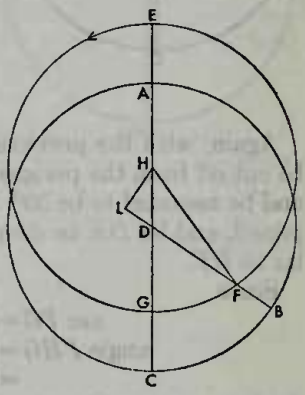
$$\text{arc } DK = 2°27'.$$

And so

$$\text{angle } DFK = 2°27' \text{ to 2 rt.}$$
$$\fallingdotseq 1°14'.$$

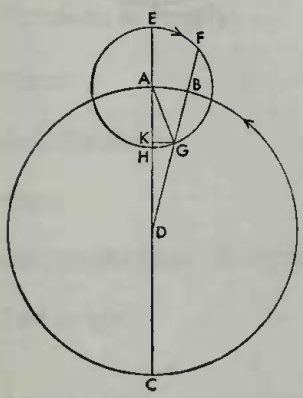
Therefore the anomalistic difference is 1°14′. And since angle FHG is assumed to be 30°, the whole angle BDC (that is, arc BC of the ecliptic) will be 31°14′.

And in the same way now, with BD produced and HL drawn perpendicular to it, if we give arc BC of the ecliptic (that is, angle HDL), then the ratio of DH to HL is also given; and since the ratio of HD to HF was given from the beginning, the ratio of FH to HL is also given. And therefore we have angle HFD given, that is, the anomalistic difference; and angle FHD (that is, arc GF) on the eccentric circle.



And if we give the anomalistic difference (that is, angle HFD) then conversely the ratio of FH to HL is also given; and since the ratio of FH to DH has been given from the beginning, the ratio of DH to HL is also given. And therefore we have angle HDL given (that is, arc BC of the ecliptic) and angle FHG (that is, arc FG of the eccentric circle).

In the same way, with the previous construction of the concentric circle and epicycle, let arc HG be cut off from the perigee H at 30°. And let AG and DGB be joined, and GK drawn from G perpendicular to AD.



Then since again

$$\text{arc } GH = 30°,$$
$$\text{angle } HAG = 30°$$
$$= 60° \text{ to 2 rt.}$$

And so, on the circle about right triangle GKA,

$$\text{arc } GK = 60°$$

and, the remainder of the semicircle,

$$\text{arc } AK = 120°.$$

And therefore

$$\text{chord } GK = 60^{p},$$
$$\text{chord } AK = 103^{p}55'$$

where

$$\text{hypt. } AG = 120^{p}.$$

And therefore

$$GK = 1^{p}15',$$
$$AK = 2^{p}10',$$
$$KD = 57^{p}50'$$

where

$$AG = 2^{p}30',$$
$$\text{rad. } AD = 60^{p}.$$

And since

$$\text{sq. } GK + \text{sq. } KD = \text{sq. } DG,$$

therefore, in length,

$$DG \fallingdotseq 57^{p}51'$$

where

$$GK = 1^{p}15'.$$

And therefore

$$GK = 2^{p}34'36''$$

where

$$DG = 120^{p};$$

and, on the circle about right triangle DGK,

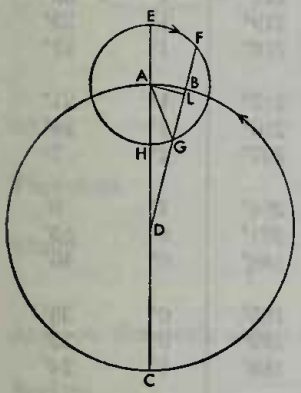
$$\text{arc } GK = 2°27'.$$

And so also

$$\text{angle } GDK = 2°27' \text{ to 2 rt.}$$
$$\fallingdotseq 1°14'.$$

Therefore the anomalistic difference (that is, arc AB) is here also very nearly 1°14′. And since angle KAG is assumed to be 30°, the whole angle BGA which contains the apparent arc of the ecliptic will be 31°14′, agreeing with the magnitudes found for the eccentric circle.

And likewise here also, with AL drawn perpendicular to DB, if we give the arc of the ecliptic (that is, angle AGL), then the ratio of AG to AL is given; and since the ratio of AG to AD has been given from the beginning, the ratio of AD to AL is also given. And therefore we have angle ADB given (that is, arc AB which is the anomalistic difference) and angle HAG (that is, arc HG of the epicycle).



And, again, if we give arc AB of the anomalistic difference (that is, angle ADB) then, conversely, the ratio of AD to AL is likewise given; and since the ratio of AD to AG has been given from the beginning, the ratio of AG to AL is also given. And therefore we shall have angle AGL given (that is, the arc of the ecliptic) and angle HAG (that is, arc HG of the epicycle). And we have shown what was proposed.

Now, although, by means of these theorems, various tables of the sections containing the anomalistic distinctions of the apparent courses can be constructed, yet that one containing the anomalistic differences arranged side by side with the regular arcs will better serve us for getting easily the magnitudes of the particular corrections, both because of its conformity with the hypotheses themselves and because of the simplicity and facility in the calculation for each section. Therefore we followed the first of the theorems set out with numbers for the particular sections, and calculated geometrically just as before the anomalistic differences corresponding to each of the regular arcs. And in general, both in the case of the sun and of the other planets, we divide the quadrants at the

apogees into 15 sections, so that the comparison is at intervals of 6°; and the quadrants at the perigees into 30 sections so that the comparison in this case is at intervals of 3°. This is done because the differences in excess at the perigees are greater than those corresponding to equal sections at the apogee.

And so we shall arrange the table of the sun's anomaly into forty-five rows and three columns. And of these columns the first two contain the numbers of the 360° of regular movement, while the first fifteen rows embrace the two quadrants at the apogee, and the other thirty those at the perigee. And the third column contains the degrees of anomalistic difference to be added or subtracted, corresponding to each of the regular numbers. And here is the table:

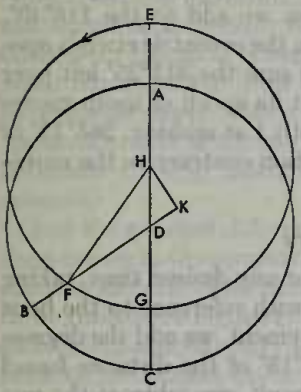
6. Table of the Sun's Anomaly

1. *Common numbers (Degrees of Regular Movement)*		*2.* *Additive—subtractive differences*		*1.* *Common numbers (Degrees of Regular Movement)*		*2.* *Additive—subtractive differences*	
6°	354°	0°	14′	120°	240°	2°	6′
12°	348°	0°	28′	123°	237°	2°	2′
18°	342°	0°	42′	126°	234°	1°	58′
24°	336°	0°	56′	129°	231°	1°	54′
30°	330°	1°	9′	132°	228°	1°	49′
36°	324°	1°	21′	135°	225°	1°	44′
42°	318°	1°	32′	138°	222°	1°	39′
48°	312°	1°	43′	141°	219°	1°	33′
54°	306°	1°	53′	144°	216°	1°	27′
60°	300°	2°	1′	147°	213°	1°	21′
66°	294°	2°	8′	150°	210°	1°	14′
72°	288°	2°	14′	153°	207°	1°	7′
78°	282°	2°	18′	156°	204°	1°	0′
84°	276°	2°	21′	159°	201°	0°	53′
90°	270°	2°	23′	162°	198°	0°	46′
93°	267°	2°	23′	165°	195°	0°	39′
96°	264°	2°	23′	168°	192°	0°	32′
99°	261°	2°	22′	171°	189°	0°	24′
102°	258°	2°	21′	174°	186°	0°	16′
105°	255°	2°	20′	177°	183°	0°	8′
108°	252°	2°	18′	180°	180°	0°	0′
111°	249°	2°	16′				
114°	246°	2°	13′				
117°	243°	2°	10′				

7. On the Epoch of the Sun's Mean Course

Since there remains to be established the epoch of the sun's regular movement for finding out its particular course at any time, we shall make the following exposition, using again in general, both in the case of the sun and of the other planets, the passages most accurately observed by ourselves, and taking the establishing of the epochs back to the beginning of the reign of Nabonassar by means of the mean movements already demonstrated. For we have ancient observations completely preserved from that period to the present.

Then let there be the circle with centre D concentric with the ecliptic; and the eccentric circle of the sun EFG with centre H; and the diameter $EAGC$ through both centres and the apogee E. And let B be supposed the fall equinox point on the ecliptic; and let BFD and FH be joined; and let HK be dropped from H perpendicular to FD.



Since the point B, the autumn equinox, is at the beginning of the sign of the Balance, and the perigee C $5\frac{1}{2}°$ within the sign of the Archer, therefore

$$\text{arc } BC = 65°30'.$$

And therefore

$$\text{angle } BDC = \text{angle } HDK = 65°30' = 131° \text{ to 2 rt.}$$

And so, on the circle about right triangle DHK,

$$\text{arc } HK = 131°,$$

and

$$\text{chord } HK = 109^{p}12'$$

where

$$\text{diam. } DH = 120^{p}.$$

Therefore

$$HK = 4^{p}33'$$

where

$$DH = 5^{p},$$

$$\text{hypt. } FH = 120^{p}.$$

And, on the circle about right triangle FHK,

$$\text{arc } HK = 4°20'.$$

And so

$$\text{angle } HFK = 4°20' \text{ to 2 rt.} = 2°10'.$$

But it was proved

$$\text{angle } BDC = 65°30',$$

and therefore the remaining angle FHG (that is, arc FG on the eccentric circle) is $63°20'$. Therefore, when the sun is at the autumn equinox, it precedes the perigee (that is, the point $5\frac{1}{2}°$ within the Archer) by $63°20'$ mean movement; and it is $116°40'$ in mean movement from the apogee (that is, $5\frac{1}{2}°$ within the Twins) in the direction contrary to the movement of the heavens.

Now that this is understood—since, of the first equinoxes observed by us, one of the most accurate occured as the autumn equinox in the year 17 of Hadrian, Egyptianwise Athyr 7, very nearly 2 hours after midday—it is clear that at that

time the sun was 116°40′ in mean movement on the eccentric circle, away from the apogee in the direction contrary to the movement of the heavens. But from the reign of Nabonassar to the death of Alexander amounts to 424 Egyptian years; and from the death of Alexander to the reign of Augustus, 294 years; and from the year 1 of Augustus, Egyptianwise Thoth 1, midday (for we establish the epochs from midday) to the year 17 of Hadrian, Athyr 7, 2 hours after midday, amounts to 161 years, 66 days, and 2 equatorial hours. And therefore from the year 1 of Nabonassar, Egyptianwise Thoth 1, midday, to the time of the autumn equinox just mentioned amounts to 879 Egyptian years, 66 days, and 2 equatorial hours.

But in that amount of time the sun makes a mean movement of very nearly 211°25′ over and above the complete circles. If then we add to the 116°40′, representing the distance of this autumn equinox from the eccentric circle's apogee, the 360° of a whole circle and subtract from the sum the 311°25′ left over from the time between, then we shall have the sun at its epoch of mean movement in the year 1 of Nabonassar, Egyptianwise Thoth 1 at midday, 265°15′ of mean movement distant from the apogee in the direction contrary to the movement of the heavens, and 0°45′ within the Fishes.

8. On Calculating the Sun

Whenever we wish to know the course of the sun for any desired time, taking the total time from the epoch to the proposed date with reference to the hour in Alexandria and taking it to the tables of mean movement, we add the degrees corresponding to the particular numbers to the 265°15′ of the distance found above; and striking the complete circles out of the result, we subtract the rest from the 5°30′ within the Twins backwards in the order of the signs [from west to east]. And wherever the number falls, there we find the mean course of the sun. Next we take the same number (that is, the number of degrees from the apogee to the mean course) to the Table of Anomaly. And, if the number falls in the first column (that is, if it is not greater than 180°), then we subtract the corresponding degrees in the third column from the position of the mean course; but, if it falls in the second column (that is, if it is greater than 180°), then we add it to the mean course. And thus we find the true and apparent sun.

9. On the Inequality of Solar Days

Now this is pretty nearly all of the theory of the sun considered by itself. But it would be well to add briefly to this something concerning the inequality of the solar days, a matter which ought to be cleared up before what follows. For each of the simple mean movements we have given receives a uniform increase, as if the solar days were equal in time; but this is contrary to true theory. Now, given that the revolution of the universe is effected regularly about the poles of the equator, if the cyclical return is taken either with respect to the horizon or the meridian (whichever point is the more easily distinguishable), it is clear that one complete turn of the cosmos is the return of the same equatorial point from some section either of the horizon or the meridian back to that same section, and the solar day considered simply is the return of the sun from some section either of the horizon or of the meridian back again to the same section. The regular [or mean] solar day is, therefore, that embracing a course of 360° equatorial time proper to one revolution of the equator plus very nearly 59′ equa-

torial time for the sun's contrary mean movement along the ecliptic. And the irregular solar day is that embracing a course of 360° equatorial time proper to one revolution of the equator plus the extra arc, either at the horizon or at the meridian, corresponding to the sun's contrary irregular movement.

And so this section of the equator which is traversed over and above the 360° equatorial time must be an unequal one because of the sun's apparent irregularity, and because equal sections of the ecliptic do not traverse the horizon or the meridian in equal periods of time. Each of these extra sections makes the difference between the regular and irregular return, in the case of one solar day, a difference indistinguishable by the senses; but a total of many solar days makes a very sensible difference.

Now, the sun's greatest anomalistic difference occurs at intervals of one mean movement of the sun to the other. For in this way the total of solar days will differ from the total of regular [or mean] solar days by very nearly 4¾° equatorial time; and the total in one interval will differ from the total in the other by twice as much, that is by 9½° equatorial time. And this is so because the apparent course of the sun is less, in the semicircle of the apogee, than the regular course by 4¾°, but greater, in the semicircle of the perigee, by the same amount.[1]

And the greatest difference of irregularity in the corresponding risings or settings occurs in the semicircles bounded by the tropic points. For then the ascensions in each of these semicircles will differ from the 180° equatorial time of those considered regularly by the difference between the shortest or longest day and the equinoctial day; and they will differ from each other by the difference between the longest day or night and the shortest day or night. But the greatest difference of inequality in the case of simultaneous culminations occurs in the intervals containing two signs, one sign on each side of the tropic or equinoctial point. For those at the tropics taken together will differ from the regular by very nearly 4½° equatorial time, but from those at the equinoctial points taken together by 9° equatorial time. For the latter are less than the mean movement, and the former are greater by nearly an equal amount.[2] And thence we establish the beginnings of the solar days at the positions of the sun as it passes the meridian, and not from the sun's risings or settings; because the difference considered with respect to horizons can amount to many hours and is not the same everywhere, but changes with the excess of the longest over the shortest day in each latitude of the sphere. But the difference at the meridian is the same everywhere and does not exceed the total time of the sun's anomalistic difference.

And the difference in the intervals to be added or subtracted is constructed out of the combination of both of the aforesaid differences: that of the sun's

[1]For the greatest anomalistic differences, it has already been proved, occur at the apparent distance of a quadrant from the apogee, which are also the points of mean movement. Now, it has been shown that the difference between the angles of the regular and irregular movements from the mean to the apogee is 2°23′ and from the apogee to the other mean again 2°23′, the regular exceeding in both cases. The same thing occurs from the mean through the perigee to the other mean, the irregular exceeding in both cases.

[2]This difference is demonstrated in the Table of Ascensions for the Right Sphere, Book II. The difference in the co-ascensions of the ecliptic and equator is additive for the ecliptic in the Twins and the Crab, and the Archer and the Goat. It is subtractive for the ecliptic in the Fishes and the Ram, and the Virgin and the Balance. The other signs are all mixed. The equator here, of course, represents the regular movement of the sun for the mean solar days; and the ecliptic represents the irregular movement of the sun for the irregular true solar days.

anomaly and that of the culminations. And the interval from the middle of the Water Bearer to the Balance is subtractive for either difference; and the interval from the Scorpion to the middle of the Water Bearer is additive. For each of these sections either adds or subtracts with respect to the solar anomaly at most very nearly $3\frac{2}{3}°$ equatorial time, and with respect to the culminations very nearly $4\frac{2}{3}°$ equatorial time. And so, at most, the difference in the solar day, gotten from the combination of the two, amounts, for either interval, to $8\frac{1}{3}°$ equatorial time over and above the regular difference—that is, $\frac{1}{2}+\frac{1}{18}$ [equatorial] hour. And with respect to each other it amounts to $16\frac{2}{3}°$ equatorial time —that is, $1\frac{1}{9}$ [equatorial] hours. Now, if this difference is neglected in the case of the sun, it perhaps would not hurt the study of its appearances to any appreciable extent; but in the case of the moon, because of the rapidity of its movement, it would produce a considerable difference even for $3\frac{1}{3}°$.

In order to reduce the solar days given over any interval of time (and by solar days I mean from midday or midnight to midday or midnight) to regular solar days, we find out for the beginning and end of the given interval at what part of the ecliptic the sun is to be found both in its regular and irregular movements. Then, taking the surplus over and above complete revolutions accumulated in the irregular interval (that is, from apparent position to apparent position) to the Table of Ascensions in the Right Sphere, we find out with how many degrees equatorial time the degrees of the irregular interval, as we said, culminate. Then we take the difference between the amount of time thus found and the degrees of the regular interval and calculate the part of the corresponding equatorial hour. And if the amount of time found in the Table of Ascensions in the Right Sphere is greater than that of the regular interval, we add this difference to the given number of solar days; if less, we subtract. And finally, in this way, we have the time expressed in mean solar days. We shall use this correction especially in the successive additions of the mean movements in the moon's tables. And it is immediately seen that, given the regular [or mean] solar days, the simply considered seasonal solar days are gotten by an addition or subtraction in the converse order.[1]

[1]If the sun moved regularly along the equator instead of moving irregularly along the ecliptic, the solar days would all be regular. Their irregularity, then, is due to two things: (1) the irregularity of the sun's movement, and (2) the fact that the sun moves on a circle oblique to the equator.

Now, as far as the solar days are concerned, it is not the arc of the ecliptic that is directly involved, but the corresponding or co-ascending arc of the equator. For the culminations which define solar days take place at the meridian, and the meridian is through the pole of the equator. Therefore we first find the arc on the equator co-ascending with the apparent arc traversed by the sun along the ecliptic.

Now if the arc of the sun's regular movement is less than the equatorial arc co-ascending with the apparent arc, then the number of actual solar days is less than the number of regular ones. But if the arc of the sun's regular movement is greater, then the number of actual solar days is greater than the number of regular ones. For when the sun moves faster than the regular, then the actual solar day is longer than the mean solar day and there are fewer of them in a given time; but, when it moves slower, then the actual solar day is shorter.

And in each case the difference between the number of actual solar days and the number of corresponding regular solar days is expressed by the difference between the equatorial arc corresponding to the apparent arc on the ecliptic and the arc of regular movement. For (1) if the regular arc is less than the equator's arc, then the apparent arc is reaching the meridian that many time-degrees later; and so just so many time-degrees have been lost to the number of regular solar days. For the same number of regular solar days as actual solar days would only take the sun the length of the arc of regular movement, if we consider it as moving on the

Now, according to the epoch we have chosen (that is, the year 1 of Nabonassar, Egyptianwise Thoth 1 at midday), the sun's mean position, as we have just shown [p. 104], was 0°45′ within the Fishes. But, considered with its anomaly, its position was very nearly 3°8′ within the Fishes.

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equator. It would perhaps be clearer if we imagined that the regular arc were less by a revolution than the equator's arc representing the apparent arc on the ecliptic. Then the number of actual solar days would be one day less than the number of regular solar days. For the sun would have moved back one whole circle, and thus would have come to the meridian one time less over the period assigned than if it had moved back according to the regular movement. And by the same argument (2) if the regular arc is greater, just so many time-degrees have been added to the number of regular solar days. Therefore, in the first case, we add the difference to the number of actual solar days to get the number of regular solar days; and, in the second case, we subtract.

BOOK FOUR

1. From What Kind of Observations an Inquiry into the Moon Must be Conducted

Having now organized in the preceding book all the incidents one could observe with respect to the sun's movement, we begin next in order with the treatise on the moon. First we think it is not proper to proceed simply and haphazardly in the use of observations for this purpose; but for the general understanding of the moon it is best to attend especially to those demonstrations which not only cover a longer period of time but also are gotten from the observations of lunar eclipses. For it is only by means of these that the positions of the moon can be found in an accurate way, since the other kinds of observations which depend either on its courses with respect to the fixed stars, or on instruments, or on the solar eclipses, can, because of the moon's parallaxes, be very deceptive. But with respect to the particular incidents the inquiry can be carried on from the other observations also.

For since the distance from the earth's centre to the lunar sphere is not as that to the ecliptic circle which is so great that the magnitude of the earth is in the ratio of a point to it, therefore the straight line drawn from the moon's centre to sections of the ecliptic, according to which the true courses of all the stars are conceived, necessarily does not everywhere sensibly coincide with the straight line according to which its apparent course is observed—that is, the straight line drawn from some part of the earth's surface or rather from the observer's eye to the moon's centre. But when the moon is directly above the observer, then only are the straight lines drawn from the earth's centre and the observer's eye to the moon's centre and the ecliptic one and the same straight line. And when it is in any way whatsoever removed from the zenith, a difference of inclination in these straight lines follows; and therefore the apparent course differs from the true one for different positions of the eye as it moves downward. For the positions of the moon seen from the earth's centre are determined proportionately to the magnitudes of the angles resulting from the inclination.

Therefore, when solar eclipses take place through the intervention and interposition of the moon, and when their occurrence shapes its passing shadow into a cone from our eye to the sun, the result is that these things are not fulfilled everywhere in the same way either in magnitudes or times, nor to all alike for the causes we have assigned; nor does the moon appear to shadow the same parts of the sun. But in the case of lunar eclipses no such difference follows from the parallaxes, since the eclipse of the moon does not involve the eye of the observer as an incidental cause. For the moon always receives its light from the sun; and, when the moon is opposite it, the whole of it appears to us lighted up, because the whole of the bright hemisphere is also at that time turned towards us. But when it is opposite the sun in such a way that it falls into the cone of the earth's shadow which is always revolving opposite the sun, then the moon be-

comes darkened proportionately to the extent it enters the cone, since the earth screens the rays of the sun. Therefore it appears to be eclipsed similarly for all parts of the earth both as to magnitudes and as to the times of the intervals.

Therefore, insofar as we are concerned with a general examination of the true positions of the moon but not of apparent ones[1] (the true ones to be ascertained because the ordered and similar thing should be necessarily preferred to disordered and dissimilar things), we say we must not use the other observations where the positions depend on the eye of the observers, but only the observations of the moon's eclipses, since there the eye is in no way involved in getting the positions. For whatever section of the ecliptic the sun is found to occupy at the midtime of the eclipse when the moon's centre is directly opposite in true longitude to the sun's centre, the section opposite that section the moon's centre will also truly occupy at the same midtime of the eclipse.

2. On the Moon's Periods of Time

Let this be our brief exposition as to what observations are to be used for general considerations of the moon. We shall try and give an account of how the ancients proceeded in their demonstrations, and how we can make a more useful differentiation of the hypotheses which conform to the appearances.

Now, since the moon appears to move irregularly both in longitude and latitude,[2] and not to cross the ecliptic nor to have a cyclical return in its latitudinal course at equal intervals of time; and since, without the discovery of the interval in which this irregularity is redeemed, necessarily the periods of the others could not be gotten; and since, from particular observations, the moon appears to have its mean, greatest, and least movements in all parts of the zodiac, and to be at its northernmost point, southernmost point, and on the ecliptic itself at all parts of the zodiac, very reasonably therefore the ancient mathematicians sought a certain time in which the moon would always move the same distance in longitude, for this alone could redeem the irregularity. Comparing, then, for the causes we have assigned, the observations of the lunar eclipses, they tried to find what multitude of lunar months would always be isochronous with other equal multitudes and would embrace an equal number of circles in longitude, either whole or in parts. Now the even more ancient ones in a more or less rough

[1] In the case of the sun, there were the mean positions and the true (ἀκριβής) positions. And the true positions were taken as coinciding with the apparent ones. For the true positions are the projections of the sun on the ecliptic by a straight line through the earth's centre and the sun's centre; the apparent ones are the projections on the ecliptic by a straight line through the observer's eye and the sun's centre. In the case of the moon, they are all three distinct by reason of the parallaxes.

[2] The moon's latitudinal movement is its movement along a great circle inclined to the ecliptic. It is not, as might be thought, its movement along an arc at right angles to the ecliptic. This inclined circle is the moon's oblique circle which is not to be confused with the ecliptic itself, the sun's oblique circle. The moon's longitudinal movement is its movement considered with reference to the ecliptic, just as the sun's is its movement with respect to the equator. The sun's movement with respect to the equator is distinguished from its movement along the ecliptic because of the great obliquity of the one to the other. In the case of the moon, the inclination of its oblique circle with respect to the ecliptic is so small as to be negligible for such considerations. But the moon's returns in latitude do not appear to correspond to its returns in longitude—that is, its successive returns to the same parallel to the ecliptic take place at different great circles through the ecliptic's poles. Hence it is necessary to suppose that the nodes of its oblique circle move with respect to the ecliptic, and therefore it is necessary to distinguish its latitudinal movement from its longitudinal.

way assumed that period of time to be 6,585⅓ days. For in this period they saw accomplished very nearly 223 lunar months, 239 restitutions of anomaly, 242 latitudinal cycles, and 241 longitudinal cycles plus the 10⅔° which the sun also adds to its 18 complete revolutions in that time, their restitutions being observed with respect to the fixed stars. And they called this interval of time "periodic" as being the least to collect approximately the differences of the movements into one complete cyclical restitution. In order to arrange it in terms of whole days, they tripled the 6,585⅓ days and got the number 19,756 which they called an evolution [ἐξελιγμός]. And likewise tripling the others, they had 669 lunar months, 717 restitutions of anomaly, 726 latitudinal cycles, and 723 longitudinal cycles plus the 32° which the sun also adds to its 54 complete revolutions.

And again Hipparchus proved by calculations from his own and Chaldean observations that these numbers were not accurate. For he shows by the observations he lays down that the least number of days in which the moment of eclipse recurs, after an equal number of months and an equal number of movements, is 126,007 days and 1 equatorial hour. And in this amount of time he finds fulfilled 4,267 lunar months, 4,573 complete restitutions of anomaly, 4,612 zodiacal revolutions less the 7½° (very nearly) which the sun also lacks of completing 345 circles, these restitutions again being observed with respect to the fixed stars. And so he finds, when the given number of days is distributed over the 4,267 lunar months, that the mean time of the lunar month amounts to very nearly 29 days $31^{i}50^{ii}8^{iii}20^{iv}$. Then he shows that in this period of time the corresponding intervals from lunar eclipse to lunar eclipse are equal, so that the restitution of irregularity becomes evident from the fact that always in this amount of time there are contained just so many lunar months, and that to the 4,611 revolutions, equal in longitude, are added 352½° in accordance with the sun's syzygies.

But if one should seek, not the number of months from lunar eclipse to lunar eclipse, but only from a conjunction or full moon to the corresponding syzygy, he would find a still smaller number for the restitution of anomaly and for the lunar months by taking their only common measure 17, which brings the result to 251 lunar months and 269 restitutions of anomaly. But this period of time was found no longer to complete the latitudinal restitution. For the return of the eclipses appeared to save the equalities only with respect to the intervals of time and of the longitudinal revolutions, but not with respect to the magnitudes and similarities of the shadows cast, from which the latitude is known.

Now, having already found the time of the return of anomaly, Hipparchus again compares the intervals of lunar months bounded by eclipses in every way similar both in the magnitude and length of time of the shadows cast and in which was no anomalistic difference, so that the latitudinal course for that reason appears to make a cyclical return. And he shows that such a revolution is completed in 5,458 lunar months and 5,923 latitudinal cycles.

Now this was somewhat the way in which our predecessors dealt with these researches. But we would consider it neither a simple nor easy way, but one demanding much methodical diligence. For, first, it would be no use for us to grant that the durations of the intervals are accurately found to be equal to each other, unless the sun effected either the same anomalistic difference or none for each of the intervals. For if this should not be so, and there should be, as I said,

some anomalistic difference of the sun, then neither will the sun, nor evidently the moon, have made equal revolutions in equal periods of time. If, for instance, each of the distances compared in addition to the complete and equal solar years takes on another half year, and if in this amount of time the sun happens to have moved in the first interval from its mean movement in the Fishes, and in the second interval from its mean movement in the Virgin, then in the first interval it will have added on less than the semicircle by nearly $4\frac{3}{4}°$; and in the second interval, more than the semicircle by the same amount. And so in the same time the moon has a surplus, over and above complete circles, of $175\frac{1}{4}°$ in the first interval; and $184\frac{3}{4}°$ in the second.

Hence we say it is first necessary that this sort of accident be characteristic of the intervals in their relationship to the sun: either (1) that it embrace whole circles; or (2) that for one interval it add on a semicircle from the apogee and for the other a semicircle from the perigee; or (3) that for both it begin from the same section; or (4) that it be equidistant on each side of either the apogee or the perigee, for the first eclipse in one interval and for the second eclipse in the other. For only in this way would its anomalistic difference be either equal or absent in each interval, so that the extra arcs would be equal either to each other, or to each other and arcs of regular movement.

And, secondly, we think that like attention should be paid to the courses of the moon. For if no distinctions are made here, it will again appear possible for the moon to be able to add on equal longitudinal arcs in equal periods of time without any restitution of its anomaly. And this will happen (1) if for each of the intervals it begins from the same course which is to be added or subtracted and does not stop at the same; or (2) if in one interval it begins from the greatest movement and stops at the least, while in the other it starts from the least and ends at the greatest; or (3) if the first course of one interval and the last course of the other are equidistant on each side of either the course of greatest movement or the course of least movement. For each of these cases, if it come about, will either again produce no anomalistic difference or the same, and therefore make the longitudinal surpluses equal, but will effect no restitution of anomaly. Therefore the intervals considered must fall under none of these cases if they are to embrace a restitution of anomaly.

But on the contrary, if complete restitutions of anomaly are not embraced by the intervals, we should choose those which can most thoroughly manifest the inequality; that is, we should choose them when they begin not only from different courses but also from ones very different either in magnitude or power; (a) in magnitude as when in one interval it begins from the least course and does not end with the greatest and when in the other interval it begins from the greatest and does not end with the least, for in this way the difference of longitudinal surplus will be greatest if whole cycles of anomaly are not completed, especially when one quadrant or three quadrants of one anomaly are left over, the intervals then being unequal by twice the anomalistic difference; and (b) in power as when in each of the intervals it begins from the mean course, and not from the same mean, but in one interval from the mean additively and in the other subtractively; for in this way, if the anomaly is not restored, the longitudinal surpluses will differ from each other the most—by twice the anomalistic difference if the surpluses are again a quadrant or three quadrants of an anomalistic cycle, or four times the difference if a half cycle.

And, therefore, we see that Hipparchus was most attentive, as he judged, in his choice of the intervals gotten for this research. And we see he used the case where the moon, in one interval, started from the greatest course and did not stop with the least, and, in the other interval, started from the least and did not stop with the greatest; and redeemed the resulting anomalistic difference of the sun, although it is short a bit because the sun's return fails of complete circles by very nearly one quarter of a sign which is not the same sign and does not produce an equal anomalistic difference in each of the intervals.

And we say these things, not to belittle this method of getting at the periods of return, but to bring to mind that, when it is done with the proper care and consequent calculation, it can rectify the difference; but that, if any at all of the incidents explained are neglected, it will completely falsify the solution sought; and that a rectification which accurately takes into account all the incidents of the observations is very difficult for those who are making a discerning choice of these observations.

Now, of the periodic returns given according to the calculations made by Hipparchus, that of the lunar month, as we said, appears to have been calculated as soundly as possible and not to differ by any sensible amount from the true one. But the periodic return in latitude and the restitution of anomaly appear to be off by an appreciable amount. So much so that we have immediately seen the flaw from the simpler and easier means employed by us in view of the same discrimination. And we shall immediately demonstrate these methods along with the quantity of the moon's anomaly. But first we set out, for greater service in what follows, the particular mean movements in latitude, longitude, and anomaly, in accordance with the proposed times of return of the periodic movements modified by that correction which will be demonstrated later.

3. On the Moon's Regular Movements in Detail

If now we multiply the sun's mean daily movement, proved to be very nearly $0°59^{i}8^{ii}17^{iii}13^{iv}12^{v}31^{vi}$, by the 29 days $31^{i}50^{ii}8^{iii}20^{iv}$ of one lunar month, and add to the result the 360° of one circle, then we shall have the longitudinal course of the moon's mean movement in one lunar month—that is, very nearly $389°6^{i}23^{ii}1^{iii}24^{iv}2^{v}30^{vi}57^{vii}$. Distributing these over the days of the lunar month we shall have the moon's daily movement in longitude—that is, very nearly $13°10^{i}34^{ii}58^{iii}33^{iv}30^{v}30^{vi}$.

Again, multiplying the 269 circles of the anomaly by the 360° of one circle, we shall have the number 96,840°. Distributing these over the 7,412 days $10^{i}44^{ii}51^{iii}40^{iv}$ of the 251 lunar months, we shall have a daily mean movement of anomaly of $13°3^{i}53^{ii}56^{iii}29^{iv}38^{v}38^{vi}$.

Likewise multiplying the 5,923 returns in latitude by the 360° of one circle, we shall have the number 2,132,280°. Distributing these over the 161,277 days $58^{i}58^{ii}3^{iii}20^{iv}$ of the 5,458 lunar months, we shall have the daily mean latitudinal movement of $13°13^{i}45^{ii}39^{iii}40^{iv}17^{v}19^{vi}$.

Again, subtracting the sun's daily mean movement from the moon's daily mean movement in longitude, we shall have the daily mean elongation of $12°11^{i}26^{ii}41^{iii}20^{iv}17^{v}59^{vi}$.

Through the methods we shall employ later on, as we have already said, we find that the daily mean movement in longitude remains practically unchanged and that, of course, the movement of elongation remains about the same, too;

but that the daily mean movement of anomaly is less by $11^{i}46^{v}39^{vi}$, so that it becomes $13°3^{i}53^{ii}56^{iii}17^{iv}51^{v}59^{vi}$, and that the daily mean movement in latitude is more by $8^{iv}39^{v}18^{vi}$, so that it becomes $13°13^{i}45^{ii}39^{iii}48^{iv}56^{v}37^{vi}$.

If, in accordance with these daily movements, we take one twenty-fourth of each, we shall have the hourly mean movement in longitude of $0°32^{i}56^{ii}27^{iii}26^{iv}23^{v}46^{vi}15^{vii}$; the hourly mean movement of anomaly of $0°32^{i}39^{ii}44^{iii}50^{iv}44^{v}39^{vi}57^{vii}30^{viii}$; in latitude of $0°33^{i}4^{ii}24^{iii}9^{iv}32^{v}21^{vi}32^{vii}30^{viii}$; and of elongation of $0°30^{i}28^{ii}36^{iii}43^{iv}20^{v}44^{vi}57^{vii}30^{viii}$. And if we multiply the daily movements by thirty and subtract the complete circles, we shall have the monthly mean surplus in longitude of $35°17^{i}29^{ii}16^{iii}45^{iv}15^{v}$; the monthly mean surplus of anomaly of $31°56^{i}58^{ii}8^{iii}55^{iv}59^{v}30^{vi}$; in latitude of $36°52^{i}49^{ii}54^{iii}28^{iv}18^{v}31^{vi}$; and of elongation of $5°43^{i}20^{ii}40^{iii}8^{iv}59^{v}30^{vi}$.

Again, multiplying the daily mean movements by the 365 days of an Egyptian year and subtracting the complete circles, we shall have the yearly mean surplus in longitude of $129°22^{i}46^{ii}13^{iii}50^{iv}32^{v}30^{vi}$; the yearly mean surplus of anomaly of $88°43^{i}7^{ii}28^{iii}41^{iv}13^{v}55^{vi}$; in latitude of $148°42^{i}47^{ii}12^{iii}44^{iv}25^{v}5^{vi}$; and of elongation of $129°37^{i}21^{ii}28^{iii}29^{iv}23^{v}55^{vi}$.

Next, multiplying the yearly movements by eighteen for greater facility as we said, in using the table, and subtracting the complete circles, we shall have an eighteen-year mean surplus in longitude of $168°49^{i}52^{ii}9^{iii}9^{iv}45^{v}$; an eighteen-year mean surplus of anomaly of $156°56^{i}14^{ii}36^{iii}22^{iv}10^{v}30^{vi}$; in latitude of $156°50^{i}9^{ii}49^{iii}19^{iv}31^{v}30^{vi}$; and of elongation of $173°12^{i}26^{ii}32^{iii}49^{iv}10^{v}30^{vi}$.

Then we shall construct, as in the case of the sun, 3 tables, again of 45 rows and 5 columns each. Now the first column will contain the particular times: in the first table, the eighteen-year periods; in the second table, the years and then again the hours; in the third table, the months and then again the days. And the other four columns contain the particular comparisons in degrees: the second column those in longitude, the third those of anomaly, the fourth those in latitude, and the fifth those of elongation. See tables on following pages.

4. Tables of the Mean Movements of the Moon, Table I

Col. 1	*Col. 2*	*Longitudinal Surplus 11°22′ within the Bull*[1]						*Col. 3*	*Surplus of Anomaly 268°49′*					
Years	*Degrees*	I	II	III	IV	V	VI	*Degrees*	I	II	III	IV	V	VI
18	168	49	52	9	9	45	0	156	56	14	36	22	10	30
36	337	39	44	18	19	30	0	313	52	29	12	44	21	0
54	146	29	36	27	29	15	0	110	48	43	49	6	31	30
72	315	19	28	36	39	0	0	267	44	58	25	28	42	0
90	124	9	20	45	48	45	0	64	41	13	1	50	52	30
108	292	59	12	54	58	30	0	221	37	27	38	13	3	0
126	101	49	5	4	8	15	0	18	33	42	14	35	13	30
144	270	38	57	13	18	0	0	175	29	56	50	57	24	0
162	79	28	49	22	27	45	0	332	26	11	27	19	34	30
180	248	18	41	31	37	30	0	129	22	26	3	41	45	0
198	57	8	33	40	47	15	0	286	18	40	40	3	55	30
216	225	58	25	49	57	0	0	83	14	55	16	26	6	0
234	34	48	17	59	6	45	0	240	11	9	52	48	16	30
252	203	38	10	8	16	30	0	37	7	24	29	10	27	0
270	12	28	2	17	26	15	0	194	3	39	5	32	37	30
288	181	17	54	26	36	0	0	350	59	53	41	54	48	0
306	350	7	46	35	45	45	0	147	56	8	18	16	58	30
324	158	57	38	44	55	30	0	304	52	22	54	39	9	0
342	327	47	30	54	5	15	0	101	48	37	31	1	19	30
360	136	37	23	3	15	0	0	258	44	52	7	23	30	0
378	305	27	15	12	24	45	0	55	41	6	43	45	40	30
396	114	17	7	21	34	30	0	212	37	21	20	7	51	0
414	283	6	59	30	44	15	0	9	33	35	56	30	1	30
432	91	56	51	39	54	0	0	166	29	50	32	52	12	0
450	260	46	43	49	3	45	0	323	26	5	9	14	22	30
468	69	36	35	58	13	30	0	120	22	19	45	36	33	0
486	238	26	28	7	23	15	0	277	18	34	21	58	43	30
504	47	16	20	16	33	0	0	74	14	48	58	20	54	0
522	216	6	12	25	42	45	0	231	11	3	34	43	4	30
540	24	56	4	34	52	30	0	28	7	18	11	5	15	0
558	193	45	56	44	2	15	0	185	3	32	47	27	25	30
576	2	35	48	53	12	0	0	341	59	47	23	49	36	0
594	171	25	41	2	21	45	0	138	56	2	0	11	46	30
612	340	15	33	11	31	30	0	295	52	16	36	33	57	0
630	149	5	25	20	41	15	0	92	48	31	12	56	7	30
648	317	55	17	29	51	0	0	249	44	45	49	18	18	0
666	126	45	9	39	0	45	0	46	41	0	25	40	28	30
684	295	35	1	48	10	30	0	203	37	15	2	2	39	0
702	104	24	53	57	20	15	0	0	33	29	38	24	49	30
720	273	14	46	6	30	0	0	157	29	44	14	47	0	0
738	82	4	38	15	39	45	0	314	25	58	51	9	10	30
756	250	54	30	24	49	30	0	111	22	13	27	31	21	0
774	59	44	22	33	59	15	0	268	18	28	3	53	31	30
792	228	34	14	43	9	0	0	65	14	42	40	15	42	0
810	37	24	6	52	18	45	0	222	10	57	16	37	52	30

[1]These numbers under the headings refer, of course, to the surpluses at the epochs in the year 1 of Nabonassar.

4. TABLES OF THE MEAN MOVEMENTS OF THE MOON, TABLE I—Continued

Col. 1	*Col. 4*	*Latitudinal Surplus 354°15′*						*Col. 5*	*Surplus of Elongation 70°37′*					
Years	*Degrees*	I	II	III	IV	V	VI	*Degrees*	I	II	III	IV	V	VI
18	156	50	9	49	19	31	30	173	12	26	32	49	10	30
36	313	40	19	38	39	3	0	346	24	53	5	38	21	0
54	110	30	29	27	58	34	30	159	37	19	38	27	31	30
72	267	20	39	17	18	6	0	332	49	46	11	16	42	0
90	64	10	49	6	37	37	30	146	2	12	44	5	52	30
108	221	0	58	55	57	9	0	319	14	39	16	55	3	0
126	17	51	8	45	16	40	30	132	27	5	49	44	13	30
144	174	41	18	34	36	12	0	305	39	32	22	33	24	0
162	331	31	28	23	55	43	30	118	51	58	55	22	34	30
180	128	21	38	13	15	15	0	292	4	25	28	11	45	0
198	285	11	48	2	34	46	30	105	16	52	1	0	55	30
216	82	1	57	51	54	18	0	278	29	18	33	50	6	0
234	238	52	7	41	13	49	30	91	41	45	6	39	16	30
252	35	42	17	30	33	21	0	264	54	11	39	28	27	0
270	192	32	27	19	52	52	30	78	6	38	12	17	37	30
288	349	22	37	9	12	24	0	251	19	4	45	6	48	0
306	146	12	46	58	31	55	30	64	31	31	17	55	58	30
324	303	2	56	47	51	27	0	237	43	57	50	45	9	0
342	99	53	6	37	10	58	30	50	56	24	23	34	19	30
360	256	43	16	26	30	30	0	224	8	50	56	23	30	0
378	53	33	26	15	50	1	30	37	21	17	29	12	40	30
396	210	23	36	5	9	33	0	210	33	44	2	1	51	0
414	7	13	45	54	29	4	30	23	46	10	34	51	1	30
432	164	3	55	43	48	36	0	196	58	37	7	40	12	0
450	320	54	5	33	8	7	30	10	11	3	40	29	22	30
468	117	44	15	22	27	39	0	183	23	30	13	18	33	0
486	274	34	25	11	47	10	30	356	35	56	46	7	43	30
504	71	24	35	1	6	42	0	169	48	23	18	56	54	0
522	228	14	44	50	26	13	30	343	0	49	51	46	4	30
540	25	4	54	39	45	45	0	156	13	16	24	35	15	0
558	181	55	4	29	5	16	30	329	25	42	57	24	25	30
576	338	45	14	18	24	48	0	142	38	9	30	13	36	0
594	135	35	24	7	44	19	30	315	50	36	3	2	46	30
612	292	25	33	57	3	51	0	129	3	2	35	51	57	0
630	89	15	43	46	23	22	30	302	15	29	8	41	7	30
648	246	5	53	35	42	54	0	115	27	55	41	30	18	0
666	42	56	3	25	2	25	30	288	40	22	14	19	28	30
684	199	46	13	14	21	57	0	101	52	48	47	8	39	0
702	356	36	23	3	41	28	30	275	5	15	19	57	49	30
720	153	26	32	53	1	0	0	88	17	41	52	47	0	0
738	310	16	42	42	20	31	30	261	30	8	25	36	10	30
756	107	6	52	31	40	3	0	74	42	34	58	25	21	0
774	263	57	2	20	59	34	30	247	55	1	31	14	31	30
792	60	47	12	10	19	6	0	61	7	28	4	3	42	0
810	217	37	21	59	38	37	30	234	19	54	36	52	52	30

4. Tables of the Mean Movements of the Moon, Table II

Col. 1	*Col. 2*	*Longitudinal Surplus*						*Col. 3*	*Surplus of Anomaly*					
Years	*Degrees*	I	II	III	IV	V	VI	*Degrees*	I	II	III	IV	V	VI
1	129	22	46	13	50	32	30	88	43	7	28	41	13	55
2	258	45	32	27	41	5	0	177	26	14	57	22	27	50
3	28	8	18	41	31	37	30	266	9	22	26	3	41	45
4	157	31	4	55	22	10	0	354	52	29	54	44	55	40
5	286	53	51	9	12	42	30	83	35	37	23	26	9	35
6	56	16	37	23	3	15	0	172	18	44	52	7	23	30
7	185	39	23	36	53	47	30	261	1	52	20	48	37	25
8	315	2	9	50	44	20	0	349	44	59	49	29	51	20
9	84	24	56	4	34	52	30	78	28	7	18	11	5	15
10	213	47	42	18	25	25	0	167	11	14	46	52	19	10
11	343	10	28	32	15	57	30	255	54	22	15	33	33	5
12	112	33	14	46	6	30	0	344	37	29	44	14	47	0
13	241	56	0	59	57	2	30	73	20	37	12	56	0	55
14	11	18	47	13	47	35	0	162	3	44	41	37	14	50
15	140	41	33	27	38	7	30	250	46	52	10	18	28	45
16	270	4	19	41	28	40	0	339	29	59	38	59	42	40
17	39	27	5	55	19	12	30	68	13	7	7	40	56	35
18	168	49	52	9	9	45	0	156	56	14	36	22	10	30
Hours	*Longitudinal Surplus*							*Surplus of Anomaly*						
1	0	32	56	27	26	23	46	0	32	39	44	50	44	40
2	1	5	52	54	52	47	32	1	5	19	29	41	29	20
3	1	38	49	22	19	11	18	1	37	59	14	32	14	0
4	2	11	45	49	45	35	5	2	10	38	59	22	58	40
5	2	44	42	17	11	58	51	2	43	18	44	13	43	20
6	3	17	38	44	38	22	37	3	15	58	29	4	28	0
7	3	50	35	12	4	46	23	3	48	38	13	55	12	40
8	4	23	31	39	31	10	10	4	21	17	58	45	57	20
9	4	56	28	6	57	33	56	4	53	57	43	36	42	0
10	5	29	24	34	23	57	42	5	26	37	28	27	26	40
11	6	2	21	1	50	21	28	5	59	17	13	18	11	20
12	6	35	17	29	16	45	15	6	31	56	58	8	56	0
13	7	8	13	56	43	9	1	7	4	36	42	59	40	39
14	7	41	10	24	9	32	47	7	37	16	27	50	25	19
15	8	14	6	51	35	56	33	8	9	56	12	41	9	59
16	8	47	3	19	2	20	20	8	42	35	57	31	54	39
17	9	19	59	46	28	44	6	9	15	15	42	22	39	19
18	9	52	56	13	55	7	52	9	47	55	27	13	23	59
19	10	25	52	41	21	31	38	10	20	35	12	4	8	39
20	10	58	49	8	47	55	25	10	53	14	56	54	53	19
21	11	31	45	36	14	19	11	11	25	54	41	45	37	59
22	12	4	42	3	40	42	57	11	58	34	26	36	22	39
23	12	37	38	31	7	6	43	12	31	14	11	27	7	19
24	13	10	34	58	33	30	30	13	3	53	56	17	51	59

4. Tables of the Mean Movements of the Moon, Table II—Continued

Col. 1 Single Years	*Col. 4* Degrees	*Latitudinal Surplus* I	II	III	IV	V	VI	*Col. 5* Degrees	*Surplus of Elongation* I	II	III	IV	V	VI
1	148	42	47	12	44	25	5	129	37	21	28	29	23	55
2	297	25	34	25	28	50	10	259	14	42	56	58	47	50
3	86	8	21	38	13	15	15	28	52	4	25	28	11	45
4	234	51	8	50	57	40	20	158	29	25	53	57	35	40
5	23	33	56	3	42	5	25	288	6	47	22	26	59	35
6	172	16	43	16	26	30	30	57	44	8	50	56	23	30
7	320	59	30	29	10	55	35	187	21	30	19	25	47	25
8	109	42	17	41	55	20	40	316	58	51	47	55	11	20
9	258	25	4	54	39	45	45	86	36	13	16	24	35	15
10	47	7	52	7	24	10	50	216	13	34	44	53	59	10
11	195	50	39	20	8	35	55	345	50	56	13	23	23	5
12	344	33	26	32	53	1	0	115	28	17	41	52	47	0
13	133	16	13	45	37	26	5	245	5	39	10	22	10	55
14	281	59	0	58	21	51	10	14	43	0	38	51	34	50
15	70	41	48	11	6	16	15	144	20	22	7	20	58	45
16	219	24	35	23	50	41	20	273	57	43	35	50	22	40
17	8	7	22	36	35	6	25	43	35	5	4	19	46	35
18	156	50	9	49	19	31	30	173	12	26	32	49	10	30
Hours		*Latitudinal Surplus*							*Surplus of Elongation*					
1	0	33	4	24	9	32	22	0	30	28	36	43	20	45
2	1	6	8	48	19	4	43	1	0	57	13	26	41	30
3	1	39	13	12	28	37	5	1	31	25	50	10	2	15
4	2	12	17	36	38	9	26	2	1	54	26	53	23	0
5	2	45	22	0	47	41	48	2	32	23	3	36	43	45
6	3	18	26	24	57	14	9	3	2	51	40	20	4	30
7	3	51	30	49	6	46	31	3	33	20	17	3	25	15
8	4	24	35	13	16	18	52	4	3	48	53	46	46	0
9	4	57	39	37	25	51	14	4	34	17	30	30	6	45
10	5	30	44	1	35	23	35	5	4	46	7	13	27	30
11	6	3	48	25	44	55	57	5	35	14	43	56	48	15
12	6	36	52	49	54	28	19	6	5	43	20	40	9	0
13	7	9	57	14	4	0	40	6	36	11	57	23	29	44
14	7	43	1	38	13	33	2	7	6	40	34	6	50	29
15	8	16	6	2	23	5	23	7	37	9	10	50	11	14
16	8	49	10	26	32	37	45	8	7	37	47	33	31	59
17	9	22	14	50	42	10	6	8	38	6	24	16	52	44
18	9	55	19	14	51	42	28	9	8	35	1	0	13	29
19	10	28	23	39	1	14	49	9	39	3	37	43	34	14
20	11	1	28	3	10	47	11	10	9	32	14	26	54	59
21	11	34	32	27	20	19	32	10	40	0	51	10	15	44
22	12	7	36	51	29	51	54	11	10	29	27	53	36	29
23	12	40	41	15	39	24	15	11	40	58	4	36	57	14
24	13	13	45	39	48	56	37	12	11	26	41	20	17	59

4. Tables of the Mean Movements of the Moon, Table III

Col. 1	*Col. 2*	*Longitudinal Surplus*						*Col. 3*	*Surplus of Anomaly*					
Months	*Degrees*	I	II	III	IV	V	VI	*Degrees*	I	II	III	IV	V	VI
30	35	17	29	16	45	15	0	31	56	58	8	55	59	30
60	70	34	58	33	30	30	0	63	53	56	17	51	59	0
90	105	52	27	50	15	45	0	95	50	54	26	47	58	30
120	141	9	57	7	1	0	0	127	47	52	35	43	58	0
150	176	27	26	23	46	15	0	159	44	50	44	39	57	30
180	211	44	55	40	31	30	0	191	41	48	53	35	57	0
210	247	2	24	57	16	45	0	223	38	47	2	31	56	30
240	282	19	54	14	2	0	0	255	35	45	11	27	56	0
270	317	37	23	30	47	15	0	287	32	43	20	23	55	30
300	352	54	52	47	32	30	0	319	29	41	29	19	55	0
330	28	12	22	4	17	45	0	351	26	39	38	15	54	30
360	63	29	51	21	3	0	0	23	23	37	47	11	54	0
Days		*Longitudinal Surplus*							*Surplus of Anomaly*					
1	13	10	34	58	33	30	30	13	3	53	56	17	51	59
2	26	21	9	57	7	1	0	26	7	47	52	35	43	58
3	39	31	44	55	40	31	30	39	11	41	48	53	35	57
4	52	42	19	54	14	2	0	52	15	35	45	11	27	56
5	65	52	54	52	47	32	30	65	19	29	41	29	19	55
6	79	3	29	51	21	3	0	78	23	23	37	47	11	54
7	92	14	4	49	54	33	30	91	27	17	34	5	3	53
8	105	24	39	48	28	4	0	104	31	11	30	22	55	52
9	118	35	14	47	1	34	30	117	35	5	26	40	47	51
10	131	45	49	45	35	5	0	130	38	59	22	58	39	50
11	144	56	24	44	8	35	30	143	42	53	19	16	31	49
12	158	6	59	42	42	6	0	156	46	47	15	34	23	48
13	171	17	34	41	15	36	30	169	50	41	11	52	15	47
14	184	28	9	39	49	7	0	182	54	35	8	10	7	46
15	197	38	44	38	22	37	30	195	58	29	4	27	59	45
16	210	49	19	36	56	8	0	209	2	23	0	45	51	44
17	223	59	54	35	29	38	30	222	6	16	57	3	43	43
18	237	10	29	34	3	9	0	235	10	10	53	21	35	42
19	250	21	4	32	36	39	30	248	14	4	49	39	27	41
20	263	31	39	31	10	10	0	261	17	58	45	57	19	40
21	276	42	14	29	43	40	30	274	21	52	42	15	11	39
22	289	52	49	28	17	11	0	287	25	46	38	33	3	38
23	303	3	24	26	50	41	30	300	29	40	34	50	55	37
24	316	13	59	25	24	12	0	313	33	34	31	8	47	36
25	329	24	34	23	57	42	30	326	37	28	27	26	39	35
26	342	35	9	22	31	13	0	339	41	22	23	44	31	34
27	355	45	44	21	4	43	30	352	45	16	20	2	23	33
28	8	56	19	19	38	14	0	5	49	10	16	20	15	32
29	22	6	54	18	11	44	30	18	53	4	12	38	7	31
30	35	17	29	16	45	15	0	31	56	58	8	55	59	30

4. Tables of the Mean Movements of the Moon, Table III—Continued

Col. 1	*Col. 4*	*Latitudinal Surplus*						*Col. 5*	*Surplus of Elongation*					
Months	*Degrees*	I	II	III	IV	V	VI	*Degrees*	I	II	III	IV	V	VI
30	36	52	49	54	28	18	30	5	43	20	40	8	59	30
60	73	45	39	48	56	37	0	11	26	41	20	17	59	0
90	110	38	29	43	24	55	30	17	10	2	0	26	58	30
120	147	31	19	37	53	14	0	22	53	22	40	35	58	0
150	184	24	9	32	21	32	30	28	36	43	20	44	57	30
180	221	16	59	26	49	51	0	34	20	4	0	53	57	0
210	258	9	49	21	18	9	30	40	3	24	41	2	56	30
240	295	2	39	15	46	28	0	45	46	45	21	11	56	0
270	331	55	29	10	14	46	30	51	30	6	1	20	55	30
300	8	48	19	4	43	5	0	57	13	26	41	29	55	0
330	45	41	8	59	11	23	30	62	56	47	21	38	54	30
360	82	33	58	53	39	42	0	68	40	8	1	47	54	0
Days	*Latitudinal Surplus*							*Surplus of Elongation*						
1	13	13	45	39	48	56	37	12	11	26	41	20	17	59
2	26	27	31	19	37	53	14	24	22	53	22	40	35	58
3	39	41	16	59	26	49	51	36	34	20	4	0	53	57
4	52	55	2	39	15	46	28	48	45	46	45	21	11	56
5	66	8	48	19	4	43	5	60	57	13	26	41	29	55
6	79	22	33	58	53	39	42	73	8	40	8	1	47	54
7	92	36	19	38	42	36	19	85	20	6	49	22	5	53
8	105	50	5	18	31	32	56	97	31	33	30	42	23	52
9	119	3	50	58	20	29	33	109	43	0	12	2	41	51
10	132	17	36	38	9	26	10	121	54	26	53	22	59	50
11	145	31	22	17	58	22	47	134	5	53	34	43	17	49
12	158	45	7	57	47	19	24	146	17	20	16	3	35	48
13	171	58	53	37	36	16	1	158	28	46	57	23	53	47
14	185	12	39	17	25	12	38	170	40	13	38	44	11	46
15	198	26	24	57	14	9	15	182	51	40	20	4	29	45
16	211	40	10	37	3	5	52	195	3	7	1	24	47	44
17	224	53	56	16	52	2	29	207	14	33	42	45	5	43
18	238	7	41	56	40	59	6	219	26	0	24	5	23	42
19	251	21	27	36	29	55	43	231	37	27	5	25	41	41
20	264	35	13	16	18	52	20	243	48	53	46	45	59	40
21	277	48	58	56	7	48	57	256	0	20	28	6	17	39
22	291	2	44	35	56	45	34	268	11	47	9	26	35	38
23	304	16	30	15	45	42	11	280	23	13	50	46	53	37
24	317	30	15	55	34	38	48	292	34	40	32	7	11	36
25	330	44	1	35	23	35	25	304	46	7	13	27	29	35
26	343	57	47	15	12	32	2	316	57	33	54	47	47	34
27	357	11	32	55	1	28	39	329	9	0	36	8	5	33
28	10	25	18	34	50	25	16	341	20	27	17	28	23	32
29	23	39	4	14	39	21	53	353	31	53	58	48	41	31
30	36	52	49	54	28	18	30	5	43	20	40	8	59	30

5. That, in the Case of the Simple Hypothesis of the Moon, both the Hypothesis of Eccentricity and of the Epicycle Produce the Same Appearances

And after these things there follows an explication of the manner and size of the moon's anomaly. At present we shall develop the explication as if there were that one anomaly belonging to it to which alone nearly all our predecessors appear to have turned their attention; I mean to that anomaly completed in the period of return already mentioned. Afterwards we shall show that the moon effects a second anomaly in its elongations from the sun,[1] an anomaly which becomes greatest at the two quarters and is twice redeemed in the month's time, at conjunctions and full moons.

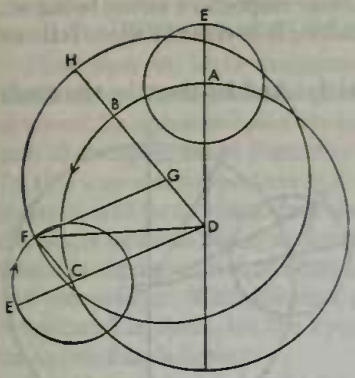
We shall use this order of demonstration because the second can never be found without the first's being implied by it. But the first can be found without the second since it is derived from lunar eclipses where there is no sensible difference due to the anomaly relative to the sun. And in the first proof we shall follow the methods we see Hipparchus used. For considering three lunar eclipses, we shall demonstrate the size of the greatest difference at the point of mean movement and the position of the apogee; we shall show that considered in itself this first anomaly can be effected both by the hypothesis of the epicycle and the hypothesis of eccentricity and the appearances will be the same; but that this last hypothesis would be more properly applied to the second anomaly relative to the sun in the combination of both anomalies.

And so, in making this research, we shall see that again in the case of the simple anomaly of the moon the appearances are the same in either of these hypotheses even if the time-periods of the two returns—(1) that relative to the anomaly and (2) that relative to the ecliptic—are not equal to each other as in the case of the sun, but are unequal as in the case of the moon, with only the ratios being again supposed the same. Since, then, the moon effects its return with respect to the ecliptic more rapidly than its return with respect to this anomaly, therefore it is evident that, on the hypothesis of the epicycle, in equal times the epicycle will move on the circle concentric with the ecliptic through an arc greater than the one similar to that traversed by the moon on the epicycle itself; but that, on the hypothesis of eccentricity, the moon will traverse an arc on the eccentric circle similar to that traversed on the epicycle, and the eccentric will turn about the ecliptic's centre, in the same direction as the moon,[2] through an arc equal to the excess of the longitudinal course over the anomalistic course—that is, the excess of the concentric circle's arc over the epicycle's. For in this way not only the sameness of the ratios but also the equality of the times of each of the movements will be saved in both hypotheses.

Now, these things being supposed as necessary and immediate consequences,

[1]This second anomaly of the moon with respect to the sun is the first of those appearances which tie the planets to the sun instead of the earth. As we shall see later, each of the five planets has an anomaly with respect to the sun besides its other anomalies. And thus even on the geocentric hypothesis of Ptolemy, there are manifested inevitably heliocentric appearances. And so it is no wonder the Greeks had heliocentric theories. Nor is the appearance of the Copernican theory anything very extraordinary; Copernicus had only to look at Ptolemy. Ptolemy himself sees all this when he says, in Book I, that the heliocentric theory would be simpler as far as the planets are concerned.

[2]That is, both movements of the eccentric hypothesis—(1) that of the moon on the ec-



let there be the circle ABC concentric with the ecliptic, with centre D and diameter AD; and the epicycle EF with centre C. And let it be assumed that, when the epicycle was at A, the moon was at E, the epicycle's apogee; and that in the same time the epicycle has traversed the arc AC and the moon the arc EF. And let DE and CF be joined. And since the arc AC is greater than the arc similar to EF, let BC be taken similar to EF and let BD be joined. It is clear, then, that in the same time the eccentric circle has moved through angle ADB, the excess of one of the courses over the other, and that its centre and apogee have come to be along the straight line BD. This being so, let DG be laid off equal to CF, and let FG be joined. And with centre G and radius FG, let the eccentric circle FH be drawn.

I say that

$$FG : DG :: CD : CF;$$

and that, according to this last hypothesis also, the moon will be at point F—that is, arc FH will also be similar to arc EF.

For since

$$\text{angle } BDC = \text{angle } ECF,$$

CF is parallel to DG. And

$$CF = DG;$$

and therefore

$$FG = CD,$$

and is parallel to it. And

$$FG : DG :: CD : CF.$$

Again, since CD is parallel to FG,

$$\text{angle } BDC = \text{angle } FGH.$$

But it was supposed

$$\text{angle } BDC = \text{angle } ECF.$$

And so also arc FH is similar to arc EF. Therefore, in the same time according to either hypothesis, the moon has arrived at the point F, since it has moved through the epicycle's arc EF and the eccentric's arc FH, which have been proved similar, and since the epicycle's centre has moved through arc AC and the eccentric's centre through arc AB, the excess of arc AC over arc EF. Which was to be proved.

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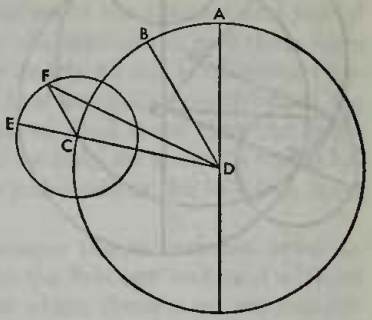
centric, and (2) that of the eccentric about the ecliptic's centre or the earth—are from west to east in the general direction of the moon's movement on the ecliptic or of the epicycle's centre on the concentric circle.

The equivalent epicyclic hypothesis is, of course, the case where the epicycle's centre moves counter-clockwise while the epicycle itself turns clockwise. In the case of the moon, Ptolemy does not explicitly state, as he does with the other five planets, why it is necessary to take one epicyclic hypothesis and not the other. The reason is obvious. By the tables of observations it was known that the time from the moon's mean movement in longitude through its greatest movement back to the mean is always less than the time from its mean movement through its least movement back to the mean.

And, even if only the ratios are equal without their respective terms being so, and the eccentric circle is not equal to the concentric, it is clear to us as follows that the result will be the same.

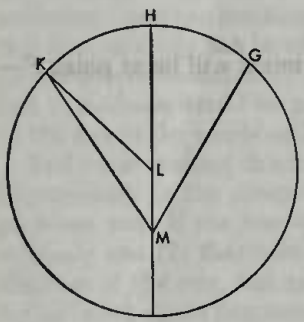
For let each of the hypotheses be drawn separately. And let there be the circle *ABC* concentric with the ecliptic, with centre *D* and diameter *AD;* and the epicycle *EF* with centre *C*. And let the moon be at *F*. And again let there be the eccentric circle *GHK* with centre *L* and diameter *HLM*. And on this diameter let there be the ecliptic's centre *M;* and let the point *K* be the moon. And in the first figure let *DCE*, *CF*, *DF* be joined; and in the second figure *GM*, *KM*, *KL*.



And let

$$CD : CE :: HL : LM.$$

And let the epicycle have moved through angle *ADC* and the moon through angle *ECF* in the same time. And let the eccentric circle have moved through angle *GMH* and the moon in turn through angle *HLK* in the same time. Then, because of the relations assumed to hold between the movements,



$$\text{angle } ECF = \text{angle } HLK$$

and

$$\text{angle } ADC = \text{angle } GMH + \text{angle } HLK.$$

This being so, I say that, again on either hypothesis, in the same time the moon will appear to have traversed equal arcs; that is,

$$\text{angle } ADF = \text{angle } GMK.$$

For the moon at the beginning of the interval is at the apogees, and appears along the straight lines *AD* and *GM;* and at the end of the interval is at the points *F* and *K*, and appears along the straight lines *DF* and *KM*.

For again let arc *BC* be laid out similar to each of the arcs *HK* and *EF*, and let *BD* be joined. Then, since

$$CD : CF :: KL : LM,$$

and since the sides about the equal angles at *C* and *L* are for that reason proportional, therefore the triangle *CDF* is equiangular with the triangle *KLM* and the angles opposite the proportional sides are equal. Therefore

$$\text{angle } CFD = \text{angle } KML.$$

But also

$$\text{angle } BDF = \text{angle } CFD$$

since it is supposed

$$\text{angle } ECF = \text{angle } BDC$$

and since, therefore, *CF* and *BD* are parallel. Therefore

$$\text{angle } BDF = \text{angle } KML.$$

But angle *ADB*, the excess of one movement over the other, is assumed equal to angle *GMH*, the passage of the eccentric. And therefore, by addition,

$$\text{angle } ADF = \text{angle } GMK.$$

Which was to be proved.

6. Demonstration of the Moon's First and Simple Anomaly

Now that we have understood this much, we shall demonstrate this lunar anomaly on the hypothesis of the epicycle for the reason we assigned, first using three of the oldest eclipses we have which seem to have been faithfully recorded, and then again three from the present very accurately observed by ourselves. In this way our inspection will extend over the longest time possible; and especially it will be evident that the anomalistic difference comes to be very nearly the same in both cases, and that the surplus of the mean movements is always found to agree with that computed from the periods of time already given with our corrections. With respect, then, to the demonstration of the first anomaly considered in itself, let the hypothesis of the epicycle, the one we have chosen, be applied as follows:

Let the concentric circle be conceived on the lunar sphere and lying in the same plane with the ecliptic. And let another circle be conceived inclined to this one proportionately to the quantity of the moon's latitudinal course, and borne from east to west around the ecliptic's centre at a regular speed equal to the excess of the latitudinal movement over the longitudinal. Then on this oblique circle we suppose the epicycle to be moving regularly but from west to east according to the latitudinal return, which (considered with respect to the ecliptic) produces, of course, the longitudinal movement. And on the epicycle we suppose the moon to be passing through the arc of the apogee from east to west, according to the anomalistic return. Yet in this demonstration we shall not be concerned with the latitudinal advance nor with the obliquity of the lunar circle since no appreciable difference results in the longitudinal movement from an inclination of this amount.

I.—Then of the three ancient eclipses observed in Babylon, of which we spoke, the first is recorded as having taken place in the year 1 of Mardokempad, Egyptianwise Thoth 29-30. And the eclipse began, it is stated, more than one hour after the rise of the moon, and the eclipse was total. Now, since the sun was very nearly at the end of the Fishes, and the night was very nearly 12 equatorial hours, evidently the beginning of the eclipse was $4\frac{1}{2}$ equatorial hours before midnight, and the middle of the eclipse, since it was complete, was $2\frac{1}{2}$ hours before midnight. Therefore, in Alexandria, the midtime of this eclipse occurred $3\frac{1}{3}$ equatorial hours before midnight. For we establish hour-positions with respect to its meridian, and the meridian through Alexandria is west of that through Babylon by $\frac{1}{2}+\frac{1}{3}$ equatorial hour. And at that hour the sun's true position was very nearly $24\frac{1}{2}°$ within the Fishes.

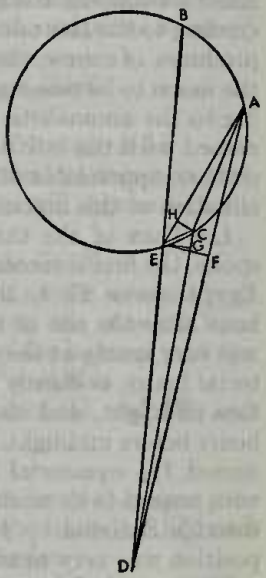
The second of the eclipses is recorded as having occurred in the year 2 of Mardokempad, Egyptianwise Thoth 18-19. And there was an eclipse, it says, of 3 digits[1] from the southern end at midnight. Since, then, the middle of the eclipse appears to have occurred in Babylon at midnight, it must have occurred in Alexandria $\frac{1}{2}+\frac{1}{3}$ of an hour before midnight. And at that hour the sun's true position was $13\frac{3}{4}°$ within the Fishes.

The third of the eclipses is recorded as having taken place in the same year 2 of Mardokempad, Egyptianwise Phamenoth 15-16. And the eclipse began, it says, after the rise of the moon, and there was an eclipse of more than half from the northern end. Now, since the sun was near the beginning of Virgin, the night

[1]A digit is 1/12 of the moon's apparent diameter.

in Babylon was very nearly 11 equatorial hours, and half the night $5\frac{1}{2}$ hours. Therefore the beginning of the eclipse occurred, at most, 5 equatorial hours before midnight since it began after the moon's rise; and the middle of the eclipse $3\frac{1}{2}$ hours before midnight, since the whole time of an eclipse of this magnitude must have been very nearly 3 hours. Therefore in Alexandria, in turn, the middle of the eclipse was effected $4\frac{1}{3}$ equatorial hours before midnight. And at this hour the sun's true position was very nearly $3\frac{1}{4}°$ within the Virgin.

It is thus clear that, from the middle of the first eclipse to that of the second, the sun had moved, and the moon too, 349°15′ over and above the complete circles; and from the middle of the second eclipse to that of the third 169°30′. But the interval of time between the first and second contains 354 days and $2\frac{1}{2}$ equatorial hours for those considering them simply, but relative to the calculation of regular solar days $2+\frac{1}{2}+\frac{1}{15}$ hours; and between the second and third 176 days and again $20\frac{1}{2}$ equatorial hours simply considered, but $20\frac{1}{5}$ hours accurately considered. And the moon moves regularly (for in this amount of time there will be no sensible difference even if one follow revolutions very close upon the true ones)[1] in 354 days and $2+\frac{1}{2}+\frac{1}{15}$ equatorial hours, 306°25′ of its cycle of anomaly over and above the complete circles, and 345°51′ in longitude. And in 176 days and $20\frac{1}{5}$ equatorial hours, 150°26′ of its cycle of anomaly, and very nearly 170°7′ in longitude. It is clear, then, that the 306°25′ of the epicycle's first interval have added 3°24′ to the moon's mean movement; and that the 150°26′ of the second interval have subtracted 37′ from the mean movement.

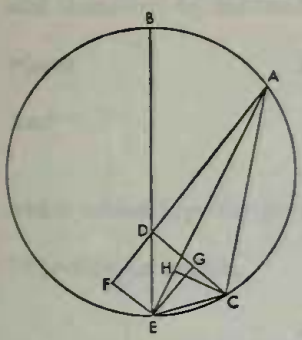


These things being assumed, let there be the moon's epicycle *ABC*. And let point *A* be the moon's position in the middle of the first eclipse, and *B* its position in the middle of the second, and *C* its position in the middle of the third. Let the moon's movement on the epicycle be thought as taking place from *B* to *A* and from *A* to *C* in such a way that the arc *ACB* of 306°25′, which it has traveled from the first eclipse to the second, adds 3°24′ to the mean movement; and that the arc *BAC* of 150°26′, which it has traveled from the second eclipse to the third, subtracts 37′ from the mean movement. And therefore in such a way that the passage from *B* to *A* of 53°35′ subtracts the 3°24′ from the mean movement, and the passage from *A* to *C* of 96°51′ adds 2°47′ to the mean movement.[2]

[1]In establishing the mean positions for the moon in each of the two series of eclipses, the lengths of the time-intervals are so small that there is no sensible difference if one use the averages of Hipparchus or the corrected ones given by Ptolemy in the tables. This remark is necessary because it is from a comparison of the second eclipse in the old series and in the new series, in Chapter 7 of this Book, an interval of some 854 Egyptian years, that the corrections are gotten—very small, indeed, but appreciable. Otherwise Ptolemy would be pulling himself up by his boot-straps.

[2]Whatever *A* to *B* adds, *B* to *A* must subtract, since *A* to *A* being a complete cycle neither adds nor subtracts. Likewise if *B* to *C* subtracts 37′ and *B* to A 3°24′, then *A* to *C* must add the difference between 37′ and 3°24′, or 2°47′.

Now, that it is impossible for the epicycle's perigee to be on the arc BAC is clear from the fact that this arc is subtractive and less than a semicircle. But the greatest movement is assumed to be at the perigee. Since, then, it must be on the arc BEC, let the centre of the ecliptic and of the circle carrying the epicycle's centre be taken, and let it be D. And let the straight lines DA, DEB, DC, from the center of the three points of eclipse, be joined.



Then, generally—in order that we may easily apply this theorem in similar demonstrations, both in case we prove them by the hypothesis of the epicycle as here, and by the hypothesis of eccentricity when the centre D is taken within the circle—let one of the three straight lines we just joined be produced to the opposite arc, as here we have DEB drawn from the point B of the second eclipse to E. And let a straight line, in this case AC, join the other two points of eclipse. And from the intersection made by the line produced, E for instance, let straight lines be joined to the other two points, in this case the lines EA and EC. And let perpendiculars be dropped on the straight lines from these other two points to the centre of the ecliptic, that is EF on AD and EG on CD. From one of these two points, in this case C, let the perpendicular be drawn to the line joining the other point, in this case A, to the principal intersection made by the line produced, here E—that is, CH on AE. And from whatever intersection we extend the construction, we shall always find the same ratios resulting for the numbers of the demonstration; and the choice is made only with a view to facility.

Since, then, arc AB was proved to subtend 3°24′ of the ecliptic, therefore, considered as an angle at the ecliptic's centre,

$$\text{angle } ADB = 3°24'$$
$$= 6°48' \text{ to 2 rt.}$$

And so also, on the circle about right triangle DEF,

$$\text{arc } EF = 6°48',$$
$$\text{chord } EF = 7^{p}7'$$

where

$$\text{hypt. } DE = 120^{p}.$$

Likewise, since

$$\text{arc } AB = 53°35',$$

therefore, being on the circumference, also

$$\text{angle } BEA = 53°35' \text{ to 2 rt.}$$

But it was seen

$$\text{angle } ADB = 6°48' \text{ to 2 rt.}$$

and therefore, by subtraction,

$$\text{angle } EAF = 46°47' \text{ to 2 rt.}$$

And so, on the circle about right triangle AEF,

$$\text{arc } EF = 46°47',$$
$$\text{chord } EF = 47^{p}38'30''$$

where

$$\text{hypt. } AE = 120^{p}.$$

And

$$AE = 17^{p}55'32''$$

where

$$EF = 7^{p}7',$$

and

$$DE = 120^{p}.$$

Again, since arc BAC subtends $37'$ of the ecliptic, considered as an angle at the centre of the ecliptic,

$$\text{angle } BDC = 37'$$
$$= 1°14' \text{ to 2 rt.}$$

And so, on the circle about right triangle DEG,

$$\text{arc } EG = 1°14',$$
$$\text{chord } EG = 1^{p}17'30''$$

where

$$\text{hypt. } DE = 120^{p}.$$

Likewise, since

$$\text{arc } BAC = 150°26',$$

being on the circumference,

$$\text{angle } BEC = 150°26' \text{ to 2 rt.}$$

But it was seen

$$\text{angle } BDC = 1°14' \text{ to 2 rt.}$$

and therefore, by subtraction,

$$\text{angle } DCE = 149°12' \text{ to 2 rt.}$$

And so, on the circle about right triangle CEG,

$$\text{arc } EG = 149°12',$$
$$\text{chord } EG = 115^{p}41'21''$$

where

$$\text{hypt. } CE = 120.^{p}$$

And therefore

$$CE = 1^{p}20'23''$$

where

$$EG = 1^{p}17'30'',$$

and

$$DE = 120^{p},$$

and where it was proved

$$AE = 17^{p}55'32''.$$

Again, since it was proved

$$\text{arc } AC = 96°51',$$

therefore, being on the circumference,

$$\text{angle } AEC = 96°51' \text{ to 2 rt.}$$

And so, on the circle about right triangle CEH,

$$\text{arc } CH = 96°51',$$

and, as remainder of the semicircle,

$$\text{arc } EH = 83°9'.$$

And therefore

$$\text{chord } CH = 89^{p}46'14'',$$
$$\text{chord } EH = 79^{p}37'55''$$

where

$$\text{hypt. } CE = 120^{p}.$$

And therefore

$$CH = 1^{p}0'8'',$$
$$EH = 0^{p}53'21''$$

where

$$CE = 1^{p}20'23'',$$

and where it was proved

$$AE = 17^{p}55'32'';$$

and therefore, by subtraction,

$$AH = 17^{p}2'11''$$

where

$$CH = 1^{p}0'8''.$$

And

$$\text{sq. } AH = 290^{p}14'19'',$$
$$\text{sq. } CH = 1^{p}0'17''$$

which added together give

$$\text{sq. } AC = 291^{p}14'36''.$$

Therefore, in length,

$$AC = 17^{p}3'57'',$$
$$CE = 1^{p}20'23''$$

where

$$DE = 120^{p}.$$

And

$$\text{chord } AC = 89^{p}46'14''$$

where

$$\text{epic. diam.} = 120^{p};$$

for it subtends arc AC and

$$\text{arc } AC = 96°51'.$$

And therefore

$$DE = 631^{p}13'48'',$$
$$CE = 7^{p}2'50''$$

where

$$AC = 89^{p}46'14''$$

and

$$\text{epic. diam.} = 120^{p}.$$

And so

$$\text{arc } CE = 6°44'1''.$$

But it is assumed

$$\text{arc } BAC = 150°26',$$

and therefore, by addition,

$$\text{arc } BCE = 157°10'1'',$$
$$\text{chord } BE = 117^{p}37'32''$$

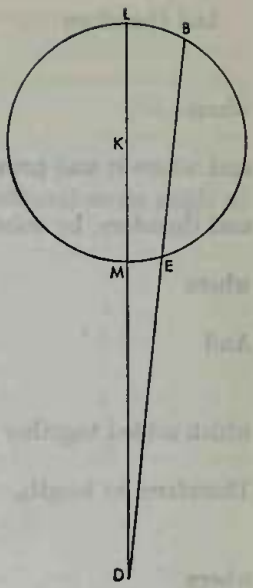
where

$$\text{epic. diam.} = 120^{p},$$
$$DE = 631^{p}13'48''.$$

If, then, this line BE were equal to the epicycle's diameter, the epicycle's centre would fall on it, and the ratio of the diameters would be immediately evident. But since it is less than the diameter, and arc BCE is less than a semicircle, it is clear that the epicycle's centre will fall outside the sector $BACE$.

Then let the point K be taken as centre, and let the straight line $DMKL$ be

drawn from D, the centre of the ecliptic, through K in such a way that point L becomes the epicycle's apogee, and M the perigee.



Since then

rect. BD, DE = rect. LD, DM, [Eucl. III, 36]

and since we have proved

$$BE = 117^{p}37'32'',$$
$$DE = 631^{p}13'49'',$$

and, by addition,

$$BD = 748^{p}51'20''$$

where, as the epicycle's diameter,

$$LKM = 120^{p},$$

therefore

rect BD, DE = rect. $LD, DM = 472{,}700^{p}5'32''$.

And again since

rect. LD, DM + sq. KM = sq. DK, [Eucl. II, 6]

and since, as radius of the epicycle,

$$KM = 60^{p},$$

therefore

$$\text{sq. } DK = 472{,}700^{p}5'32'' + 3{,}600^{p}$$
$$= 476{,}300^{p}5'32''.$$

And therefore, as the radius of the circle concentric with the ecliptic and carrying the epicycle,

$$DK = 690^{p}8'42''$$

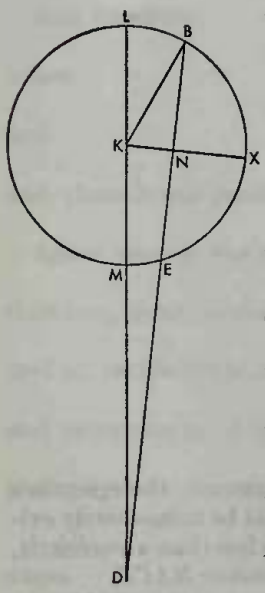
where, as the epicycle's radius,

$$KM = 60^{p}.$$

And so

$$\text{epic. rad.} = 5^{p}13'$$

where the radius of the circle carrying the epicycle and concentric with the eye is 60^{p}.



And in a similar figure let fall KNX from the centre K perpendicular to line BE, and let BK be joined.

Since then

$$DE = 631^{p}13'48'',$$
$$EN = \text{half } BE$$
$$= 58^{p}48'46'',$$

where

$$DK = 690^{p}8'42''$$

so that, by addition,

$$DEN = 690^{p}2'34'',$$

therefore

$$DN = 119^{p}58'57''$$

where

$$DK = 120^{p}.$$

And, on the circle about right triangle DNK,

$$\text{arc } DN = 178°2'.$$

And so also

$$\text{angle } DKN = 178°2' \text{ to 2 rt.}$$
$$= 89°1'.$$

And therefore, on the epicycle

$$\text{arc } MX = 89°1',$$

and, as remainder of the semicircle,

$$\text{arc } LBX = 90°59'.$$

And

$$\text{arc } BX = \text{half arc } BXE = 78°35',$$

since it was proved

$$\text{arc } BXE = 157°10'.$$

And therefore, by subtraction, the arc BL of the epicycle, which is the moon's distance from the apogee at the middle of the second eclipse, is 12°24′.

And likewise, since it was proved

$$\text{angle } DKN = 89°1',$$

therefore, by subtraction, angle KDN, which subtends the arc subtracted from the mean longitudinal course and resulting from the anomaly due to the epicycle's arc BL, is its complement 59′. Therefore the moon's mean longitudinal position at the middle of the second eclipse was 14°44′ within the Virgin, since indeed its true position was 13°45′ within the Virgin, with the sun 13°45′ within the Fishes.

II.—Again, of the three eclipses we have chosen from those most carefully observed by us in Alexandria, the first occurred in the year 17 of Hadrian, Egyptianwise Paÿni 20-21; and we accurately calculated the middle of it to have occurred ¾ equatorial hour before midnight. And the eclipse was total. At this hour the sun's true position was very nearly 13¼° within the Bull.

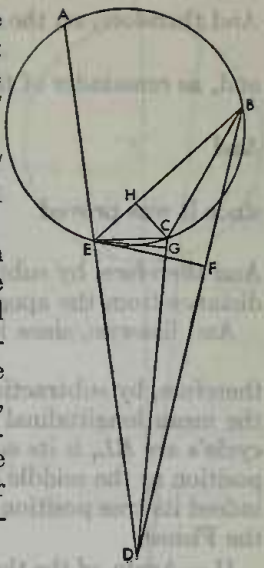
The second occurred in the year 19 of Hadrian, Egyptianwise Choïac 2-3; and we calculated the middle of it to have occurred 1 equatorial hour before midnight. And there was an eclipse to the extent of ½+⅓ of the diameter from the northern side. And at this hour the sun's true position was very nearly 25⅙° within the Balance.

The third of the eclipses occurred in the year 20 of Hadrian, Egyptianwise Pharmouthi 19-20; and we calculated the middle of it to have occurred 4 equatorial hours after midnight. And there was an eclipse to the extent of ½ of the diameter from the northern side. And at that hour the sun's true position was very nearly 14°12′ within the Fishes.

Now, it is clear that the moon and also the sun have moved, from the middle of the first eclipse to the middle of the second, 161°55′ over and above the complete circles; and from the middle of the second to the middle of the third, 138° 55′. The time of the first interval is 1 Egyptian year 166 days and 23¾ equatorial hours simply considered, but 23⅝ hours accurately considered. And the time of the second interval is again 1 Egyptian year 137 days and 5 equatorial hours simply considered, but 5½ hours accurately considered.

Now in 1 year, 166 days, and 23⅝ equatorial hours, the moon makes a mean movement of 110°21′ of its anomalistic cycle over and above the complete circles, and very nearly 169°37′ in longitude; and in 1 year, 137 days, and 5½ equatorial hours, 81°36′ of its anomalistic cycle, and very nearly 137°34′ in longitude. It is clear, then, that the 110°21′ on the epicycle of the first interval subtract 7°42′ from the mean longitudinal course, and that the 81°36′ of the second interval add 1°21′ to the mean longitudinal course.

Now, these things being assumed, let there again be the moon's epicycle ABC.



And let the point A be taken as the moon's position in the middle of the first eclipse; B that of the second eclipse; and C that of the third. And let the movement of the moon be conceived as passing from A to B and then to C in such a way that arc AB of 110°21′ subtracts, as we said, 7°42′ from the mean longitudinal course; and that arc BC of 81°36′ adds 1°21′ to the mean longitudinal course; and that the remaining arc CA of 168°3′ adds 6°21′.

That the apogee must be on the arc AB is clear from the fact that it can neither be on arc BC nor arc CA, since each of them is additive and less than a semicircle. And yet, as if this were not assumed, let the centre of the ecliptic and of the circle on which the epicycle is borne be taken, and let it be D. Let the straight lines DEA, DB, DC be joined from the centre to the three points of eclipse. And with BC joined, let the straight lines EB and EC be drawn from E to B and C; and EF and EG perpendicular to BD and DC; and again from C let CH be drawn perpendicular to BE.

Since then arc AB subtends 7°42′ of the ecliptic, therefore, being at the centre of the ecliptic,

$$\text{angle } ADB = 7°42'$$
$$= 15°24' \text{ to 2 rt.}$$

And so also, on the circle about right triangle DEF,

$$\text{arc } EF = 15°24',$$
$$\text{chord } EF = 16^{p}4'42''$$

where

$$\text{hypt. } DE = 120^{p}.$$

Likewise, since

$$\text{arc } AB = 110°21',$$

therefore, being on the circumference,

$$\text{angle } AEB = 110°21' \text{ to 2 rt.}$$

But

$$\text{angle } ADB = 15°24' \text{ to 2 rt.}$$

Therefore, by subtraction,

$$\text{angle } EBD = 94°57' \text{ to 2 rt.}$$

And so, on the circle about right triangle BEF,

$$\text{arc } EF = 94°57',$$
$$\text{chord } EF = 88^{p}26'17''$$

where

$$\text{hypt. } BE = 120^{p}.$$

And therefore

$$BE = 21^{p}48'59''$$

where

$$EF = 16^{p}4'42'',$$
$$DE = 120^{p}$$

Again, since arc AEC was shown to embrace 6°21′ of the ecliptic, therefore being at the centre of the ecliptic

$$\text{angle } ADC = 6°21'$$
$$= 12°42' \text{ to 2 rt.}$$

And so, on the circle about right triangle DEG,

$$\text{arc } EG = 12°42',$$
$$\text{chord } EG = 13^p16'19''$$

where

$$\text{hypt. } DE = 120^p.$$

Likewise, since, by addition,

$$\text{arc } ABC = 191°57',$$

therefore, being on the circumference,

$$\text{angle } AEC = 191°57' \text{ to 2 rt.}$$

But

$$\text{angle } ADC = 12°42' \text{ to 2 rt.}$$

Therefore, by subtraction,

$$\text{angle } ECD = 179°15' \text{ to 2 rt.}$$

And so also, on the circle about right triangle CEG,

$$\text{arc } EG = 179°15',$$
$$\text{chord } EG = 119^p59'50''$$

where

$$\text{hypt. } CE = 120^p.$$

And therefore

$$CE = 13^p16'20''$$

where

$$EG = 13^p16'19'',$$

and

$$DE = 120^p,$$

and where it was proved

$$BE = 21^p48'59''.$$

Again, since

$$\text{arc } BC = 81°36',$$

therefore, being on the circumference,

$$\text{angle } BEC = 81°36' \text{ to 2 rt.}$$

And so, on the circle about right triangle CEH,

$$\text{arc } CH = 81°36',$$

and, as remainder of the semicircle,

$$\text{arc } EH = 98°24'.$$

And therefore

$$\text{chord } CH = 73^p24'37'',$$
$$\text{chord } EH = 90^p50'22''$$

where

$$\text{hypt. } CE = 120^p.$$

And therefore

$$CH = 8^p40'20'',$$
$$EH = 10^p2'49''$$

where

$$CE = 13^p16'20''$$

and

$$BE = 21^p48'59''.$$

Therefore, by subtraction,

$$BH = 11^p46'10''$$

where

$$CH = 8^{p}40'20''.$$

And

$$\text{sq. } BH = 138^{p}31'11'',$$
$$\text{sq. } CH = 75^{p}12'27'';$$

and

$$\text{sq. } BC = 138^{p}31'11'' + 75^{p}12'27''$$
$$= 213^{p}43'38''.$$

Therefore, in length,

$$BC = 14^{p}37'10''$$

where

$$DE = 120^{p},$$
$$CE = 13^{p}16'20''.$$

But also

$$BC = 78^{p}24'37''$$

where

$$\text{epic. diam.} = 120^{p},$$

for it subtends arc BC which is 81°36′. And therefore

$$DE = 643^{p}36'39'',$$
$$CE = 71^{p}11'4''$$

where

$$BC = 78^{p}24'37'',$$
$$\text{epic. diam.} = 120^{p}.$$

And so, on the epicycle,

$$\text{arc } CE = 72°46'10''.$$

And it is assumed

$$\text{arc } CEA = 168°3'.$$

Therefore, by subtraction,

$$\text{arc } AE = 95°16'50'';$$

and

$$\text{chord } AE = 88^{p}40'17''$$

where

$$\text{epic. diam.} = 120^{p},$$
$$DE = 643^{p}36'39''.$$

Now again, since arc AE was shown to be less than a semicircle, it is clear that the epicycle's centre will fall outside the segment AE. Then let it be taken and let it be K; and let $DMKL$ be joined so that again point L becomes the apogee and M the perigee. Now since

$$\text{rect. } AD, DE = \text{rect. } LD, DM,$$

and since we have shown that

$$AE = 88^{p}40'17'',$$
$$DE = 643^{p}36'39'',$$

and, by addition,

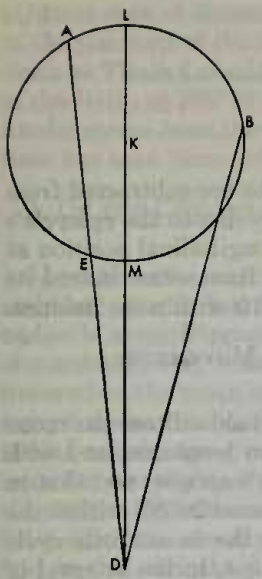
$$AD = 732^{p}16'56''$$

where, as epicycle's diameter,

$$LKM = 120^{p},$$

therefore

$$\text{rect. } LD, DM = 471{,}304^{p}46'17''.$$



And again, since

rect. LD, DM + sq. KM = sq. DK,

and since KM, the epicycle's radius, when squared, is 3,600^p, therefore

sq. DK = 471,304^{p}46′17″ + 3600^p
= 474,904^{p}46′17″.

And therefore, as the radius of the circle bearing the epicycle and concentric with the ecliptic,

DK = 689^{p}8′

where, as the epicycle's radius,

KM = 60^p.

And so

epic. rad. = 5^{p}14′

where the straight line between the epicycle's centre and the ecliptic's centre is 60^p. And this is very nearly the same ratio as that demonstrated a little while ago with the other eclipses.

Again with the same figure, let the straight line KNX be drawn perpendicular to DEA from the centre K, and let AK be joined.

Now since

DE = 643^{p}36′39″,

and

EN = half AE = 44^{p}20′8″

so that, by addition,

DEN = 687^{p}56′47″

where

DK = 689^{p}8′,

therefore also

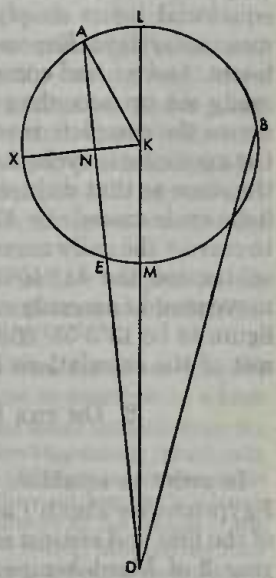
DN = 119^{p}47′36″

where

hypt. DK = 120^p;

and, on the circle about right triangle DKN,

arc DN ≒ 173°17′.



And so

angle DKN = 173°17′ to 2 rt.
= 86°38′30″.

And therefore, on the epicycle,

arc MEX = 86°38′30″

and, as remainder of the semicircle,

arc LAX = 93°21′30″.

And

arc AX = half arc AE ≒ 47°38′30″,

and therefore, by subtraction,

arc AL = 45°43′,

But it was assumed

arc AB = 110°21′.

And therefore the remainder arc BL, the distance of the moon from the apogee at the middle of the second eclipse, is 64°38′.

Likewise, since it was proved

angle $DKN = 86°38'$

and, as its complement,

angle $KDN = 3°22'$,

and since it was assumed

angle $ADB = 7°42'$,

therefore, by subtraction, angle LDB, which subtends the arc subtracted from the mean longitudinal course resulting from the anomaly due to the epicycle's arc BL, will be $4°20'$. And therefore the moon's mean longitudinal position at the middle of the second eclipse was $29°30'$ within the Ram, since indeed its true position was $25°10'$ within the Ram with the sun $25°10'$ within the Balance.

7. On the Correction of the Moon's Mean Movements of Longitude and Anomaly

Since we showed that in the middle of the second of the old eclipses the moon was situated $14°44'$ within the Virgin according to mean longitude, and with respect to the anomalistic cycle $12°24'$ from the epicycle's apogee; and that in the middle of the second of our own eclipses it was situated $29°30'$ within the Ram according to mean longitude, and with respect to the anomalistic cycle $64°38'$ from the epicycle's apogee, therefore it is clear that, in the interval of time between the two eclipses, the moon has run in mean movement over and above complete revolutions $224°46'$ in longitude and $52°14'$ of its anomalistic cycle. But the interval of time from the year 2 of Mardokempad, Thoth 18-19, $\frac{1}{2}+\frac{1}{3}$ equatorial hours before midnight to the year 19 of Hadrian, Choïac 2-3, 1 equatorial hour before midnight is 854 Egyptian years, 73 days, and $23\frac{1}{2}+\frac{1}{3}$ equatorial hours simply calculated, but $23\frac{1}{3}$ hours accurately calculated in mean solar days. Expressed entirely in days this is 311,783 days $23\frac{1}{3}$ equatorial hours. And we find corresponding to them, for the daily movements we have already set up according to the hypotheses preceding the correction, over and above the complete revolutions surpluses of $224°46'$ in longitude and $52°31'$ in the anomalistic cycle. And so the longitudinal surplus, as we said, is found to be the same as that deduced from our observations, but the surplus in the anomalistic cycle exceeds by $17'$. And therefore, before setting out the tables, in order to correct the daily movements, we distributed the $17'$ over the number of days, subtracted the $11^{iv}46^{v}39^{vi}$ corresponding to each day from the daily mean movement of anomaly calculated before the correction, and found the corrected figure to be $13°3^{i}53^{ii}56^{iii}17^{iv}51^{v}59^{vi}$. And in accordance with this we made the rest of the calculations for the tables.

8. On the Epoch of the Moon's Mean Movements of Longitude and Anomaly

In order to establish the epochs of two cycles in the year 1 of Nabonassar, Egyptianwise Thoth 1 at midday, we take the time from this date to the second of the first and nearest series of three eclipses, which, as we said, occurred in the year 2 of Mardokempad, Egyptianwise Thoth 18-19, $\frac{1}{2}+\frac{1}{3}$ equatorial hour before midnight. And this makes a total of 27 Egyptian years, 17 days, and very nearly $11\frac{1}{6}$ hours considered both simply and accurately. And corresponding to this interval of time there is a surplus, over and above the complete revolutions, of $123°22'$ in longitude and a surplus of $103°35'$ of the anomalistic cycle. If we

subtract each of these numbers from each of the corresponding numbers proper to the positions of the second eclipse, then, in the year 1 of Nabonassar, Egyptianwise Thoth 1 at midday, we shall have the moon's mean position 11°22′ within the Bull and 268°49′ from the epicycle's apogee in the anomalistic cycle. And its elongation from the sun will be clearly 70°37′, since the sun's position at that time has been demonstrated to be 0°45′ within the Fishes.

9. On the Correction of the Moon's Mean Latitudinal Movements and their Epochs

Now this is the way we have established the cycles of longitude and anomaly and their epochs. But in the case of the latitudinal cycles we were quite wrong earlier in using Hipparchus' assumption that the moon measures its own circle of revolution very nearly 650 times, and the circle of [the earth's] shadow 2½ times when the moon is at its mean distance at the syzygies.[1] For with these assumptions and the inclination of the moon's oblique circle, the limits of its particular eclipses are given. Then, taking the intervals of eclipse and calculating from the size of the shadows cast in mid-eclipse the true latitudinal courses on the oblique circle from either node, and distinguishing by means of the anomalistic difference already demonstrated the periodic from the true courses,[2] we find in this way the positions of the latitudinal cycle at mid-eclipse and the surplus over and above the complete revolutions which accrues in the time between. But by using more elegant methods which need none of the former hypotheses for getting what is desired, we have found that the latitudinal course is quite faulty. And from the course obtained now without them we have proved that these hypotheses concerning magnitudes and distances were not right, and we have made corrections.

We have done likewise in the case of the hypotheses concerning Saturn and Mercury, changing certain things not altogether accurately determined before us by having fallen later upon surer observations. For it behooves those going forward with this theory inquiringly and for the love of truth to use the new and surest methods found, not only for the correction of the old hypotheses but also of their own if they need it; and not to think it disgraceful (for it is a great and divine profession) even if they happen upon a correction for greater accuracy due to others and not only to themselves.

Later in this treatise we shall explain in the proper places how we prove each of these things. At present, for the sake of sequence, we shall turn to the demonstration of the latitudinal course. And here is the method.

Now, in the correction of the mean course we first looked for lunar eclipses from among those accurately recorded, as far apart in time as possible, in which

[1]By the moon's mean distance at the syzygies is here meant the linear distance from the earth of the centre of the moon's epicycle at the syzygies. This is speaking strictly; practically it is the distance from the earth to the moon when it is at the point of mean movement on its epicycle at the syzygies. For it will be seen later, in Book V, that the moon's epicycle is borne on a moving eccentric in such a way that the epicycle's centre is always at the apogee of the eccentric at the syzygies. Hence the mean distance at the syzygies is strictly the distance from the earth to the apogee of the moon's eccentric circle. These distances are calculated in detail in Chapter 13 of Book V.

[2]The true latitudinal return is the latitudinal return with respect to the moon; and the periodic latitudinal return is the latitudinal return with respect to the centre of the moon's epicycle. Hence, as we shall see, when the anomalistic difference is known it is possible to find the latitudinal position of the epicycle's centre.

the magnitudes of the shadows were equal, near the same node with both shadows either on the southern or northern side, and in which, moreover, the moon was at the same distance from the earth. All this being so, it necessarily follows that the moon's centre in each of the eclipses is equidistant on the same side of the same node; and, therefore, that the true course of the moon embraces complete latitudinal cycles in the time between the observations.

We then took, first, the eclipse observed in Babylon in the year 31 of Darius, Egyptianwise Tybi 3-4 at the middle of the sixth hour; and the moon was eclipsed to a breadth of 2 digits from the southern side.

Second, we took that observed in Alexandria in the year 9 of Hadrian, Egyptianwise Pachom 17-18 at $3\frac{3}{5}$ equatorial hours before midnight; and the moon was eclipsed likewise to the extent of $\frac{1}{6}$ of its diameter from the southern side.

And the latitudinal course of the moon was in each eclipse near the descending node. For this is taken from more general hypotheses. And the distance was very nearly equal and a little towards the perigee from the mean. For this becomes clear from the previous demonstrations of the anomaly. Now since, whenever the moon is eclipsed from the southern side, its centre is north of the ecliptic, it is clear the moon, in each of the eclipses, precedes the descending node by an equal amount.

But at the first eclipse the moon was 100°19′ from the epicycle's apogee, for the middle of this eclipse occurred in Babylon $\frac{1}{2}$ hour before midnight, and in Alexandria $1\frac{1}{3}$ hours before midnight. And the time from its epoch in the reign of Nabonassar totals 256 years, 122 days, and $10\frac{2}{3}$ equatorial hours simply considered, but $10\frac{1}{4}$ hours with respect to mean solar days. Therefore, the true course was less than the periodic course by 5°[1]. In the second eclipse the moon was 251°53′ from the epicycle's apogee. Here the time from the epoch to the middle of the eclipse totals 871 years, 256 days, and $8\frac{3}{5}$ equatorial hours considered simply, but $8\frac{1}{12}$ accurately considered. Therefore, the true course was more than the mean course by 4°53′. Consequently, in the interval of time between the two eclipses, which contains 615 Egyptian years, 133 days, and $21+\frac{1}{2}+\frac{1}{3}$ equatorial hours, the true latitudinal course of the moon consists of complete cycles; but the periodic falls short of complete cycles by a total of 9°53′ from the two anomalistic differences. But calculated from the mean courses set out according to Hipparchus' hypotheses, the periodic course falls short of complete restitutions by very nearly 10°2′. Therefore the mean latitudinal course is 9′ greater than his hypotheses allow.

Now, distributing these over the given number of days which is very nearly 224,609, and adding the resulting $8^{iv}39^{v}18^{vi}$ to the daily mean movement already demonstrated according to those hypotheses, we found the corrected daily mean movement to be $13°13^{i}45^{ii}39^{iii}48^{iv}56^{v}37^{vi}$. And again, in accordance with this result, we made out the rest of the additions in the tables.

Once the periodic latitudinal movement had been demonstrated in this way, in order to establish its position we again looked for an interval between two well-observed eclipses having the same incidents as the former—that is, where the distances of the moon were very nearly equal, and the shadows cast were equal, being in both cases either on the northern side or on the southern side,

[1]This is taken from the Table of the Moon's First and Simple Anomaly which is Chapter 10 of this Book. This table is only for the anomalies at the syzygies because of the later use of an eccentric deferent.

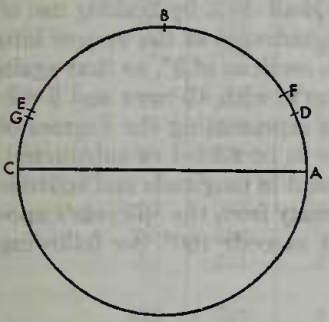
but where the nodes were no longer the same but opposite.

The first of these eclipses is the one we used in connection with the demonstration of the anomaly, which occurred in the year 2 of Mardokempad, Egyptianwise Thoth 18-19, at midnight in Babylon, and $\frac{1}{2}+\frac{1}{3}$ equatorial hour before midnight in Alexandria. And the moon was observed to be eclipsed 3 digits' breadth from the south.

The second is the one Hipparchus used, occurring in the year 20 of Darius, successor of Cambyses, Egyptianwise Epiphi 28-29, $6\frac{1}{3}$ equatorial hours after nightfall. And here likewise the moon was eclipsed to the extent of a quarter of its diameter from the southern side. The middle of the eclipse in Babylon was at $\frac{2}{3}$ equatorial hour before midnight, since a half-night at that time was very nearly $6+\frac{1}{2}+\frac{1}{4}$ equatorial hours, and in Alexandria $1\frac{1}{4}$ equatorial hours before midnight.

Each of these eclipses occurred when the moon was near the apogee, but the first at the ascending node and the second at the descending node, so that the moon's centre was an equal distance north of the ecliptic in these eclipses also.

Then let there be the oblique circle of the moon ABC with diameter AC; and let point A be supposed the ascending node, C the descending node, and B the northernmost limit. And let equal arcs AD and CE be cut off from each of the nodes A and C, in the direction of the northern limit B, so that in the first eclipse the moon's centre is at D, and in the second at E.



But the time from the epoch to the first eclipse is 27 Egyptian years, 17 days, and $11\frac{1}{6}$ equatorial hours considered both simply and accurately. And, therefore, the moon was 12°24′ from the epicycle's apogee and the periodic course was 59′ greater than the true one, and the time from the epoch to the second eclipse is likewise 245 Egyptian years, 327 days, and $10+\frac{1}{2}+\frac{1}{4}$ equatorial hours simply considered and $10\frac{1}{4}$ accurately considered. The moon hence was 2°44′ from the epicycle's apogee, and the periodic course was 13′ greater than the true one. The time between the observations (amounting as it does to 218 Egyptian years, 309 days, and $23\frac{1}{2}$ equatorial hours) gives, according to the mean latitudinal movement just demonstrated, a surplus of 160°4′.

Now, with these things in mind let the mean course of the moon's centre be in the first eclipse at F, and in the second at G. And since

$$\text{arc } FBG = 160°4',$$
$$\text{arc } DF = 59',$$
$$\text{arc } EG = 13',$$

therefore

$$\text{arc } ED = 160°50'.$$

And therefore

$$\text{arc } AD + \text{arc } CE = 19°10'$$

the rest of the semicircle. And since

$$\text{arc } AD = \text{arc } CE,$$

therefore

$$\text{arc } AD = \text{arc } CE = 9°35'.$$

This is the amount by which the moon's true course, in the first eclipse, lay east of the ascending node, and, in the second eclipse, lay west of the descending node. Therefore, by addition and subtraction,

$$\text{arc } AF = 10°34',$$
$$\text{arc } CG = 9°22'.$$

And so the moon's periodic course, in the first eclipse, lay east of the ascending node 10°34′, and was 280°34′ from the northern limit *B;* and, in the second eclipse, lay west of the descending node 9°22′, and was 80°38′ from the same northern limit.

Finally, since the time from the epoch to the middle of the first eclipse embraces a latitudinal surplus of 286°19′, therefore, if we subtract this surplus from a circle plus the 280°34′ representing the position of the first eclipse, we shall have the epoch of the latitudinal cycle in the year 1 of Nabonassar, Egyptianwise Thoth 1 at midday; and it will be 354°15′ from the northern limit.

As for the determinations of the calculations for the conjunctions and full moons—since for these passages we need take no heed of the second anomaly to be demonstrated later—we shall set out the table of the particular sections geometrically again as in the case of the sun. And we shall do it by making use of the ratio of 60 to 5¼, and dividing as before the quadrants at the apogee into sections of 6° and the quadrants at the perigee into sections of 3°, so that again the scheme of the table is similar to that of the sun's, with 45 rows and 3 columns. The first two columns contain the numbers representing the degrees of anomaly, and the third the corresponding degrees to be added or subtracted. They are to be subtracted in computing the movement in longitude and latitude when the total number representing the arc of anomaly from the epicycle's apogee is less than 180°, and are to be added when it exceeds 180°. See following table:

10. Table of the Moon's First and Simple Anomaly

Common numbers		*Additive—subtractive*		*Common numbers*		*Additive—subtractive*	
1	2	3		1	2	3	
6	354	0	29	120	240	4	31
12	348	0	57	123	237	4	24
18	342	1	25	126	234	4	16
24	336	1	53	129	231	4	7
30	330	2	19	132	228	3	57
36	324	2	44	135	225	3	46
42	318	3	8	138	222	3	35
48	312	3	31	141	219	3	23
54	306	3	51	144	216	3	10
60	300	4	8	147	213	2	57
66	294	4	24	150	210	2	43
72	288	4	38	153	207	2	28
78	282	4	49	156	204	2	13
84	276	4	56	159	201	1	57
90	270	4	59	162	198	1	41
93	267	5	0	165	195	1	25
96	264	5	1	168	192	1	9
99	261	5	0	171	189	0	52
102	258	4	59	174	186	0	35
105	255	4	57	177	183	0	18
108	252	4	53	180	180	0	0
111	249	4	49				
114	246	4	44				
117	243	4	38				

11. That the Quantity of the Moon's Anomaly as Given by Hipparchus did not Differ because of the Dissimilarity in Hypotheses, but because of the Calculations

These things having been demonstrated in this way, it would be reasonable to try and find out why, in the case of the lunar eclipses compared by Hipparchus in his research on this anomaly, the ratio does not tally with that demonstrated by us; and why the first ratio, demonstrated according to the hypothesis of eccentricity, does not agree with the second calculated according to the hypothesis of the epicycle. For according to the first demonstration, he finds the ratio of the eccentric's radius to the line between its centre and the ecliptic's centre to be very nearly 3,144 to 327⅔, which is the same as the ratio of 60^{p} to 6^{p}15′. And according to the second demonstration, he finds the ratio of the line from the

ecliptic's centre to the epicycle's centre to the epicycle's radius to be 3,122½ to 247½, which is the same as the ratio of 60ᵖ to 4ᵖ46′. And the ratio of 60 to 6¼ makes the greatest anomalistic difference 5°49′; and the ratio of 60ᵖ to 4ᵖ46′ makes the greatest difference 4ᵖ34′. But in our calculations the ratio of 60 to 5¼ makes the greatest difference very nearly 5°.

Now, it became evident to us a little way back, from the fact that the same appearances would result interchangeably from either hypothesis, that this discrepancy does not follow, as some think, from the difference of the hypotheses; and that if we should make our calculations with numbers we would find the same ratio resulting from either hypothesis if one should follow the same appearances for each, and not different ones like Hipparchus. For in this way, with different eclipses supposed, it is possible for the error to have occurred, either because of the observations themselves or because of the calculations of the intervals.

In these eclipses we find the syzygies well observed and we find that they occurred according to the hypotheses of regular and irregular movements we have just demonstrated. But we find that the calculations of the intervals by which the ratio is demonstrated were not made with the greatest care possible. And beginning with the first three eclipses we shall go through each one of these calculations.

Now he says these three eclipses were given out by those crossing over from Babylon as having been observed there, that the first of them occurred in the Athenian magistracy of Phanostratus in the month of Poseideon, and that the moon was eclipsed to the extent of a small bit of its circle on the side of the summer rising point, the night failing a half hour of being finished. And he says it was still eclipsed when setting. Now, this date is the year 366 of Nabonassar, Egyptianwise, as he himself says, Thoth 26-27, 5½ seasonal hours after midnight, since there remained a half hour of night. But when the sun is at the end of the Archer in Babylon, the night hour is 18° equatorial time. For then the night is 14⅖ equatorial hours. Therefore the 5½ seasonal hours come to 6⅗ equatorial hours. The beginning of the eclipse hence occurred 18⅗ equatorial hours after midday of Thoth 26. But since a small part was shadowed, the whole time of the eclipse must have been very nearly 1½ hours. Therefore in Alexandria the middle of the eclipse must have occurred 18½ equatorial hours after midday of Thoth 26. And the time from the epoch in the year 1 of Nabonassar to the eclipse in question is 365 Egyptian years, 25 days, and 18½ equatorial hours considered simply, but 18¼ hours considered accurately. And calculating according to our hypotheses we find the sun's true position at this time to be 28°18′ within the Archer; and find the moon's mean position to be 24°20′ within the Twins, but true position to be 28°17′, since according to its anom, alistic cycle it is 227°43′ from the epicycle's apogee.

Again, he says, the next eclipse occurred in the Athenian magistracy of Phanostratus in the month of Scirophorion, but Egyptianwise Phamenoth 24-25. And the moon was eclipsed, he says, on the side of the summer rising point at the end of the first hour of the night. Now, this date is the year 366 of Nabonassar, Phamenoth 24-25, at most 5½ seasonal hours before midnight. But when the sun is at the extremity of the Twins, the night-hour in Babylon is 12° equatorial time. Therefore 5½ seasonal hours make 4⅖ equatorial hours, and the beginning of the eclipse occurred 7⅗ equatorial hours after midday of Phame-

noth 24. But since the whole eclipse is recorded as having lasted 3 hours, the middle of the eclipse evidently occurred $9\frac{1}{10}$ equatorial hours after midday. In Alexandria, therefore, it must have occurred very nearly $8\frac{1}{4}$ equatorial hours after midday. And again the time from the epochs to the middle of the eclipse is 365 Egyptian years, 203 days, and $8\frac{1}{4}$ equatorial hours simply considered, but $7+\frac{1}{2}+\frac{1}{3}$ hours accurately considered. And we find the sun's true position at this time to be 21°46′ within the Twins; and the moon's mean position to be 23°58′ within the Archer, but true position 21°48′, since according to its anomalistic cycle it is 27°37′ from the epicycle's apogee. And the interval from the first to the second eclipse comes to 177 days and $13\frac{3}{5}$ equatorial hours, and 173°28′ of the sun's movement, while Hipparchus proceeded as if the interval were 177 days and $13+\frac{1}{2}+\frac{1}{4}$ equatorial hours, and $172\frac{7}{8}$° of the sun's movement.

And he says the third eclipse occurred in the Athenian magistracy of Evandrus on the first day of the month of Poseideon, Egyptianwise Thoth 16-17. And the eclipse, he says, was total, beginning from the direction of the summer rising point at the end of the fourth hour of the night. Now, this date is the year 367 of Nabonassar, Thoth 16-17, at most $2\frac{1}{2}$ hours before midnight. But when the sun is about two-thirds the way within the Archer, the night-hour in Babylon is nearly 18° equatorial time. Therefore $2\frac{1}{2}$ seasonal hours make 3 equatorial hours. And so the beginning of the eclipse occurred 9 equatorial hours after midday of Thoth 16. But since the eclipse was total, the whole time of the eclipse was very nearly 4 equatorial hours, and the middle of it was evidently 11 hours after midday. In Alexandria, therefore, the middle of the eclipse must have occurred $10\frac{1}{6}$ equatorial hours after midday of Thoth 16. And the time from the epochs to the middle of this eclipse is 366 Egyptian years, 15 days, and again $10\frac{1}{6}$ equatorial hours simply considered, but $9+\frac{1}{2}+\frac{1}{3}$ hours accurately considered. At this time we find the sun's true position to be 17°30′ within the Archer; and the moon's mean position 17°21′ within the Twins, but its true position 17°28′ because, according to its anomalistic cycle, it is 181°12′ from the epicycle's apogee. And the interval from the second to the third eclipse comes to 177 days and 2 equatorial hours, and 175°44′ of the sun's movement, while Hipparchus again assumes the interval to be 177 days and $1\frac{2}{3}$ hours, and 175°8′ of the sun's movement.

Now, in his calculations of the intervals he appears to have made an error, in the case of the days, of $\frac{1}{6}$ equatorial hour and $\frac{1}{3}$ equatorial hour; and, in the degrees of the sun's movement, to have made an error of very nearly $\frac{3}{5}$° in each of them. And this is enough to exclude just a chance disagreement in the quantity of the ratio.

And next we shall pass to the three later eclipses set out by him, which he says were observed in Alexandria. He says the first of these occurred in the year 54 of the Second Callippic Period, Egyptianwise Mesore 16, in which the moon began to be eclipsed $\frac{1}{2}$ hour before rising and returned to its full size in the middle of the third hour. Therefore the mid-eclipse occurred at the beginning of the second hour or 5 seasonal hours before midnight, and about as many equatorial hours since the sun was near the end of the Virgin. And so the middle of the eclipse occurred in Alexandria 7 equatorial hours after midday of Mesore 16. The time from the epochs in the year 1 of Nabonassar is 546 Egyptian years, 345 days, and 7 equatorial hours simply considered, but $6\frac{1}{2}$ hours accurately considered.

And again we find that at this time the sun's true position is 26°6′ within the Virgin; and the moon's mean position 22° within the Fishes, and its true position 26°7′ because, according to its anomalistic cycle, it is 300°13′ from the epicycle's apogee.

The next eclipse occurred, he says, in the year 55 of the same period, Egyptianwise Mechir 9; and it began after the night had run 5⅓ hours, and the eclipse was total. Therefore the beginning of the eclipse occurred 11⅓ equatorial hours after midday of Mechir 9, since the sun was now near the end of the Fishes. And the middle of the eclipse was 13⅓ equatorial hours after midday since the eclipse of the moon was total. The time from the epochs to the middle of the eclipse is 547 Egyptian years, 158 days, and very nearly 13⅓ equatorial hours both simply and accurately computed. Likewise we find that at this time the sun's true position is 26°17′ within the Fishes; and the moon's mean position 1°7′ within the Balance, and its true position 26°16′ within the Virgin, since according to its anomalistic cycle it is 109°28′ from the apogee. And the interval from the first to the second eclipse comes to 178 days and 6+½+⅓ equatorial hours, and 180°11′ of the sun's movement, while Hipparchus proceeded as if the interval were 178 days and 6 equatorial hours, and 180°20′ of the sun's movement.

And he says the third eclipse occurred in the same year 55 of this Second Period, Egyptianwise Mesore 5; and it began after the night had run 6⅔ hours, and the eclipse was total. The middle of the eclipse, he says, occurred at about 8⅓ hours in the night, that is 2⅓ seasonal hours after midnight. But when the sun is about the middle of the Virgin, the night-hour in Alexandria is 14⅖° equatorial time. Therefore the 2⅓ seasonal hours make very nearly 2¼ equatorial hours. And so the middle of the eclipse occurred 14¼ equatorial hours after midday of Mesore 5. Again the time from the epochs to the middle of this eclipse is 547 Egyptian years, 334 days, and 14¼ equatorial hours simply considered, but 13+½+¼ accurately considered. And we find that the sun's true position at this time is 15°12′ within the Virgin; and the moon's mean position 10°24′ within the Fishes, but true position 15°13′, since according to its anomalistic cycle it is 249°9′ from the epicycle's apogee. And the interval from the second to the third eclipse comes to 176 days and ⅖ equatorial hour, and 168°55′ of the sun's movement, while Hipparchus again assumes this interval to be 176 days and 1⅓ equatorial hours, and 168°33′ of the sun's movement.

Therefore it here appears that in the degrees of the sun's movement he has made an error of very nearly (1/6+1/5)°, and an error in days of very nearly ½+⅓+1/10 equatorial hour. These errors can work an appreciable difference in the ratio of the hypothesis.

Now we have seen the cause of this disagreement. We have also seen that with still more confidence we can use the ratio of anomaly demonstrated by us for the moon's syzygies, for their eclipses are found likewise to agree most completely with our hypotheses.

BOOK FIVE

1. On the Construction of the Astrolabe

We now find that the hypothesis set up with reference to the first and simple anomaly is sufficient for the moon's syzygies with respect to the sun, both the synodical and the plenilunar, as well as for the eclipses which accompany them, even if we take this anomaly by itself. But, as regards the particular passages of the moon's other positions relative to the sun, it would not be found sufficient. For, as we said, a second anomaly of the moon is discovered relative to its elongations from the sun, an anomaly which is reduced to the first at the syzygies, and is greatest at the first and third quarters. We were brought to this examination and opinion by the courses of the moon observed and recorded by Hipparchus, and by those we ourselves obtained with an instrument we constructed for this purpose. And here is the kind of thing it is.

We take two circles accurately turned with four perpendicular surfaces each, similar in magnitude, and in every way equal and similar to each other. And we fit them at right angles to each other with a common diameter, so that one of them is conceived to be the ecliptic and the other becomes the meridian through the poles of the ecliptic and of the equator [solstitial colure]. On this circle we take, with the side of the inscribed square, the points defining the poles of the ecliptic, and we place at both of them pivots projecting from the inside and outside surfaces. And on the outside pivots we place another circle having its concave surface exactly touching everywhere the convex surface of the circles already fitted together, and able to revolve in longitude about these poles of the ecliptic. Likewise on the inside pivots we place another circle having its convex surface everywhere exactly touching the concave surface of the two circles and likewise revolving in longitude about the same poles as the outside circle. Now, dividing this inside circle and the one used for the ecliptic each into the usual 360 parts and these in turn into as many parts as practicable, we fit in under the inner circle another little circle with diametrically opposite projecting sights. And we fit it in so that it can move in the same plane with this inner circle with respect to each of the poles, for latitudinal observations. With this done, on the circle we supposed through both sets of poles, we lay off, from each of the ecliptic's poles, the arc we computed to lie between the ecliptic's poles and the equator's. We place the limits of these arcs diametrically opposite each other on the meridian like that exhibited in the beginning of this treatise relative to the observations of the meridian arc between the tropics, so that, when this meridian has been placed in the same position (in other words when it has been placed perpendicular to the plane of the horizon and in accordance with the height of the pole of the place in question and parallel to the plane of the natural meridian) then the inner circles may move about the equator's poles from east to west in accord with the prime movement of the universe.

Now, arranging the instrument in this way, whenever
could appear together above the earth, we placed the outside
labe as nearly as possible at the sun's degree at that hour. A
colure until, with the intersection of the circles at the sun's
cisely to the sun itself, the two circles, the ecliptic and th
poles, obscured each other; or, if it were a star being sighted,
placed at one side of the outside circle, already correctly arra
proper section of the ecliptic, the star as if stuck to both su
the opposite side in the plane through both circles. We move
side circle of the astrolabe towards the moon or any other s
until, simultaneously with the sun or star in question, the m
be examined is lined up with both sights of the little circle f
side circle.

In this way, we find out, from the inside circle's intersect
the circle corresponding to the ecliptic, what longitudinal se
they occupy and how many degrees distant they are either n
circle through the ecliptic's poles. And this last reading is ma
division of the inner circle, from the interval between the r
sight of the little circle fitted under it and the ecliptic.

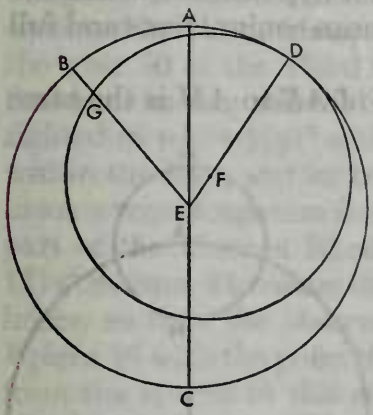
2. On the Hypothesis of the Moon's Double

Now, with this kind of observation, the moon's distances
sun, both those recorded by Hipparchus and those observed
at times to agree with the calculations made according to the
advanced, and at other times to disagree and differ, somet
times a great deal. With a continuously greater and more n
of our own, we learned concerning the order of this irregula
conjunctions and full moons there was either little or no appr
and only such as the moon's parallaxes could account for; and
the first and third quarters, there is very little or no discrepa
is at the epicycle's apogee or perigee, but the most when th
courses effects the greatest difference of first anomaly. We fo
ever of the two semicircles the first anomaly is subtractive,
is found to be advanced even less than the result of this first
but, when it is additive, it is likewise found to be more a
occurs in proportion to the magnitude of the first addition-
this order we see at a glance that it is necessary to suppose
is borne on an eccentric circle with its apogee at the conjuncti
and its perigee at the first and third quarters. And this would

eccentric circle, which is taken in this same plane always bearing centre, and turning it from east to west at a speed equal to the the moon's elongation from the sun over the latitudinal movemer excess of the moon's mean longitudinal movement over the sun

And so, for example, in one day the epicycle's centre appears nearly 13°14′ in latitude from west to east, traversing 13°11′ in cause of the oblique circle's retrogression of the 3′ excess from ea in turn the eccentric's apogee is rotated in that same opposite east to west 11°9′, by which amount 24°23′ (double the moon's ceeds 13°14′. Thus, because of the two movements' retrogression direction about the ecliptic's centre, as we said, the movemen epicycle's centre will differ from that of the eccentric's by the a 13°14′ plus 11°9′, which is very nearly double the 12°11½′ of Therefore the epicycle will traverse the eccentric circle's circumf a mean month's time, and its return to the eccentric's apogee is a place at mean conjunctions and full moons.[1]

But to see the details of the hypothesis more thoroughly, aga



conceived the circle $ABCD$ concer ecliptic and lying in the moon's obliqu E as centre and AEC as diameter. At suppose the point A to be the ecce the epicycle's centre, the northern li ning of the Ram, and the mean sun.

Then, I say, in one day's course t moves from east to west, or from A t ly 3′ about centre E, so that the no comes to be 29°57′ within the Fishes. the two contrary regular movements straight line similar to EA again abou tic's centre, I say that, likewise in a the straight line similar to EA throu tric's centre and revolving westward at a regular speed to ED centric's apogee to D, describes the eccentric circle DG about makes an arc AD of 11°9′; but that the straight line through the ep revolving about E, this time eastward, at a regular speed to EB^2 cycle's centre to G and makes an arc AB of 13°14′. And so G, the tre, appears to be 13°14′ in latitudinal movement distant from

[1]The mean conjunctions and full moons or mean syzygies are the conjunct tions of the centre of the moon's epicycle and the mean sun.

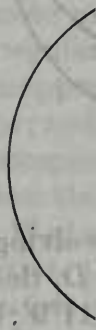
limit A; but 13°11′ within the Ram in longitude, because the
29°57′ within the Fishes; and to be 24°23′, the sum of arcs A
the eccentric's apogee, which is double the daily movement

In this way, since the two movements together through
make a complete return to each other in half a mean month
that in a quarter or three quarters of a mean month's time
directly opposite each other—that is, at the first and third
mean movement—and that, when the epicycle's centre a
opposite the eccentric's apogee along ED it will be at the ec

It is clear that, with things thus disposed, as regards the ec
dissimilarity of arcs DB and DG there is no difference in the
since the straight line EB, because of its rotation about E
centric's centre, traverses at a regular speed, not arc DG of
but arc DB of the ecliptic. But only as regards the difference
itself will there be a change. For the epicycle when nearer the
crease the anomalistic difference, the additive equally with
angle at the eye intercepting it being made greater in position

There will be no general difference, then, for the first hyp
epicycle's centre is at the apogee A. This happens at mean c
moons.

For if we draw the epicycle MN about A, the ratio of AE
as that demonstrated by means of eclipses. And the
difference will be greatest when the epicycle is passing G, the eccentric's perigee—as, for example, the one drawn through the points X and O. This occurs at the first and third quarters considered in terms of mean movement. For the ratio of XG to GE is greater than all those constructed at the other positions, since XG, the epicycle's radius, is always equal and the same, and the line EG from the earth's centre is less than all the others drawn to the eccentric circle.



3. On the Size of the Moon's Solar Anomaly

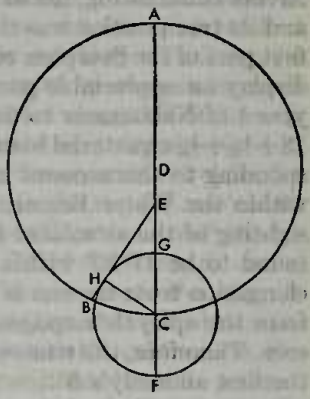
To see, then, how much the greatest anomalistic difference comes to when the epicycle happens to be borne at the eccentric's perigee, we observed those of the moon's elongations sighted with the sun in which the moon's courses approached the mean (for

tions, we sighted the sun and moon in the year 2 of Antonine, I
Phamenoth 25, after sunrise, or 5¼ equatorial hours before noon.
sighted (18+½+⅓)° within the Water Bearer and the point ¼
Archer culminating, the moon's apparent position was 9⅔° within t
and its true position was the same, since in Alexandria, when the m
first part of the Scorpion very nearly 1½ hours west of the meridia
display an appreciable parallax in longitude. The time from the e
year 1 of Nabonassar to this observation is 885 Egyptian years, 2
18+½+¼ equatorial hours, both simply and accurately considere
sponding to this amount of time we find the sun's mean position
within the Water Bearer, and its true position 18°50′ which agr
sighting of the astrolabe. From the first hypothesis, the moon at
found to be 17°20′ within the Scorpion in mean longitude, so th
elongation from the sun is very nearly a quadrant, and in anomal
from the epicycle's apogee, near which the greatest anomalistic d
curs. Therefore, the true course was less than the regular one by 7⅔
the first anomaly's 5°.

Again, in order to have before us the difference in like cases from
observed by Hipparchus, we shall first set out one which he says he
the year 50 of the Third Callippic Period, Egyptianwise Epiphi 1
first hour having passed. Now the course, he says, was mean.[1] And
sighted (8+½+1/12)° within the Lion, the moon's apparent positio
within the Bull, and its true position was very nearly the same. T
moon's true elongation from the sun was 86°15′. But when the sun i
part of the Lion, in Rhodes where the observation occurred, the
17⅓° in time. Therefore the 5⅓ seasonal hours before noon make 6⅓
hours, so that the observation occurred 6⅙ equatorial hours bef
Epiphi 16 with the point 9° within the Bull culminating. Consequen
from the epochs to this observation totals 619 Egyptian years, 31
17+½+⅓ equatorial hours simply considered, but 17+½+¼ acc
sidered. And according to our hypotheses (since the meridian thro
is the same as that through Alexandria), for this amount of time
sun's mean position to be 10°27′ within the Lion, but its true po
8°20′. And we find the moon's mean longitudinal position to be 4°2
Bull so that its mean elongation from the sun is again nearly a quad
anomaly is 257°47′ from the epicycle's apogee, very near which the
ference of epicyclic anomaly occurs. Therefore, the distance from
mean position to the sun's true position totals 93°55′. But the moon

4. On the Lunar Circle's Ratio of Eccentricity

Now this being so, let there be the moon's eccentric circle ABC about D as centre and ADC as diameter. And on this diameter let E be taken as the ecliptic's centre so that A is the eccentric's apogee and C its perigee. And let there be drawn the moon's epicycle FGH with centre C, and let the tangent EHB be drawn to it, and let CH be joined.



Then, since the greatest anomalistic difference is produced on this tangent of the moon's epicycle (this being shown to total $7\frac{2}{3}°$), therefore, being at the ecliptic's centre,

$$\text{angle } CEH = 7°40'$$
$$= 15°20' \text{ to 2 rt.}$$

And therefore, on the circle about right triangle CEH,

$$\text{arc } CH = 15°20',$$
$$\text{chord } CH = 16^{p}$$

where

$$\text{hypt. } CE = 120^{p}.$$

And so CE, the line from the ecliptic's centre to the eccentric's perigee, will also be $39^{p}22'$ to the already-demonstrated $5^{p}15'$ of the epicycle's radius and the 60^{p} of AE, the line from the ecliptic's centre to the eccentric's apogee. And, therefore, the whole diameter AC will be $99^{p}22'$, the eccentric's radius AD $49^{p}41'$, and DE the line between the centres of the ecliptic and eccentric $10^{p}19'$. So we have shown the ratio contained by the eccentricity.[1]

5. On the Inclination of the Moon's Epicycle

Now this number of circles would be added to the hypotheses for an explanation of the appearances at the syzygies and again at the first and third quarters. But from a consideration of the particular courses where it is either crescent or gibbous and when its epicycle is between the eccentric's apogee and perigee, in the case of the moon we find a peculiar incident concerning the epicycle's inclination. For in general it is necessary to suppose one and the same point on epicycles with respect to which the returns of the movements on them are accomplished. We call this the regular apogee from which we begin the numbers of the epicyclic movement, as the point F in the preceding figure; and this point is determined by the line of centres, such as DEC, when the epicycle is at the apogees and perigees of the eccentric circles. Now, for all the other hypotheses, we see absolutely nothing in the appearances contrary to this supposition: that the diameter through the epicycle's apogee (FCG, for instance) always and in

[1]From this theory of eccentricity it follows that the moon's distance from the earth varies by as much as 34 to 65, or nearly 1 to 2. And since, for small angles, the tangents are nearly proportional to the angles, the size of the moon's diameter (as Copernicus points out in Chapter 2 of Book IV, *On the Revolutions of the Heavenly Spheres*) will vary as 1 to 2. But, by observation, this is not true. The diameter varies very nearly as 55 to 65, just as the epicylic theory without the eccentric theory allows. Hence at this point the Ptolemaic theory, in saving the appearances of the angular distances, is unable to save the appearances of the diameter's variations.

the other courses of the epicycles keeps the same direction as the line which revolves the epicycle's centre at a regular speed (in this case EC); and that, consequently, it always points to the centre of revolution at which in equal times are intercepted the equal angles of the regular movement.

But in the case of the moon, the appearances are opposed to the epicyclic diameter, FG's being inclined towards E the centre of revolution and keeping the same direction as EC in the passages between A and C. True enough, we find it, always pointing to one and the same point on the diameter AC; not to the ecliptic's center E nor to the eccentric's center D, however, but to the point the same distance from E on the same side of the eccentric's perigee as DE, the line between the centres.[1]

Again we shall show this is so by choosing from several observations two which can bring this to light most efficiently—that is, two observations where the epicycle was in the mean distances, and the moon either at the epicycle's perigee or apogee. For it is in such passages that the greatest difference in these inclinations occurs.

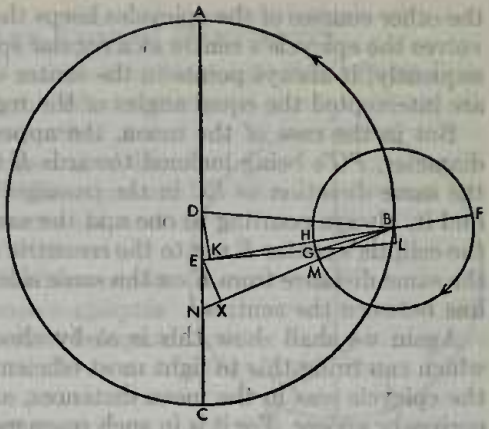
Hipparchus writes that he observed the sun and moon with instruments in Rhodes, in the year 197 after Alexander's death, Egyptianwise Pharmouthi 11, at the beginning of the second hour. And he says that, with the sun sighted $(7+\frac{1}{2}+\frac{1}{4})°$ within the Bull, the apparent position of the moon's centre was $21\frac{2}{3}°$ within the Fishes, and its true position $(21+\frac{1}{3}+\frac{1}{8})°$ within the Fishes. Therefore, at that time, the true moon's eastern elongation from the true sun was very nearly 313°42′. But since the observation occurred at the beginning of the second hour or very nearly 5 seasonal hours before noon of Pharmouthi 11, and since in Rhodes at that time the seasonal hours were equivalent to very nearly $5\frac{2}{3}$ equatorial hours, therefore the time from our epoch to this observation totals 620 Egyptian years, 219 days, and $18\frac{1}{3}$ equatorial hours simply considered, but only 18 hours accurately considered. Corresponding to this time, we find the regular sun's position to be 6°41′ within the Bull, but the true sun's position to be 7°45′; and the regular moon's longitudinal position to be 22°13′ within the Fishes, and in anomaly the moon to be 185°30′ from the epicycle's mean apogee.[2] And so the regular moon's elongation from the true sun totals 314°28′.

Now all this being granted, let ABC be the moon's eccentric circle with centre

[1]Again in Chapter 2 of Book IV, *On the Revolutions of the Heavenly Spheres*, Copernicus sharply criticises this third lunar anomaly where the regular movement of the moon on the epicycle is no longer tied to the regular movement of the epicycle on its deferent. For the result of this contriving of movements is that the moon moves irregularly about the epicycle's centre. Copernicus says that the initial violation of the strict principle of celestial mechanics, with respect to the epicycle's movement on its deferent, necessitates this further irregularity.

But Ptolemy might well reply that even if these movements, so contrived, turn out to move the stars on their epicycles irregularly with respect to the centres of the epicycles, and the epicycles irregularly with respect to the centres of their deferents, yet these irregular movements are such as can be computed and mastered by means of circles and regular movements, and this is all that is necessary.

[2]The apparent apogee and apparent perigee are those defined in the normal way, by a straight line through the epicycle's centre and the ecliptic's centre, or the earth. The mean apogee and mean perigee are those defined in another regular way to fit the particular appearances. As Ptolemy says, in the case of the moon, they are defined by a line through the epicycle's centre and a point on the line of apsides other than the earth. It is evident that the apparent and mean apogees coincide at the quarters and syzygies. Furthermore, the apparent and regular apogees always coincide in the case of the moon.



D and diameter ADC, on which let there be the ecliptic's centre E. And with B as centre let the moon's epicycle FGH be drawn. Let the epicycle be revolved eastward as from B to A, and let the moon be revolved on the epicycle from F to G and H, letting DB and $EHBF$ be joined.

Now, since a mean lunar month's time embraces two returns of the epicycle with respect to the eccentric, and since in the foregoing position the mean moon was 315°32′ from the mean sun, it follows that if we double this amount and subtract a complete circle we shall have, for that time, the epicycle's distance of 271°4′ eastward from the eccentric's apogee. And so, by subtraction,

$$\text{angle } AEB = 88°56'.$$

Then let DK be drawn from D perpendicular to BE.

Now, since

$$\text{angle } DEB = 88°56' = 177°52' \text{ to 2 rt.},$$

therefore also, on the circle about right triangle DEK,

$$\text{arc } DK = 177°52',$$

and, by subtraction from the semicircle,

$$\text{arc } EK = 2°8'.$$

And therefore

$$\text{chord } DK = 119^{p}59', \quad \text{chord } EK = 2^{p}14'$$

where

$$\text{diam. } DE = 120^{p}.$$

And therefore

$$DK = 10^{p}19', \quad EK = 0^{p}12'$$

where again

$$\text{1. betw. c. } DE = 10^{p}19'$$

and

$$\text{ecc. rad. } BD = 49^{p}41'.$$

And since

$$\text{sq. } BD - \text{sq. } DK = \text{sq. } BK,$$

therefore

$$BK = 48^{p}36',$$

and, by addition,

$$BE = 48^{p}48'.$$

Again, since the mean moon's elongation from the true sun was 314°28′, and in the observation the true moon's elongation from the true sun was 313°42′ so that the moon's anomalistic difference subtracts 46′, and since the moon's regu-

lar course is sighted along the line EB, therefore let the moon be supposed at the point G, for it was near the epicycle's perigee. And with EG and BG joined, let BL be drawn from B perpendicular to EG produced.

Since it contains the moon's anomalistic difference,

$$\text{angle } BEL = 0°46'$$
$$= 1°32' \text{ to 2 rt.}$$

And so also, on the circle about right triangle BEL,

$$\text{arc } BL = 1°32',$$
$$\text{chord } BL = 1^{p}36'$$

where

$$\text{hypt. } BE = 120^{p}.$$

And so

$$BL = 0^{p}39'$$

where

$$BE = 48^{p}48'$$

and

$$\text{epic. rad. } BG = 5^{p}15'.$$

And therefore

$$BL = 14^{p}52'$$

where

$$\text{epic. rad. } BG = 120^{p}.$$

And, on the circle about right triangle BGL,

$$\text{arc } BL = 14°14'.$$

And so

$$\text{angle } BGL = 14°14' \text{ to 2 rt.,}$$

and, by subtraction,

$$\text{angle } EBG = 12°42' \text{ to 2 rt.}$$
$$= 6°21'.$$

Therefore, on the epicycle,

$$\text{arc } GH = 6°21'$$

which is the distance from the moon to the true perigee.

But since the moon, at the time of the observation, was 185°30′ distant from the epicycle's mean apogee, it is evident that the mean perigee precedes the moon or point G. Let it be M, and let BMN be drawn and EX from E perpendicular to it.

Then, since it was shown

$$\text{arc } GH = 6°21'$$

and it is assumed

$$\text{arc } GM = 5°30'$$

from the perigee so that, by addition,

$$\text{arc } HM = 11°51',$$

therefore

$$\text{angle } EBX = 11°51'$$
$$= 23°42' \text{ to 2 rt.}$$

And so also, on the circle about right triangle BEX,

$$\text{arc } EX = 23°42';$$

chord $EX = 24^{p}39'$

where

hypt. $BE = 120^{p}$.

And therefore

$EX = 10^{p}2'$

where

$BE = 48^{p}48'$.

Again, since

angle $AEB = 177°52'$ to 2 rt.

and

angle $EBN = 23°42'$ to 2 rt.,

therefore, by subtraction,

angle $ENB = 154°10'$ to 2 rt.

And so also, on the circle about right triangle ENX,

arc $EX = 154°10'$;

chord $EX = 116^{p}58'$

where

hypt. $EN = 120^{p}$.

And therefore

$EN = 10^{p}18'$

where

$EX = 10^{p}2'$

and

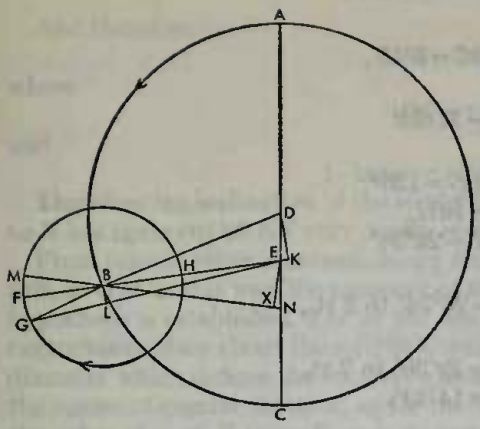
1. betw. c. $DE = 10^{p}19'$.

Therefore, the inclination of the straight line BM through the mean perigee to N cuts off EN very nearly equal to DE.

Likewise—to show that the same thing occurs also for the opposite parts of the eccentric and epicycle—we have again taken from among the intervals observed by Hipparchus in Rhodes, as we said, the one sighted in the same year 197 after Alexander's death, Egyptianwise Paÿni 17, at $9\frac{1}{3}$ hours. At that time, he says, with the sun sighted $10\frac{9}{10}°$ within the Crab, the moon's apparent position was about 29° within the Lion. And this was its true position also, since in Rhodes towards the end of the Lion, at very nearly one hour after noon, the moon does not display a parallax in longitude. Therefore, at that time, the true moon's eastern elongation from the true sun was 48°6′. But since the observation occurred $3\frac{1}{3}$ seasonal hours after noon of Paÿni 17, and in Rhodes at that date these came to very nearly 4 equatorial hours, the time from this observation to our epoch is 620 Egyptian years, 286 days, and 4 equatorial hours simply considered, but $3\frac{2}{3}$ hours accurately considered. Likewise corresponding to this time we find the regular sun's position to be 12°5′ within the Crab, but the true sun's position 10°40′; and the regular moon's position in longitude to be 27°20, within the Lion so that the regular moon's elongation from the true sun totals 46°40′, in anomaly the moon being 333°12′ from the epicycle's mean apogee.

Now, with these things granted, again let there be the moon's eccentric circle ABC with centre D and diameter ADC on which let E be taken as the ecliptic's centre. And let the moon's epicycle FGH be described about the point B, and let DB and $EHBF$ be joined.

Since, then, double the moon's mean elongation from the sun contains 90°30′,



therefore for reasons already given

angle $AEB = 90°30'$
$= 181°$ to 2 rt.

Therefore if we extend the line BE and drop DK from D perpendicular to it, then, as supplement,

angle $DEK = 179°$ to 2 rt.

And so also, on the circle about right triangle DEK,

arc $DK = 179°$

and, as remainder of the semicircle,

arc $EK = 1°$.

And therefore

chord $DK = 119^p59'$,
chord $EK = 1^p3'$

where

hypt. $DE = 120^p$.

And so

$DK = 10^p19'$,
$EK = 0^p5'$

where

1. betw. c. $DE = 10^p19'$,
ecc. rad. $BD = 49^p41'$.

And since

sq. BD − sq. DK = sq. BK,

we shall have

$BK = 48^p36'$

and, by subtraction,

$BE = 48^p31'$.

Again, since the regular moon's elongation from the true sun was 46°40′, but the true moon's elongation was 48°6′ (so that the anomalistic difference has added 1°26′), let the moon, since it was near the epicycle's apogee, be supposed at point G. And with EG and BG joined, let BL be drawn from B perpendicular to EG.

Since, then,

angle $BEL = 1°26'$
$= 2°52'$ to 2 rt.

therefore also, on the circle about right triangle BEL,

arc $BL = 2°52'$;
chord $BL = 2^p59'$

where

hypt. $BE = 120^p$.

And therefore

$BL = 1^p12'$

where

$BE = 48^p31'$

and

epic. rad. $BG = 5^p15'$.

And so

$BL = 27^p34'$

where

hypt. $BG = 120^p$;

and, on the circle about right triangle BGL,

arc $BL = 26°34'$

And therefore

angle $BGL = 26°34'$ to 2 rt.,

and, by addition,

angle $FBG = 29°26'$ to 2 rt.
$= 14°43'$.

Therefore, on the epicycle,

arc $FG = 14°43'$,

which is the moon's distance to the true apogee.

But since at the time of the observation the moon was $333°12'$ from the mean apogee, if we suppose the mean apogee at M and, joining MBN, drop EX from E perpendicular to it, then

arc $GFM = 26°48'$

the rest of the circle, and, by subtraction,

arc $FM = 12°5'$.

And so

angle MBF = angle $EBX = 12°5'$
$= 24°10'$ to 2 rt.

And also, on the circle about right triangle BEX,

arc $EX = 24°10'$;
chord $EX = 25^p7'$

where

hypt. $BE = 120^p$.

And therefore

$EX = 10^p8'$

where

$BE = 48^p31'$

and

1. betw. c. $DE = 10^p19'$.

Again, since it is assumed

angle $AEB = 181°$ to 2 rt.,

and it was proved

angle $EBN = 24°10'$ to 2 rt.

so that by subtraction

angle $ENB = 156°50'$ to 2 rt.,

therefore also, on the circle about right triangle ENX,

arc $EX = 156°50'$;
chord $EX = 117^p33'$

where

hypt. $EN = 120^p$.

And therefore

$$EN = 10^{p}20'$$

where

$$EX = 10^{p}8'$$

and

1. betw. c. $DE = 10^{p}19'$.

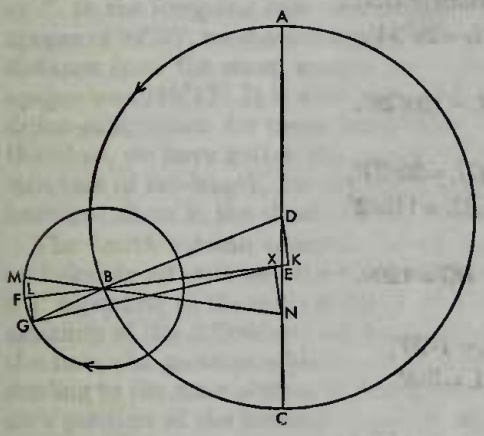
Therefore the inclination of the straight line MB through the mean apogee M to N has again cut off EN very nearly equal to DE, the line between the centres.

From several other observations we find that very nearly the same ratios result, so that in this way the property of the epicycle's inclination in the moon's hypothesis is established. For on the one hand the revolution of the epicycle's centre takes place about the ecliptic's centre E; on the other hand the epicycle's diameter which defines the epicycle's mean-apogee point no longer points to E, the centre of regular rotation, as for the other stars, but always points to N on the other side of E from D at a distance equal to DE, the line between the centres.

6. How the Moon's True Course is Gotten Geometrically from its Periodic Movements

With these things demonstrated, it would be proper next to add how, for particular courses of the moon, by taking the epochs of the mean movements we could find, from the elongation-number and the moon's epicyclic number, the addition or subtraction to the mean longitudinal course due to the anomalistic difference. This determination is gotten geometrically from theorems like those we have already expounded.

For example, in the case of the last figure we gave, let us suppose the same periodic movements of elongation and anomaly—that is, for double the elongation, 90°30′, and for anomaly 333°12′ from the epicycle's apogee. Let us draw NX perpendicular, instead of EX, and GL instead of BL. And by the same means, with the angles at the centre E given and the hypotenuses DE and NE given equal, it can be shown that



$$DK \fallingdotseq 10^{p}19',$$
$$NX \fallingdotseq 10^{p}19'$$

where

ecc. rad. $BD = 49^{p}41'$,
epic. rad. $BG = 5^{p}15'$,

and

$$EK = EX = 0^{p}5.'$$

And, therefore, as we showed before,

$$BK = 48^{p}36',$$
$$BE = 48^{p}31',$$

and, by subtraction,

$$BX = 48^{p}26'.$$

And so, since

$$\text{sq. } BX + \text{sq. } NX = \text{sq. } BN,$$

we shall have in length

$$BN = 49^{p}31'$$

where

$$NX = 10^{p}19'.$$

And therefore

$$NX = 25^{p}$$

where

$$\text{hypt. } BN = 120^{p};$$

and, on the circle about right triangle BNX,

$$\text{arc } NX = 24°3'.$$

And so also

$$\text{angle } NBX = \text{angle } FBM = 24°3' \text{ to 2 rt.},$$
$$= 12°1'.$$

And therefore, on the epicycle, also

$$\text{arc } FM = 12°1'.$$

But since the point G, the moon, is the remainder of the circle or 26°48′ distant from the mean apogee M, therefore we shall have, by subtraction,

$$\text{arc } FG = 14°47'.$$

And so also

$$\text{angle } FBG = 14°47'$$
$$= 29°34' \text{ to 2 rt.}$$

And also, on the circle about right triangle GBL,

$$\text{arc } GL = 29°34',$$

and, as remainder of the semicircle,

$$\text{arc } BL = 150°26'.$$

And therefore

$$\text{chord } GL = 30^{p}37',$$
$$\text{chord } BL = 116^{p}2'$$

where

$$\text{hypt. } BG = 120^{p}.$$

And so

$$GL = 1^{p}20',$$
$$BL = 5^{p}5'$$

where

$$\text{epic. rad.} = 5^{p}15'$$

and where it was proved

$$BE = 48^{p}31'.$$

And therefore, by addition,

$$EBL = 53^{p}36'$$

where

$$GL = 1^{p}20'.$$

And again since

$$\text{sq. } EBL + \text{sq. } GL = \text{sq. } EG,$$

therefore, we shall have in length

$$EG = 53^{p}37'.$$

And so

$$GL = 2^{p}59'$$

where

$$\text{hypt. } EG = 120^{p};$$

and, on the circle about right triangle EGL,

$$\text{arc } GL = 2°52'.$$

And therefore, as the anomalistic difference,

$$\text{angle } GEL = 2°52' \text{ to 2 rt.}$$
$$= 1°26'.$$

Which it was required to show.

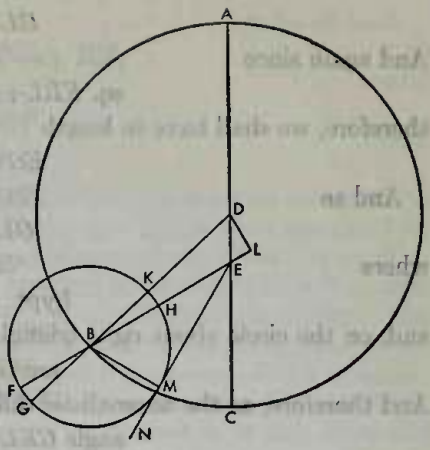
7. Construction of the Table of General Lunar Anomaly

To have a methodical determination of the particular addition-subtractions set out in a table, we have again used the same geometrical figures and filled out the table we constructed according to the simple hypothesis, with columns which readily correct for the second anomaly. For after the first two columns containing the numbers, we have added a third column containing the addition-subtractions corresponding to the anomalistic number for reducing this number, calculated by the mean passages from the mean apogee or M, to the true apogee or F. In the foregoing case of a distance between the moon and the eccentric's apogee of 90°30′, we showed the arc FM to be 12°1′ so that, although the moon's distance from the mean apogee was only 333°12′, its distance from the true apogee was 345°13′. It is with respect to this last number that the epicyclic addition-subtraction for mean longitudinal movement must be taken. In brief, therefore, we have gotten the quantities of addition-subtraction, at convenient intervals of arc-length, for the other numbers of the moon's distance, and we have put them in the third column beside the proper number.

The fourth column contains the anomalistic differences due to the epicycle and already set out; here the greatest addition-subtraction comes to very nearly 5°1′ according to the ratio of 60^{p} to $5^{p}15'$. The fifth column contains the excess amounts of the differences resulting from the second anomaly over those from the first, the greatest addition-subtraction from the second totaling 7⅔°, according to the ratio of 60 to 8. And so the fourth column provides for the epicycle's position at the eccentric's apogee near the syzygies; but the fifth column consists of the excess amounts gotten from the anomaly produced at the eccentric's perigee near the first and third quarters.

To find proportionately the parts of these excess amounts corresponding to the passages between these two positions on the eccentric, we have added a sixth column containing the sixtieth part of that difference for each number of elongation; and this must be added to the first anomaly's addition-subtraction laid out in the fourth column. And we have gotten them in this way:

Let there be the moon's eccentric circle ABC with centre D and diameter ADC on which let the ecliptic's centre E be taken. And with arc AB cut off and with the epicycle $FGHK$ described about B, let EBF be drawn. By way of example, let the moon's elongation be given as 60° so that, for the same reasons already demonstrated, angle AEB is double the elongation of 120°. And let DL be drawn from D perpendicular to BE produced, and let $GBKD$ be drawn through; let the straight line (in this case EMN) from the centre E to the moon be supposed tangent to the epicycle so as to have the greatest anomalistic difference; and let BM be joined.



Now, since it is assumed

angle $AEB = 120°$
$= 240°$ to 2 rt.,

therefore, as supplement,

angle $DEL = 120°$ to 2 rt.

And so, on the circle about right triangle DEL,

arc $DL = 120°$,

and, as remainder of the semicircle,

arc $EL = 60°$.

And therefore

chord $EL = 60^{p}$,
chord $DL = 103^{p}55'$

where

hypt. $DE = 120^{p}$.

And therefore

$EL = 5^{p}10'$,
$DL = 8^{p}56'$

where

$DE = 10^{p}19'$,
$BD = 49^{p}41'$.

And

sq. BD − sq. DL = sq. BL;

therefore, in length,

$BEL = 48^{p}53'$,

and, by subtraction,

$BE = 43^{p}43'$

where

epic. rad. $BM = 5^{p}15'$.

And therefore

$BM = 14^{p}25'$

where

hypt. $BE = 120^{p}$;

and, on the circle about right triangle BEM,

$$\text{arc } BM = 13°48'.$$

Therefore, as the angle which contains the greatest anomalistic difference,

$$\text{angle } BEM = 13°48' \text{ to 2 rt.}$$
$$= 6°54'.$$

The anomalistic difference for this distance of elongation thus differs from the 5°1′ produced at the apogee by 1°53′. But the complete difference up to the perigee is 2°39′. And therefore the difference of 1°53′ will be 42°48′ to the greatest difference's 60°. We shall put this 42°38′ in the sixth column in the row with the number 120° which here indicates the distance of the epicycle from the eccentric's apogee.

Likewise for the other sections we have calculated by the same means the parts so taken of the excess of one anomaly over the other, placing beside the proper numbers the sixtieth part of this excess corresponding to each one. And, of course, the whole 60 such parts have been put in a row with double the number 90°, the elongation, for the resulting 180° is at the eccentric's perigee.

We have added a seventh column containing the latitudinal passages of the moon on a circle through the ecliptic's poles corresponding to each part of the ecliptic; or, in other words, the arcs intercepted on the circle through the ecliptic's poles between the ecliptic and the moon's oblique circle. And for this we have used the same proof as that by which we calculated the arcs on the circle through the equator's poles, and which lie between the ecliptic and the equator. But in this case the arc on the great circle through the poles of the ecliptic and of the moon's oblique circle, lying between the ecliptic and the northern or southern limit of the oblique circle, is very nearly 5°. For, according to both Hipparchus and ourselves in calculating the appearances pertaining to the northernmost and southernmost courses, the moon's greatest passage was found to be just about that much on either side of the ecliptic. Nearly all things pertaining to lunar observation, considered with respect to the stars and by means of instruments, agree with greatest latitudinal courses of this size. And things to be demonstrated later will agree also. And the following is the table of general lunar anomaly.[1]

[1]This table is fairly complicated and condensed, and Ptolemy's explanation is not too clear. We shall here give a clearer and more analytical summary.

In the first place, the first two columns are called "common numbers" because these numbers play different roles for the different columns that follow.

With respect to column 3, columns 1 and 2 contain the numbers of arcs on the eccentric indicating the distance of the epicycle's centre from the eccentric's apogee. And column 3 contains the corresponding addition-subtractions to be made in the moon's distances from the epicycle's mean apogee, in order to determine its distance from the epicycle's true apogee.

With respect to column 4, columns 1 and 2 contain the numbers of arcs on the epicycle indicating the moon's distance from the epicycle's true apogee. And column 4 contains the corresponding addition-subtractions in longitude and latitude which correct for the anomaly when the epicycle is at the eccentric's apogee. These last numbers correspond to the addition-subtractions in the table of the first lunar anomaly, but they are here given their proper place within the general scheme of lunar anomaly.

With respect to column 5, columns 1 and 2 again contain the numbers of arcs on the epicycle indicating the moon's distances from the epicycle's true apogee. And column 5 contains the corresponding additions-subtractions in longitude due to the second lunar anomaly when the epicycle is at the eccentric's perigee, and which are over and above those due to the first anomaly. In other words, these are the differences between the first and general anomalies.

With respect to column 6, columns 1 and 2 contain the numbers of arcs on the eccentric

8. Table of the General Lunar Anomaly

1	2	3		4		5		6		7		
Common Numbers		Addition—subtractions for distances from eccentric's apogee		Addition—subtractions in longitude and latitude for epicycle		Epicyclic difference		Difference in sixtieths		Latitude		
6	354	0	53	0	29	0	14	0	12	4	58	*Northern limit*
12	348	1	46	0	57	0	28	0	24	4	54	
18	342	2	39	1	25	0	42	1	20	4	45	
24	336	3	31	1	53	0	56	2	16	4	34	
30	330	4	23	2	19	1	10	3	24	4	20	
36	324	5	15	2	44	1	23	4	32	4	3	
42	318	6	7	3	8	1	35	6	25	3	43	
48	312	6	58	3	31	1	45	8	18	3	20	
54	306	7	48	3	51	1	54	10	22	2	56	
60	300	8	36	4	8	2	3	12	26	2	30	
66	294	9	22	4	24	2	11	15	5	2	2	
72	288	10	6	4	38	2	18	17	44	1	33	
78	282	10	48	4	49	2	25	20	34	1	3	
84	276	11	27	4	56	2	31	23	24	0	32	
90	270	12	0	4	59	2	35	26	36	0	0	
93	267	12	15	5	0	2	37	28	12	0	16	
96	264	12	28	5	1	2	38	29	49	0	32	
99	261	12	39	5	0	2	39	31	25	0	48	
102	258	12	48	4	59	2	39	33	1	1	3	
105	255	12	56	4	57	2	39	34	37	1	17	
108	252	13	3	4	53	2	38	36	14	1	33	
111	249	13	6	4	49	2	38	37	50	1	48	
114	246	13	9	4	44	2	37	39	26	2	2	
117	243	13	7	4	38	2	35	41	2	2	16	
120	240	13	4	4	32	2	32	42	38	2	30	
123	237	12	59	4	25	2	28	44	3	2	43	
126	234	12	50	4	16	2	24	45	28	2	56	
129	231	12	36	4	7	2	20	46	53	3	8	
132	228	12	16	3	57	2	16	48	18	3	20	
135	225	11	54	3	46	2	11	49	32	3	32	
138	222	11	29	3	35	2	5	50	45	3	43	
141	219	11	2	3	23	1	58	51	59	3	53	
144	216	10	33	3	10	1	51	53	12	4	3	
147	213	10	0	2	57	1	43	54	3	4	11	
150	210	9	22	2	43	1	35	54	54	4	20	
153	207	8	38	2	28	1	27	55	45	4	27	
156	204	7	48	2	13	1	19	56	36	4	34	
159	201	6	56	1	57	1	11	57	15	4	40	
162	198	6	3	1	41	1	2	57	55	4	45	
165	195	5	8	1	25	0	52	58	35	4	50	
168	192	4	11	1	9	0	42	59	4	4	54	
171	189	3	12	0	52	0	31	59	26	4	56	
174	186	2	11	0	35	0	21	59	37	4	58	
177	183	1	7	0	18	0	10	59	49	4	59	*Southern limit*
180	180	0	0	0	0	0	0	60	0	5	0	

9. On General Calculations of the Moon

Now whenever we wish, by means of this tabular set up, to calculate the lunar anomaly we take the moon's mean movements, for the date in question in Alexandria, and we take them in longitude, elongation, anomaly, and latitude in the way we have shown. Always doubling the first number of elongation calculated, and subtracting a whole circle from the product if we can, we carry the result to this Table of Anomaly. We then add the degrees in the third column corresponding to it to the mean degrees of anomaly, if the number doubled runs up to 180°; but, if it exceeds 180°, we subtract it. Again we carry the resulting true number of anomaly to this same table, and we note down the addition-subtraction corresponding to it in the fourth column, and also the corresponding difference in the fifth column.

After this we again carry the double of the number indicating the mean elongation to the same first two columns; and, as many sixtieths in the sixth column as correspond to it, just so many do we take of the difference from the fifth column which we have already noted down. We always add the result to the addition-subtraction we found in the fourth column. If the true anomalistic number is not above 180°, we subtract this sum from the mean longitude and latitude; but, if it is above 180°, we add it. Of the two resulting numbers we take the one in longitude and add it to the mean position calculated at the epoch; and whatever it comes out to, we shall say the moon is truly at that place.

We carry the latitudinal number, computed from the northern limit, to this

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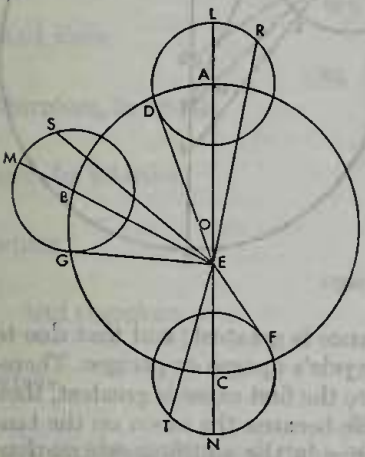
indicating the distances of the epicycle's centre from the eccentric's apogee. And column 6 contains the corresponding differences between the first anomaly and the general anomaly when the moon lies on the tangent from the earth to the epicycle or when the epicyclic addition-subtraction is greatest. But these differences are here expressed in terms of the greatest difference among them, which is taken as 60. It is then assumed that, for all practical purposes, the ratio of a difference for a given position of the epicycle to the greatest difference when the epicycle is at the eccentric's perigee is the same for all positions on the epicycle. In other words, taking the eccentric about O and the centre of the ecliptic E, and drawing equal epicycles at the apogee and perigee, A and C, and another equal epicycle at a chance position B, let the tangents from E to the three epicycles be drawn, and let equal arcs LR, MS, and NT be taken from their apogees. Then Ptolemy is assuming that, for all practical purposes, we can use the following proportion:



angle CEF — angle AED :
angle BEG — angle AED : :
angle NET — angle LER :
angle SEM — angle LER.

Now, column 6 gives us the first ratio for different positions of B. And therefore, knowing the differences between angle NET and angle LER, for different lengths of LR and TN, from column 5, we can find the differences between angle SEM and angle LER.

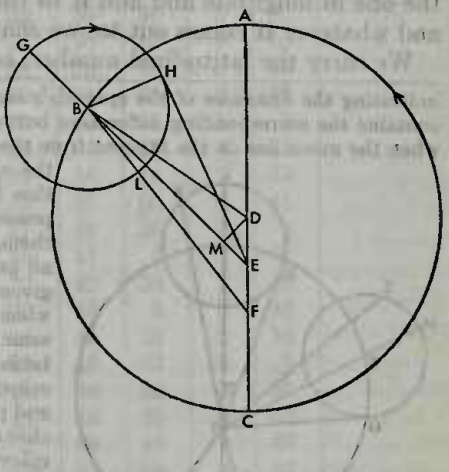
With respect to column 7, columns 1 and 2 contain the numbers of the arcs of the moon's oblique circle, beginning at the northern limit and ending at the southern. And column 7 contains the corresponding numbers of arcs intercepted on circles through the poles of the ecliptic, between the ecliptic and the moon's oblique circle.

same table. And whatever number of degrees in latitude in the seventh column correspond to it, by just so many is the moon's centre distant from the ecliptic along the great circle drawn through the ecliptic's poles. If the number carried is in the first 15 rows, this distance is to the north of the ecliptic; but if it is in the rows below, the distance is to the south of the ecliptic. The first column of numbers embraces the movement from north to south, and the second column the movement from south to north.

10. That no Appreciable Difference is Produced at the Syzygies by the Moon's Eccentric Circle

Now, it is reasonable for some to suspect that at times an appreciable difference occurs at the conjunctions, full moons, and eclipses accompanying them, because of the moon's eccentric circle. For the epicycle's centre does not always fall exactly on the eccentric's apogee at these periods, but can miss it to the extent that the return of the epicycle's centre to the eccentric's apogee is effected at the mean syzygies, but true conjunctions and full moons are taken with the anomaly of each of the two luminaries. Therefore we shall try and show that this difference can produce no error worth mentioning in the appearances at the syzygies, even if the difference due to the circle's eccentricity is neglected.

For let *ABC* be the moon's eccentric circle with centre *D* and diameter *ADC* on which let the point *E* be taken as the ecliptic's centre and *F* on the side opposite *D* as the centre of epicyclic inclination. And with arc *AB* cut off from the apogee *A*, let the epicycle *GHL* be described about *B*, and let *BD* and *GBE* and also *BLF* be joined.



Now, the magnitude of the anomaly can differ in two ways from that proper to the epicycle's position at the apogee *A:* (1) by cutting off a greater angle at *E* because of its position towards the perigee, and (2) by having the diameter through the mean apogee and perigee not directed to the center *E* but to the point *F*.

The difference due to the first cause is greatest when the moon's anomalistic difference is greatest; and that due to the second cause, when the moon is at the epicycle's apogee or perigee. Therefore it is evident that, when the difference due to the first cause is greatest, then that due to the second will be entirely negligible because the moon on the tangents to the epicycle makes very little difference in the addition-subtraction; and it is evident that the true syzygy will possibly differ from the mean by as much as the anomalistic differences of both luminaries put together, either by addition or by subtraction. But when (according to the second cause) the difference in inclination is the greatest, then the difference from the first cause is negligible because the entire anomaly is either nothing or very little when the moon is at the epicycle's apogee or perigee; and the true syzygy will differ from the mean syzygy by the difference of solar anomaly alone.

(1) Then let the sun be assumed to effect the greatest addition of 2°23′, and also the moon itself to effect first the greatest subtraction of 5°1′ so that angle AEB contains 14°48′, the double of the 7°24′ of the two together. And let the tangent EH from E to the epicycle be drawn, and the perpendicular BH be joined; and also let DM be drawn from D perpendicular to BE.

Since, then,

$$\text{angle } AEB = 14°48′ = 29°36′ \text{ to 2 rt.},$$

therefore also, on the circle about right triangle DEM,

$$\text{arc } DM = 29°36′,$$

and, as remainder of the semicircle,

$$\text{arc } EM = 150°24′.$$

And

$$\text{chord } DM = 30^{p}39′,$$
$$\text{chord } EM = 116^{p}1′$$

where

$$\text{hypt. } DE = 120^{p}.$$

And so

$$DM = 2^{p}38′,$$
$$EM = 9^{p}59′$$

where

$$1. \text{ betw. c. } DE = 10^{p}19′$$

and

$$\text{ecc. rad. } BD = 49^{p}41′.$$

And since

$$\text{sq. } BD - \text{sq. } DM = \text{sq. } BM,$$

therefore, in length,

$$BM = 49^{p}37′,$$

and, by addition,

$$BME = 59^{p}36′$$

where

$$\text{epic. rad. } BH = 5^{p}15′.$$

And therefore

$$BH = 10^{p}34′$$

where

$$\text{hypt. } BE = 120^{p};$$

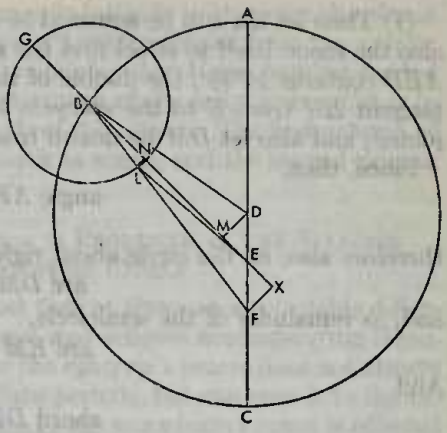
and, on the circle about right triangle BEH,

$$\text{arc } BH = 10°6′.$$

And therefore, as the angle of greatest anomalistic difference,

$$\text{angle } BEH = 10°6′ \text{ to 2 rt.} = 5°3′$$

instead of the 5°1′ produced when the epicycle is at the apogee A. Therefore from this cause the anomalistic difference diverges by 2′, which is an error of less than $\frac{1}{16}$ of an hour.



(2) Again let the moon be supposed at L the mean perigee, so that angle AEB clearly contains very nearly the double of the sun's anomaly alone, that is 4°46′. And with EL joined in a figure similar to the last, let the perpendiculars LN, DM, and FX be dropped from L, D, and F to BE produced. And, just as before, since

angle at $E = 4°46'$
$= 9°32'$ to 2 rt.,

therefore, on the circles about right triangles EDM and EFX,

arc DM = arc $FX = 9°32'$.

And, as remainders of semicircles,

arc EM = arc $EX = 170°28'$.

And therefore

chord DM = chord $FX = 9^p58'$,
chord EM = chord $EX = 119^p35'$

where

hypt. DE = hypt. $EF = 120^p$.

And so

$DM = FX = 0^p51'$,
$EM = EX = 10^p17'$

where

$DE = EF = 10^p19'$

and

ecc. rad. $DB = 49^p41'$.

And since

sq. BD − sq. DM = sq. BM,

therefore, in length,

$BM = 49^p41'$.

And so

$BE = 59^p58'$

and, by addition,

$BX = 70^p15'$

where

$FX = 0^p51'$.

And, in the same way also,

hypt. $BF = 70^p15'$.

And

$BF : FX :: BL : LN$
$BF : BX :: BL : BN$.

And so

$LN = 0^p4'$,
$BN = 5^p15'$,

and, by subtraction,

$EN = 54^p43'$

where

epic. rad. $BL = 5^p15'$

and it was proved

$$BE = 59^{p}58'.$$

And since, therefore, the hypotenuse EL differs inappreciably from $54^{p}43'$, it is inferred that

$$LN \doteq 0^{p}8'$$

where

$$\text{hypt. } EL = 120^{p};$$

and that, on the circle about right triangle ELN,

$$\text{arc } LN = 0°8'.$$

And, therefore, angle BEL, by which the moon differs because of the inclination to F, will also be $0°8'$ to 2 right angles' $360°$, or $0°4'$.

So in this case the moon's anomalistic difference has diverged by $4'$, which produces no appreciable error in the appearances at the syzygies, and hardly amounts to $\frac{1}{8}$ of an hour. And it is not extraordinary for such an error to occur often in the observations themselves.

We have added all this, not because it is impossible to calculate these differences along with the examination of the syzygies, but because there is no appreciable error for us in the proofs by means of the lunar eclipses we have set out, even if we have not used the hypothesis of eccentricity in the developed form we give it in what follows.

11. On the Moon's Parallaxes

Now, the means used in getting the true passages of the moon would be generally speaking these: Since in the case of the moon it turns out that the apparent passage is not sensibly the same as the true passage (because, as we said, the earth does not have the ratio of a point to the radius of the moon's sphere) it would be necessary, and consequent, both for other appearances and especially for those observed about the solar eclipses, to get the measure of the parallaxes from which one will be able to determine the passages considered from the observer's eye—that is, from some part of the earth's surface—by means of the true passages conceived with respect to the centre of the earth and ecliptic; and, conversely, to determine the true passages by means of the apparent.

Since it is a necessary condition of this research that the particular magnitudes of the parallaxes cannot be worked out without one's having as given the ratio of the linear distance nor the ratio of the linear distance without some parallax's being given, it is clear that in the case of those stars having no sensible parallax (that is, those with respect to which the earth is in the ratio of a point) getting the ratio of the distance would be impossible. But only in the case of those which display a parallax, as the moon, would it be possible to find the ratio of the distance by means of some first-given parallax, because a parallactic observation can be gotten in itself, but not the size of the distance.

Now, Hipparchus made this examination chiefly from the sun. Since, from certain other incidents of the sun and moon about which we shall give an account later on, it follows that when the linear distance for one of the luminaries is given the distance for the other is also given, he tries by conjecturing the sun's distance to demonstrate the moon's. He first supposes the sun to display only the least sensible parallax in order to get its distance. And then by means of the solar eclipse recorded by him, from calculations with the sun—first as if displaying no sensible parallax and secondly as if displaying a sufficient parallax—

he arrives at the ratios of the lunar distance, different for eac
ses, although not only is the magnitude of the sun's parallax
but also whether it displays any parallax at all.

12. On the Construction of an Instrum for Determining Parallaxes

To record nothing unclear in this research, we built an in
we could observe as accurately as possible how much and
tance from the zenith on the great circle through the moo
poles the moon displays a parallax.

We made two four-sided rods, not less than four cubits i
divide them into a great many parts, and proportionate in c
be bent by their length, but to be extended very true and i
each of their sides. Then we drew straight lines on each of th
the wider side. At each end of one of the rods on the centr
square right prisms equal and parallel, each having a hole p
middle—the one for the eye having a small hole, the one fo
hole—so that, when an eye is placed at the prism with th
whole moon can appear through the other hole in a straight

Then we bored evenly each of the rods on the centre lin
extremities; in the case of the rod with the prisms, near th
the prism with the larger hole. Through them both we fitted
of the rods with the lines would be bound together as if by a
the prisms could thus be turned in any direction without v
rod, the one without prisms, we fixed on a base. On the cen
we took points equidistant from the lower extremities and as
the axle-centre, dividing the line so defined on the fixed ro
each of these into as many as possible. And on the back of th
extremities, we placed small prisms having their sides in the
straight line with each other and everywhere equidistant fro
line, so that the rod could be stood straight and without inc
zon's plane by hanging a plumb-line through them.

Having the meridian line already established in a plane
zon's, we stood the instrument upright in a shadowless place
the rods, where they were joined together by the axle, were
parallel to the meridian line, and so that the rod with the
without inclination and fixed without deviation, while the
revolved around the axle in the plane of the meridian, subje
added also another small thin rod, fitted on straight for revo

until its centre was sighted through both holes at the centre of and marking on the fine small rod the distance between the ends lines on the other rods and applying it to the line of the upright been cut into 60 parts, we found out how many parts the chord has to the 60 of the radius of the circle described by the revolutic ian plane. And getting the arc subtended by a chord of such l had the arc by which the moon's apparent centre was distant f along the great circle drawn through the poles of the horizon an centre—a great circle which at that time was the same as the m the poles of the equator and of the ecliptic.

For observing the moon's greatest latitudinal passage, we u especially when it was at the summer tropic point and again a most limit of the moon's oblique circle. This was done becaus points for some distance sensibly the same latitudinal passage is because the moon, at those times being near the zenith in the p andria where we made the observations, has very nearly the position as true position. In every case the moon's centre was ob passages to be very nearly $2\frac{1}{8}°$ from the zenith so that, from th its greatest latitudinal passage on either side of the ecliptic is d be 5°. And this is almost exactly the difference between the 23°5 the equator to the summer tropic point, and the 30°58′ from th equator in Alexandria less $2\frac{1}{8}°$.

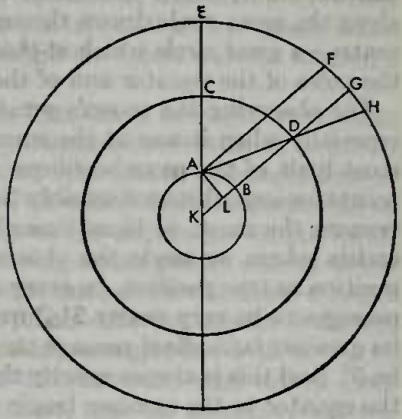
But for getting the inquiry into the parallaxes done, we again i observed the moon about the winter tropic for reasons already cause also (being at that time away from the zenith by a simila the meridian) it provides a greater and more easily observed para many parallaxes observed by us in such passages, we shall again means of which we shall at once both present the manner of demonstrate the other things as immediate consequences.

13. Demonstration of the Moon's Distance

For we observed the moon culminating in the year 20 of Hadr wise Athyr 13, $5+\frac{1}{2}+\frac{1}{3}$ equatorial hours after noon, as the su set; its centre appeared to us through the instrument at a distan $\frac{1}{3}+\frac{1}{12})°$ from the zenith. The distance on the small light rod w parts to the 60 parts of the radius of the circle of revolution, and length subtends an arc of $(50+\frac{1}{2}+\frac{1}{3}+\frac{1}{12})°$. But the time fro the year 1 of Nabonassar, up to this observation, is 882 Egyptian and $5+\frac{1}{2}+\frac{1}{3}$ equatorial hours simply considered, but $5\frac{1}{3}$ h

circle. And the equator is 30°58′ from the zenith in Alexandria, and likewise south of it. Therefore the moon's centre was truly 49°48′ from the zenith, but it appeared to be 50°55′ from it. Therefore the moon displayed a parallax, for the distance at this passage, of 1°7′ along the great circle drawn through the moon and the horizon's poles, its true position being 49°48′ from the zenith.

Now that this is clear, in the plane of the circle through the horizon's poles and the moon let the following great circles be drawn concentric: the earth's great circle *AB;* that through the moon's centre at the observation, *CD;* and the circle *EFGH* to which the earth has the ratio of a point. And let *K* be the common centre of them all, and *KACE* the straight line through the zenith points. And let the moon's true position be supposed at the point *D* 49°48′ from the zenith as in the preceding instance; and let *KDG* and *ADH* be joined. Moreover, from *A*, which is the observer's eye, let *AL* be drawn perpendicular to *KB*, and *AF* parallel to *KG*.



It is clear that the moon displays a parallax of arc *GH* for those observing from *A*, so that

$$\text{arc } GH = 1°7'$$

as gotten by observation. But since arc *FH* is not appreciably greater than *GH* because of the whole earth's being in the ratio of a point to the circle *EFGH*, therefore

$$\text{arc } FGH = 1°7'.$$

And so, again because the point *A* is indistinguishable from the centre with respect to circle *FH*,

$$\text{angle } FAH = 1°7' = 2°14' \text{ to 2 rt.}$$

And

$$\text{angle } ADL = \text{angle } FAH = 2°14' \text{ to 2 rt.}$$

And therefore, on the circle about right triangle *ADL*,

$$\text{arc } AL = 2°14',$$
$$\text{chord } AL = 2^p21'$$

where

$$\text{hypt. } AD = 120^p.$$

And *DL* is indistinguishably less than *AD*. Therefore

$$DL = 120^p$$

where

$$AL = 2^p21'.$$

Again, since it is assumed

$$\text{arc } CD = 49°48',$$

therefore, being at the centre,

$$\text{angle } CKD = 49°48' = 99°36' \text{ to 2 rt.}$$

And so, on the circle about right triangle ALK,

arc $AL = 99°36'$,

and, as remainder of the semicircle,

arc $LK = 80°24'$.

And therefore

chord $AL = 91^{p}39'$,
chord $KL = 77^{p}27'$

where

hypt. $AK = 120^{p}$.

And so

$AL = 0^{p}46'$,
$KL = 0^{p}39'$

where, as the earth's radius,

$AK = 1^{p}$.

But it was shown that

$DL = 120^{p}$

where

$AL = 2^{p}21'$,

and therefore

$DL = 39^{p}6'$

where

$AL = 0^{p}46'$.

And, of the same parts,

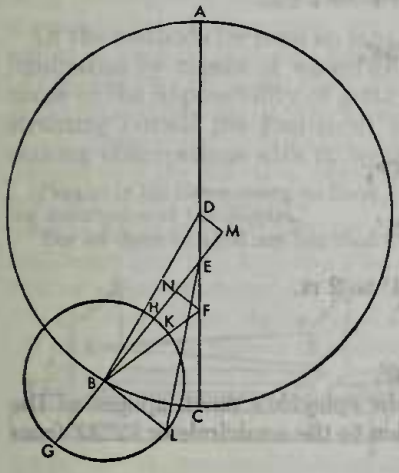
$KL = 0^{p}39'$

and, as the earth's radius,

$AK = 1^{p}$.

Therefore the whole line KLD, embracing the moon's distance at the observation, will be $39^{p}45'$ to the 1^{p} of AK, the earth's radius.

Now that this has been shown, let there be the moon's eccentric circle ABC with centre D and diameter ADC, on which



let there be the ecliptic's centre E, and the epicycle's point of inclination F. With the epicycle $GHKL$ drawn about the point B, let the straight lines $GBHE$, BD, and BKF be joined. And let the moon in this observation be supposed at the point L. Let LE and LB be joined, and let line DM be drawn from point D, and FN from point F, both perpendicular to BE.

Since, then, at the time of the observation the number of the elongation was $78°13'$, therefore, through things already seen,

angle $AEB = 156°26'$,

and, as supplementary angles,

angle FEN = angle $DEM = 23°34'$
$= 47°8'$ to 2 rt.

And so, on circles about these right triangles

arc DM = arc FN = 47°8′

because

$DE = EF$;

and

arc EM = arc EN = 132°52′.

And therefore

chord DM = chord FN = 47^{p}59′

and

chord EM = chord EN = 110^{p}

where

hypt. DE = hypt. EF = 120^{p}.

And so

$DM = FN = 4^{p}8'$,
$EM = EN = 9^{p}27'$

where

$DE = EF = 10^{p}19'$

and

ecc. rad. $BD = 49^{p}41'$.

And since

sq. BD − sq. DM = sq. BM,

we shall have, in length,

$BM = 49^{p}31'$,

and likewise

$BE = 40^{p}4'$

and, by subtraction,

$BN = 30^{p}37'$

where

$FN = 4^{p}8'$.

And since

sq. BN + sq. FN = sq. BF,

we shall have, in length,

hypt. $BF = 30^{p}54'$.

And so

$FN = 16^{p}2'$

where

hypt. $BF = 120^{p}$;

and, on the circle about right triangle BFN,

arc $FN = 15°21'$.

And therefore

angle $FBN = 15°21'$ to 2 rt.
$\fallingdotseq 7°40'$.

Therefore, on the epicycle,

arc $HK = 7°40'$.

Again, since the moon was 262°20′ from the epicycle's mean apogee at the time of the observation, and clearly the addition to the semicircle or 82°20′ from the mean perigee K, therefore also

arc $KL = 82°20'$,

and, by addition,

arc $HKL = 90°$.

Therefore

angle HBL = rt. angle.

And so, since it was shown that

$BE = 40^p4'$

where

ecc. rad. $BD = 49^p41'$

and

epic. rad. $BL = 5^p15'$,

and since

sq. BL + sq. BE = sq. EL,

we shall also have, in length,

$EL = 40^p25'$.

The moon's linear distance at the observation is hence $40^p25'$ to the assumed $5^p15'$ of the epicycle's radius BL; to the 60^p of EA, the radius from the earth's centre to the eccentric's apogee; and to the $39^p22'$ of EC, the radius from the earth's centre to the eccentric's perigee.

But it was shown that the moon's distance at the observation (that is, the straight line EL) was $39^p45'$ to the 1^p of the earth's radius. And, therefore, the straight line EA or the mean distance at the syzygies is 59^p; and EC or the mean distance at the first and third quarters is $38^p43'$; and the epicycle's radius $5^p10'$ to the $39^p45'$ of the straight line EL, or the moon's distance at the observation. Which things it was required to show.

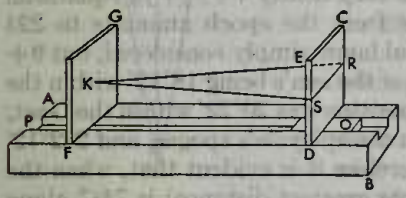
And now that the moon's distances have been shown in the manner described, it would next be in order to demonstrate at the same time the sun's distance too, since it is readily accessible geometrically if, in addition to the moon's distances at the syzygies, there should be given the magnitudes of the angles at the eye in the syzygies, subtended by the diameters of the sun, moon, and shadow.

14. On the Magnitude of the Apparent Diameters of the Sun, Moon, and Shadow during the Syzygies

Of the methods for such an inquiry, we rejected all those which measure these luminaries by means of waterclocks or the times of equatorial ascensions, because of the impossibility of getting what is proposed by such means. But constructing ourself the four-cubit rod dioptra described by Hipparchus,[1] and making observations with it, we find the sun's diameter everywhere contained

[1]Pappus in his *Commentary* on Book v gives a description of the waterclock, and the following description of the dioptra.

"For let there be a rod not less than 4 cubits in length and of width and thickness sufficient to keep the rod rigid. Let its upper lateral surface be conceived as the parallelogram AB. And let there be a groove on it along the straight line PO through the middle of it, so that a little block is easily moved as we wish along the whole length of the rod without falling from it. And let a suitable prism CD be added on, perpendicular to the rod and stable, having RSD as its side toward the block and the straight line CE at the top of the prism. Let there be another suitable perpendicular prism FG at one end of the rod, and let it have a small hole, not at the rod, but in the middle like K, so that, when our eye is placed at it for



by very nearly the same angle with no variation worthy of mention resulting from its distances. But we find the moon's diameter contained by the same angle as the sun's, then, only when during the full moons it is at its greatest distance from the earth, being at the epicycle's apogee; and not when it is at the mean distance as in the hypotheses of older astronomers. Moreover, we also found these angles a good deal smaller than those handed down, and that without using the measurement on the rod, but by calculating with lunar eclipses. For it was easy to see when each of the diameters subtends the same angle, from the fact that no comparative measurement is available in such a situation. But how large they were seemed very doubtful to us since, even when the comparison is greatest in the to-and-fro movements of the covering width along the length from the eye to the prism, it can be off the true one. But once the moon at its greatest distance appeared to make an angle at the eye equal to the sun's, by means of the lunar eclipses observed at that distance we calculated the angle subtended by the moon, and immediately we had that of the sun also. And, again, by two eclipses set out below, we shall render the method of this general theory very understandable.

For in the year 5 of Nabopollassar (which is the year 127 of Nabonassar, Egyptianwise Athyr 27-28 at the end of the eleventh hour) the moon began to be eclipsed in Babylon; and the greatest extent of the eclipse was ¼ of the diameter from the south. Since, then, the beginning of the eclipse took place 5 seasonal hours after midnight, and the middle very nearly 6 hours after midnight which in Babylon amounted to 5+½+⅓ equatorial hours because of the sun's true position's being 27°3′ within the Ram, therefore it is clear that the middle of the eclipse, when the greatest part of the diameter fell within the shadow, took place in Babylon 5+½+⅓ equatorial hours after midnight, but in Alexandria only 5 hours after. And the total time from the epoch amounts to 126 Egyptian years, 86 days, and 17 equatorial hours simply considered, but in terms of mean solar days 16+½+¼ equatorial hours. The moon's mean longitudinal passage was thus 25°32′ within the Balance, and its true position 27°5′; also it was 340°7′ from the epicycle's apogee and 80°40′ from the northern limit of the oblique circle. And it is evident that, when the moon's centre (the moon now being near its greatest distance) is 9⅓° along the oblique circle from the nodes, and when the shadow's centre lies on the great circle drawn through the moon at right angles to the oblique circle in which position the greatest obscurations take place, then a quarter of the diameter falls within the shadow.

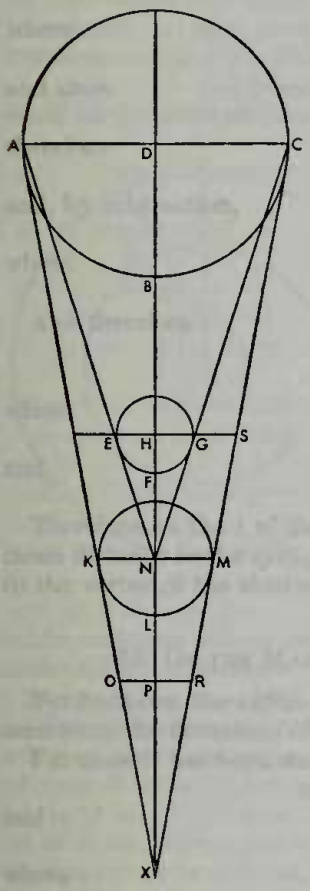
Again, in the year 7 of Cambyses (which is the year 225 of Nabonassar, Egyptianwise Phamenoth 17-18 one hour before midnight) the moon was eclipsed in Babylon to the extent of a half of its diameter from the north. Therefore the eclipse also took place in Alexandria very nearly 1+½+⅓ equatorial hours before midnight. And the total time from the epoch amounts to 224 Egyptian years, 196 days, and 10⅙ equatorial hours simply considered, but 9+½+⅓ hours accurately considered, because of the sun's being 18°12′ within the Crab. And so the moon's mean longitudinal position was 20°22′ within the Goat, but its true one 18°14′; also it was 28°5′ from the epicycle's apogee, and 262°12′ from the oblique circle's northern limit. Therefore it is evident that, when the moon's centre (the moon being near the same greatest distance) is 7⅘° along

use, the straight lines drawn from it to the moving prism *CD* through its side edges can contain the whole apparent diameter of the sun, touching it at its extremities."

the oblique circle from the nodes, and the shadow's centre has the position we just described with respect to the moon's centre, one half of the moon's diameter falls within the shadow.

But if the moon's centre is $9\frac{1}{3}°$ along the oblique circle from the nodes, it is $0°48\frac{1}{2}'$ from the ecliptic along the great circle drawn through itself at right angles to the oblique circle. And when it is $7\frac{1}{5}°$ along the oblique circle from the nodes, it is $0°40\frac{2}{3}'$ from the ecliptic along the great circle drawn through itself at right angles to the oblique circle. Since, then, the difference between the two eclipses embraces a quarter of the moon's diameter, and since the difference between the two distances of its centre from the ecliptic, that is from the shadow's centre, is $(7+\frac{1}{2}+\frac{1}{3})'$, therefore it is evident the whole diameter of the moon subtends an arc of a great circle amounting to $0°31\frac{1}{3}'$.

It is immediately easy to see that the radius of the shadow at the moon's greatest distance subtends $0°40\frac{2}{3}'$. For, when the moon's centre was that distance from the shadow's centre, it was tangent to the shadow's circle, half of the moon's diameter being eclipsed. And the shadow's radius is very little less than $2\frac{3}{5}$ times as great as the moon's radius which is $0°15\frac{2}{3}'$. By means of several other such observations we got very nearly these same magnitudes, and we have used them for other things about eclipses. We now use one of them which Hipparchus also followed, for the demonstration of the sun's distance to be made in this same way, since the circles intercepted by the cones on the sun, moon, and earth are not appreciably different from the great circles themselves described on those spheres and their diameters.[1]



15. On the Sun's Distance and Demonstrations Related Thereto

Now that these things are given, and seeing that the moon's greatest distance is $64^p10'$ to the 1^p of the earth's radius because it has been shown the mean distance is 59^p and the epicycle's radius $5^p10'$, let us find out also how great is the sun's distance.

For let there be the great circles of the spheres in the same plane: *ABC* the sun's with centre *D*, *EFG* the moon's at its greatest distance with centre *H*, and *KLM* the earth's with centre *N*. Of the planes through the centres, let *AXC* be that containing the earth's and the sun's, and *ANC* that containing the sun's and the moon's. Let there be the common axis *DHNX;* and let there be the parallel lines through the points of tangency, clearly equal to the diameters as far as the senses are con-

[1]This demonstration in all its details for the case of the moon is given by Aristarchus in his *Treatise on the Sizes and Distances of the Sun and Moon.*

cerned: that of the sun's circle, *ADC;* that of the moon's circle, *EHG;* that of the earth's circle, *KNM;* and *OPR* of the shadow's circle into which the moon falls at its greatest distance. And so

$$HN = NP = 64^{p}10'$$

where, as earth's radius,

$$LN = 1^{p}.$$

Then it is required to find what ratio the line *DN*, the sun's distance, has to *LN*, the earth's radius.

Now let *EGS* be produced. And since we have shown that the moon's diameter at the given greatest distance in the syzygies subtends an arc of 0°31′20″ on the circle drawn through the moon about the earth's centre, it follows that

$$\text{angle } ENG = 0°31'20''$$

and, as half of it,

$$\text{angle } HNG = 0°31'20'' \text{ to 2 rt.}$$

And so, on the circle about right triangle *HGN*,

$$\text{arc } GH = 0°31'20''$$

and, as remainder of the semicircle,

$$\text{arc } HN = 179°28'40''.$$

And therefore

$$\text{chord } GH = 0^{p}32'48''$$

where

$$\text{diam. } GN = 120^{p}$$

and

$$\text{chord } HN = 120^{p}.$$

And so also

$$GH = 0^{p}17'33''$$

where

$$HN = 64^{p}10'$$

and

$$\text{earth's rad. } MN = 1^{p}.$$

But since, very nearly,

$$PR : GH :: 2^{p}36' : 1^{p},$$

therefore

$$PR = 0^{p}45'38''.$$

Therefore

$$GH + PR = 1^{p}3'11''$$

where

$$MN = 1^{p}.$$

But

$$HS + PR = 2^{p},$$

because

$$HS + PR = 2MN.$$

For, as we said, they are all parallel, and

$$NP = HN.$$

And therefore, by subtraction,

$$GS = 0^{p}56'49''$$

where

$$MN = 1^{p}.$$



And

$$MN : GS :: CN : CG :: DN : DH.$$

Therefore also

$$DH = 0^{p}56'49''$$

and, by subtraction,

$$HN = 0^{p}3'11''$$

where

$$DN = 1^{p}.$$

And so we shall have also, as the sun's distance,

$$DN = 1210^{p}$$

where

$$HN = 64^{p}10'$$

and

$$MN = 1^{p}.$$

And since likewise it was shown that

$$PR = 0^{p}45'38''$$

where

$$MN = 1^{p},$$

and since

$$MN : PR :: NX : PX,$$

therefore

$$PX = 0^{p}45'38'',$$

and, by subtraction,

$$NP = 0^{p}14'22''$$

where

$$NX = 1^{p}.$$

And therefore

$$PX = 203^{p}50',$$
$$NX = 268^{p}$$

where

$$NP = 64^{p}10'$$

and

earth's rad. $MN = 1^{p}$.

Therefore, to the 1 of the earth's radius, we have concluded that the moon's mean distance in the syzygies is 59, the sun's 1210, and from the earth's centre to the vertex of the shadow's cone 268.

16. On the Magnitudes of the Sun, Moon, and Earth

Furthermore, the ratio of these solid magnitudes is easily and immediately seen from the diameters of the sun, moon, and earth.

For since it has been shown that

moon's rad. $GH = 0^{p}17'33''$,

and

$$HN = 64^{p}10'$$

where

earth's rad. $MN = 1^{p}$,

and since

$$HN : GH :: DN : CD,$$

therefore, having shown

$$DN = 1210^{p},$$

we shall also have

$$\text{sun's rad. } CD = 5\tfrac{1}{2}{}^{p}.$$

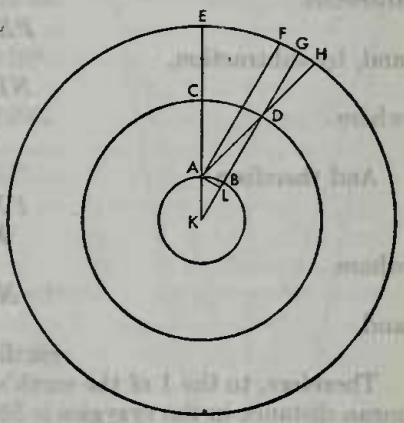
And, therefore, the ratios of the diameters will be the same.

The earth's diameter will hence be very nearly $3\frac{2}{5}$, and the sun's $18\frac{4}{5}$, to the moon's 1. Therefore the earth's diameter is $3\frac{2}{5}$ times as great as the moon's, and the sun's $18\frac{4}{5}$ times as great as the moon's, and very nearly $5\frac{1}{2}$ times as great as the earth's. Accordingly, since the cube of 1 is 1, and the cube of $3\frac{2}{5}$ is very nearly $39\frac{1}{4}$, and the cube of $18\frac{4}{5}$ likewise very nearly $6644\frac{1}{2}$, therefore we have concluded [Eucl. XII, 18] that the earth's solid magnitude is $39\frac{1}{4}$, and the sun's $6644\frac{1}{2}$, to the moon's 1. Therefore the sun's solid magnitude is very nearly 170 times the earth's.

17. On the Particular Parallaxes of the Sun and Moon

Now, with these things supposed in this way, it would next follow that we show in addition and very briefly how one could compute, from the distances of the sun and moon, their particular parallaxes—and, first of all, those considered on the great circle drawn through the zenith and themselves.

Again, then, let there be in the plane of this great circle the earth's great circle AB, the circle at the sun or moon CD, and the circle $EFGH$ to which the earth has the ratio of a point; and let K be the centre of them all, and $KACE$ the diameter through the zenith points. With arc CD cut off from the zenith C at 30° as an example, again let KDG and ADH be joined; and from A let AF be drawn parallel to KG, and AL perpendicular to it.



Now, since the distance of each of these luminaries is not always the same, and since the difference of the parallaxes resulting from this cause will be, for the sun, altogether small and imperceptible because its circle's eccentricity is small and its distance great, but since for the moon it would be quite perceptible both because of its epicyclic motion and because of the epicycle's movement on the eccentric which makes a considerable difference at either distance, we shall show the sun's parallaxes only in the one ratio (I mean the ratio of 1210 to 1) and the moon's in those four ratios which will be the more accessible to successive calculations. Of these four distances we have taken, first, those two which result from the epicycle's being at the eccentric's apogee; and, of these two, first the distance to the epicycle's apogee which we have already shown to be $64^{p}10'$ to the 1^{p} of the earth's radius, and second the distance to the epicycle's perigee found to be $53^{p}50'$. The remaining two we have taken when the epicycle is at the eccentric's perigee; and again, of these, first the distance to the epicycle's apogee which was found (as already shown) to be $43^{p}53'$ to the 1^{p} of the earth's radius, and second to the epicycle's perigee found to be $33^{p}33'$.

Since then

$$\text{arc } CD = 30°,$$

therefore also

$$\text{angle } CKD = 30° = 60° \text{ to 2 rt.}$$

And so, on the circle about right triangle AKL,

$$\text{arc } AL = 60°$$

and, as remainder of the semicircle,

$$\text{arc } KL = 120°.$$

And therefore

$$\text{chord } AL = 60^{p}$$
$$\text{chord } KL = 103^{p}55'$$

where

$$\text{diam. } AK = 120^{p}.$$

And therefore

$$AL = 0^{p}30'$$
$$KL = 0^{p}52'$$

where

$$AK = 1^{p}.$$

And, for the sun's distance,

$$KLD = 1210^{p};$$

and, for the lunar distances, at the first term

$$KLD = 64^{p}10';$$

at the second term

$$KLD = 53^{p}50';$$

at the third term

$$KLD = 43^{p}53';$$

and at the fourth term

$$KLD = 33^{p}33'.$$

Therefore, since the remainder, line LD, differs from AD by an indistinguishable amount, for the solar distance,

$$AD = 1209^{p}8';$$

and for the lunar distances, at the first term

$$AD = 63^{p}18';$$

at the second term

$$AD = 52^{p}58';$$

at the third term

$$AD = 43^{p}1';$$

and at the fourth term

$$AD = 32^{p}41'.$$

And so, in the same order to avoid repetition,

$$AL = 0^{p}2'59'',$$
$$AL = 0^{p}56'52'',$$
$$AL = 1^{p}7'58'',$$
$$AL = 1^{p}23'41'',$$

and

$$AL = 1^{p}50'9'',$$

where

$$\text{hypt. } AD = 120^{p}.$$

And, therefore, on the circle about right triangle ADL,

$$\text{arc } AL = 0°2'50'',$$
$$\text{arc } AL = 0°54'18'',$$
$$\text{arc } AL = 1°4'54'',$$
$$\text{arc } AL = 1°20',$$

and

$$\text{arc } AL = 1°45'.$$

And, since

$$\text{angle } ADB = \text{angle } FAH,$$

therefore

$$\text{angle } FAH = 0°2'50'' \text{ to 2 rt.},$$
$$= 0°1'25'';$$
$$\text{angle } FAH = 0°54'18'' \text{ to 2 rt.},$$
$$= 0°27'9'';$$
$$\text{angle } FAH = 1°4'54'' \text{ to 2 rt.},$$
$$= 0°32'27'';$$
$$\text{angle } FAH = 1°20' \text{ to 2 rt.},$$
$$= 0°40';$$

and

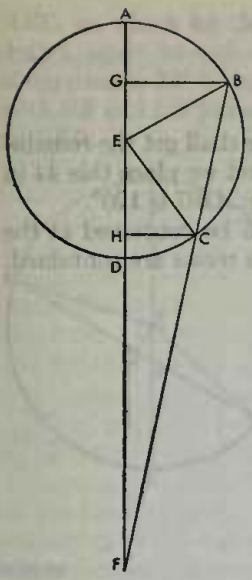
$$\text{angle } FAH = 1°45' \text{ to 2 rt.}$$
$$= 0°52'30''.$$

And so—since the point A is indistinguishable from the center K, and arc FGH is not appreciably greater than arc GH because of the whole earth's being in the ratio of a point to circle $EFGH$—arc GH of the parallax will be, for the solar distance, 0°1′25″; and for the lunar distances, at the first term 0°27′9″, at the second term 0°32′27″, at the third term 0°40′, and at the fourth term 0°52′30″. Which it was required to show.

In the same way, for the other distances from the zenith, we calculated the resulting parallaxes for each term at intervals of 6° up to the quadrant of 90°, and we drew up a table for the determination of parallaxes, again in 45 rows and 9 columns. In the first column we put the 90° of the quadrant making them increase by 2°; in the second column, the sixtieths of the solar parallaxes corresponding to each section; in the third column, the moon's parallaxes at the first term; in the fourth, the excesses of the second term's parallaxes over the first term's; in the fifth, the parallaxes at the third term; and in the sixth, the excesses of the fourth term's parallaxes over the third term's.

For example, compared to 30° there is the sun's 0°1′25″, then next the 0°27′9″ of the moon's first term, and then 0°5′18″ which is the excess of the second term over the first, then again the third term's 0°40′, and next 0°12′30″ which is the excess of the fourth term over the third. And in order to find easily and systematically the parallaxes in the distances between the apogees and perigees proportionate to the particular sections, from the parallaxes at the four terms, by comparing the sixtieths, we added the remaining three columns for comparing such differences as were calculated in the following way:

Let there be the moon's epicycle $ABCD$ about E as centre; and let F be the



centre of the earth and the ecliptic. With $AEDF$ joined, let FCB be drawn, and BE and CE joined. And let BG be drawn from B, and CH from C, perpendicular to AD. First let the moon be supposed the distance of arc AB from the true apogee at A considered relative to centre F. And let the arc be 60°, for example, so that

$$\text{angle } BEG = 60° = 120° \text{ to 2 rt.}$$

And so, on the circle about right triangle BEG,

$$\text{arc } BG = 120°$$

and, as remainder of the semicircle,

$$\text{arc } EG = 60°.$$

And therefore

$$\text{chord } BG = 103°55',$$
$$\text{chord } EG = 60^{p}$$

where

$$\text{diam. } BE = 120^{p}.$$

But, when the epicycle's centre E is at the eccentric's apogee,

$$EF : BE :: 60^{p} : 5^{p}15'.$$

And therefore

$$BG = 4^{p}33',$$
$$EG = 2^{p}38',$$
$$GEF = 62^{p}38',$$

where

$$BE = 5^{p}15'.$$

And, since

$$\text{sq. } FG + \text{sq. } BG = \text{sq. } BF,$$

therefore

$$BF = 62^{p}48'$$

where, as the first term's distance,

$$AF = 65^{p}15',$$

and, as the second term's distance,

$$DF = 54^{p}45',$$

and, as the difference between the two terms,

$$AD = 10^{p}30'.$$

And therefore the difference at B with respect to the first term's distance is $2^{p}27'$ to the whole difference's $10^{p}30'$.

And so the difference at that time will be 14 to the whole difference's 60. Therefore we shall place this 14 in column 7 in the row containing half the number 60 (that is, opposite 30) because the whole 90° in the first column of the table are only half the 180° from A to D.

In the same way, if we suppose

$$\text{arc } CD = 60°,$$

then it will be shown that

$$CH = 4^{p}33',$$
$$EH = 2^{p}38',$$

and, by subtraction,

$$FH = 57^{p}22'$$

where

$$\text{rad. } CE = 5^{p}15'.$$

And likewise

$$\text{hypt. } CF = 57^{p}33'.$$

Again subtracting these from the first term's $65^{p}15'$, we shall get the remainder $7^{p}42'$ which is 44 sixtieths of the whole difference. And we place this 44 in the same column opposite the number 60 because the arc ABC is 120°.

Again, with the same things assumed, let the centre E be conceived at the eccentric's perigee, in which position the third and fourth terms are contained. Since, then, at this position

$$EF : BE :: 60 : 8,$$

therefore, whenever it is assumed

$$\text{arc } AB = 60°$$

or

$$\text{arc } CD = 60°,$$

then

$$BG = 6^{p}56'$$

or

$$CH = 6^{p}56'$$

where

$$BE = 8^{p}$$

and

$$EF = 60^{p}.$$

And

$$EG = 4^{p}$$

or

$$EH = 4^{p}.$$

And so, with

$$FG = 64^{p}$$

and

$$FH = 56^{p},$$

likewise also

$$\text{hypt. } BF = 64^{p}23',$$
$$\text{hypt. } CF = 56^{p}26'$$

where the third term's

$$AF = 68^{p}$$

and, as difference between the third and fourth term,

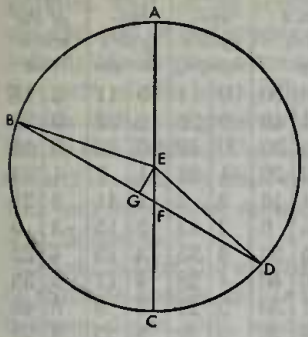
$$AD = 16^{p}.$$

Therefore, if we subtract $64^{p}23'$ from 68^{p}, we shall have left $3^{p}37'$ which is 13 and 33′ sixtieths of the whole difference's 16^{p}. And we shall place this likewise opposite the number 30 in column 8. And if we subtract $56^{p}26'$ from the same 68^{p}, we shall have left $11^{p}34'$ which is 43 and 24′ sixtieths of the whole difference's 16^{p}. And we shall likewise place this opposite the number 60 in column 8.

The differences resulting from the moon's movement on the epicycle will be set out by us in this way, and we shall systematically handle those resulting from the epicycle's passage on the eccentric in the following manner:

Let there be the moon's eccentric circle $ABCD$ with centre E and diameter

AEC, on which let there be conceived the ecliptic's centre F. And with BFD drawn, again let each of the angles AFB and CFD be 60°. This takes place if the elongation is 30° when the epicycle's centre is at B, and 120° when at D. And, with BE and DE joined, let the straight line EG be drawn from E perpendicular to BFD.



Now, since

angle $AFB = 120°$ to 2 rt.,

therefore, on the circle about right triangle EFG,

arc $EG = 120°$

and, as remainder of the semicircle,

arc $FG = 60°$.

And therefore

chord $EG = 103^p55'$,

chord $FG = 60^p$.

where

hypt. $EF = 120^p$.

And so

$EG = 8^p56'$,

$FG = 5^p10'$

where

1. betw. c. $EF = 10^p19'$

and

ecc. rad. $49^p41'$.

And since

sq. BE—sq. EG = sq. BG,

therefore

$BG = DG = 48^p53'$.

And so, by addition,

$BF = 54^p3'$

where, for the first pair of limits,

$AF = 60^p$

and, for the second pair of limits,

$CF = 39^p22'$,

and the difference between them is $20^p38'$. Also, by subtraction,

$DF = 43^p43'$.

Since, then, 60^p exceeds $54^p3'$ by $5^p57'$ which is 17 18′ sixtieths of the whole difference's $20^p38'$, and exceeds $43^p43'$ by $16^p17'$ which is 47 21′ sixtieths of $20^p38'$, we place the 17 and 18′ in column 9 opposite number 30 of the elongation and the 47 and 21′ opposite the number 120 (or, rather, 60, because the elongation of 60° is equal in distance to that of 120° when the perigee is at 90°).

For the other arcs we calculate in this same way the resulting sixtieths of the differences according to the three differences just set out, by sections of twelve which become sections of six for the numbers in the table, the 180 parts from the apogees to the perigees corresponding to the 90 parts in the table. And we have put down for each of the numbers given the corresponding sixtieths all worked out. Those corresponding to the parts in between these we have worked out according to the regular increase of the sixth parts of the difference; for there is no discrepancy worth mentioning between those so gotten and those derived geometrically for differences of this size, either in the sixtieths or in the parallaxes themselves. And here is the table:

18. Parallactic Table

1	2			3			4			5			6			7		8		9	
Numbers	Sun's Parallaxes			Parallaxes of the moon's first term			Differences of the moon's second term			Parallaxes of the moon's third term			Differences of the moon's fourth term			Sixtieths of the epicycle's apogee		Sixtieths of the epicycle's perigee		Sixtieths of the eccentric	
2	0	0	7	0	1	54	0	0	23	0	3	0	0	0	50	0	14	0	11	0	15
4	0	0	13	0	3	48	0	0	45	0	6	0	0	1	40	0	28	0	22	0	30
6	0	0	19	0	5	41	0	1	7	0	9	0	0	2	30	0	42	0	33	0	45
8	0	0	25	0	7	34	0	1	29	0	11	40	0	3	20	1	22	1	7	1	33
10	0	0	31	0	9	27	0	1	51	0	14	20	0	4	10	2	2	1	41	2	21
12	0	0	37	0	11	19	0	2	12	0	17	0	0	5	0	2	42	2	15	3	9
14	0	0	42	0	13	10	0	2	33	0	19	40	0	5	50	3	35	3	13	4	22
16	0	0	48	0	15	0	0	2	54	0	22	20	0	6	40	4	28	4	11	5	35
18	0	0	53	0	16	49	0	3	15	0	25	0	0	7	30	5	21	5	9	6	48
20	0	0	58	0	18	36	0	3	36	0	27	40	0	8	20	6	39	6	25	8	25
22	0	1	4	0	20	22	0	3	57	0	30	20	0	9	10	7	57	7	41	10	2
24	0	1	9	0	22	6	0	4	18	0	33	0	0	10	0	9	15	8	57	11	39
26	0	1	14	0	23	49	0	4	39	0	35	20	0	10	50	10	50	10	29	13	32
28	0	1	20	0	25	30	0	4	59	0	37	40	0	11	40	12	25	12	1	15	25
30	0	1	25	0	27	9	0	5	18	0	40	0	0	12	30	14	0	13	33	17	18
32	0	1	30	0	28	46	0	5	37	0	42	20	0	13	20	15	52	15	22	19	23
34	0	1	35	0	30	21	0	5	55	0	44	40	0	14	10	17	44	17	11	21	28
36	0	1	40	0	31	54	0	6	13	0	47	0	0	15	0	19	36	19	0	23	33
38	0	1	44	0	33	24	0	6	30	0	49	0	0	15	40	21	36	20	59	25	40
40	0	1	49	0	34	51	0	6	47	0	51	0	0	16	20	23	36	22	58	27	47
42	0	1	54	0	36	14	0	7	4	0	53	0	0	17	0	25	36	24	57	29	54
44	0	1	58	0	37	37	0	7	20	0	55	0	0	17	40	27	40	27	1	32	0
46	0	2	3	0	38	57	0	7	35	0	57	0	0	18	20	29	44	29	5	34	6
48	0	2	8	0	40	14	0	7	49	0	59	0	0	19	0	31	48	31	9	36	12
50	0	2	12	0	41	28	0	8	3	1	0	40	0	19	40	33	52	33	14	38	9
52	0	2	16	0	42	39	0	8	16	1	2	20	0	20	20	35	56	35	19	40	6
54	0	2	20	0	43	45	0	8	29	1	4	0	0	21	0	38	0	37	24	42	3
56	0	2	23	0	44	48	0	8	42	1	5	20	0	21	20	40	0	39	24	43	49
58	0	2	26	0	45	48	0	8	53	1	6	40	0	21	40	42	0	41	24	45	35
60	0	2	29	0	46	46	0	9	3	1	8	0	0	22	0	44	0	43	24	47	21
62	0	2	32	0	47	40	0	9	13	1	9	20	0	22	20	45	50	45	13	48	49
64	0	2	34	0	48	30	0	9	22	1	10	40	0	22	40	47	40	47	2	50	17
66	0	2	36	0	49	15	0	9	31	1	12	0	0	23	0	49	30	48	51	51	45
68	0	2	38	0	49	57	0	9	39	1	13	0	0	23	10	50	56	50	24	52	57
70	0	2	40	0	50	36	0	9	46	1	14	0	0	23	20	52	22	51	57	54	9
72	0	2	42	0	51	11	0	9	53	1	15	0	0	23	30	53	48	53	30	55	41
74	0	2	44	0	51	44	0	9	59	1	15	40	0	23	40	54	57	54	41	56	12
76	0	2	46	0	52	12	0	10	4	1	16	20	0	23	50	56	6	55	52	57	3
78	0	2	47	0	52	34	0	10	8	1	17	0	0	24	0	57	15	57	3	57	54
80	0	2	48	0	52	53	0	10	11	1	17	20	0	24	10	57	57	57	47	58	26
82	0	2	49	0	53	9	0	10	14	1	17	40	0	24	20	58	39	58	31	58	58
84	0	2	50	0	53	21	0	10	16	1	18	0	0	24	30	59	21	59	15	59	30
86	0	2	50	0	53	29	0	10	16	1	18	20	0	24	40	59	34	59	30	59	40
88	0	2	51	0	53	33	0	10	17	1	18	40	0	24	50	59	47	59	45	59	50
90	0	2	51	0	53	34	0	10	17	1	19	0	0	25	0	60	0	60	0	60	0

19. On the Determination of Parallaxes

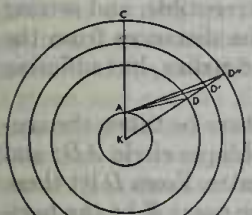
Whenever we wish to determine what parallax the moon displays in each of its passages—first along the great circle through itself and the zenith—we shall look to see how many equatorial hours it is from the meridian of the given parallel. And taking the number found to the Table of Angles of the proper parallel and of the proper sign [Chap. 13, Book II],[1] we shall find in the second column the degrees corresponding to that hour either entire or in proportion to the part of the hour. These are the degrees of the moon's distance from the zenith along the great circle drawn through them both. And carrying this number to the Table of Parallaxes, we shall look to see what row of the first column it falls in, and we shall note down each of the numbers corresponding to it in the four successive columns following the solar parallaxes—that is, columns 3, 4, 5, and 6.

Then, taking the anomalistic number determined for that hour with respect to the true apogee, either the number itself or (if it exceeds 180°) the difference between it and 360°, we carry the half of the number of the degrees so obtained to the same numbers [in column 1], and we look to see how many sixtieths in columns 7 and 8 correspond to that number. As many sixtieths as are found in column 7, just so many sixtieths shall we take of the difference in column 4, always adding it in turn to the parallax in column 3. And as many sixtieths as are found in column 8, just so many sixtieths shall we take of the difference in column 6, always adding it in turn to the parallax of column 5. And we shall set out the difference of the two parallaxes gotten in this way.

Next, taking the number of degrees of the moon's mean distance either from the sun or from the point diametrically opposite according to whichever yields the nearer distance, we shall also carry these to the numbers in column 1. And again, as many sixtieths as correspond in the ninth and last column, just so many sixtieths shall we take of the difference between the parallaxes set out, always adding the result to the lesser of the two parallaxes—that is, to the parallax determined from columns 3 and 4.[2] And we shall have the total parallax the moon displays along the great circle through itself and the zenith.

For solar eclipses, the solar parallax simply considered is immediately obtained at the similar position from the degrees in column 2 corresponding to the magnitude of the arc from the zenith.

[1]To use this table with accuracy, the moon must be supposed at the beginning of the sign and to be on the ecliptic.



[2]Here, as in the preceding paragraph, the procedure is practically true, not strictly and mathematically true. Ptolemy, here and in the preceding paragraph, is supposing that, between the smaller parallax at the greater distance and the larger parallax at the lesser distance, the differences in the parallaxes are proportional to the differences in the distances.

Thus, consider the centre of the earth K, the observer's eye A, the zenith point C, and the three distances of the moon, D, D', and D'' with the three corresponding parallactic angles ADK, $AD'K$, and $AD''K$. The supposition is that

angle $AD'K$ — angle $AD''K$: angle ADK — $AD''K$: : KD' — KD : KD'' — KD, or that

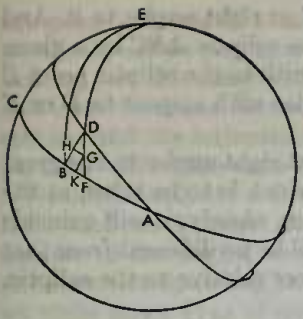
angle $D''AD'$: angle $D''AD$: : $D''D'$: $D''D$.

Now, to determine also the parallax displayed at that time with respect to the ecliptic, longitudinally and latitudinally, we shall take again the same equatorial hours of the moon's distance from the meridian to the same part of the Table of Angles. And we shall look for the number of degrees corresponding to the number of hours, in column 3 if the moon is antemeridian, but in column 4 if the moon is postmeridian. If it is within 90°, we shall write it down; if above 90°, we shall write down what is left from 180°. For the lesser of these angles about the intersection will be just so many degrees to one right angle's 90°. Then, doubling the number of degrees noted down, we carry it to the Table of Chords, the number itself and the remainder from 180°. And whatever ratio the chord subtending the arc of twice the number of degrees has to the chord subtending the remainder of the semicircle, that ratio the latitudinal parallax has to the longitudinal, since arcs of such size do not differ appreciably from their chords. Then, multiplying the numbers of the corresponding chords by the parallax found on the great circle drawn through the zenith, and dividing the result by 120, we analytically derive the degrees resulting from the division as the degrees of the particular parallax.

In general, in the case of the latitudinal parallax, whenever the zenith is farther north on the meridian than the point of the ecliptic then culminating, the parallax will be to the south of it; and whenever the zenith is farther south than the point of the ecliptic culminating, the latitudinal parallax will be to the north. In the case of the longitudinal parallaxes, since the magnitudes of the angles laid out in the table contain the northern angle of the two angles contained by the eastward section of the ecliptic to either side of it, when the latitudinal parallax is to the north (if the angle in question is greater than a right angle) the longitudinal parallax will be westward; if less than a right angle, eastward. But, contrawise, when the latitudinal parallax is to the south, if the angle in question is greater than a right angle, the longitudinal parallax is eastward; if less than a right angle, westward.

Now, we have used the things previously demonstrated concerning the sun as if it displayed no sensible parallax—not because we were unaware that its parallax when later worked out would make some difference in these matters, but because we did not think any appreciable error would follow from this with respect to the appearances, so as to make it necessary to change any of the details worked out already and without regard to this small difficulty. Likewise, with respect to the moon's parallaxes, we were satisfied with the arcs and angles made with the ecliptic by the great circle drawn through the horizon's poles in the place of those considered with respect to the moon's oblique circle. For the difference resulting at the ecliptic syzygies would be imperceptible, and setting them out requires many proofs and long calculations, the distances from the nodes not being fixed for each of the moon's passages in the zodiac, but suffering various changes in magnitude and position.

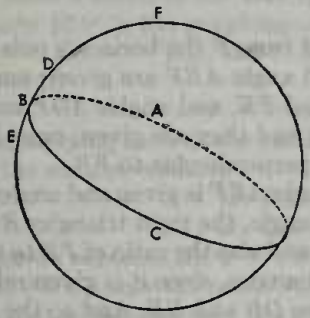
For an easy understanding of what has been said, let there be taken the section of the ecliptic *ABC*, and the section of the moon's oblique circle *AD;* and let the point *A* be supposed the node, and *D* the moon's centre. From *D* let there be drawn the arc *DB* perpendicular to the ecliptic. Let the point *E* be the horizon's pole, and let there be drawn through it *EDF* as a section of the great circle through the moon's centre, and section *EB* through *B*. And let the moon display a parallax of arc *DG*, and let the arcs *GH* and *GK* be drawn through *G* perpen-

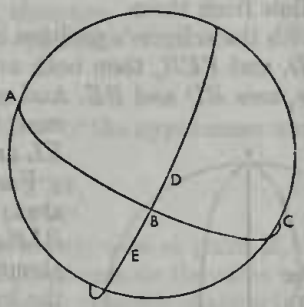


dicular to the arcs *BD* and *BF* so that, of the longitudinal distances from the node, arc *AB* is the true one, and *AK* the apparent one; and, of the latitudinal distances from the ecliptic, the true one is arc *BD* and the apparent one *KG*; so that, of the parallaxes from *DG* considered with respect to the ecliptic, the longitudinal is equal to *HG* and the latitudinal to *DH*.

Now, since the parallax *DG* is found by the means presented above when arc *ED* is given, and since each of the parallaxes *DH* and *HG* is found when angle *CFE* is given, and since we previously demonstrated the angles and arcs of the great circle through the zenith with respect to given points on the ecliptic and thus we have only point *B* of the ecliptic given, it is clear that we are using arc *EB* instead of arc *ED*, and angle *CBE* instead of angle *CFE*.

Hipparchus tried to make this correction, and he appears to have attacked the problem in an ill-considered and illogical fashion. For, first, he used one distance *AD*, and not all or several distances as would have been consequent upon his desire to be exact in small things. Then, too, he failed to notice that he was falling into several absurdities. For after he himself has already demonstrated the arcs and angles considered with respect to the ecliptic, and had





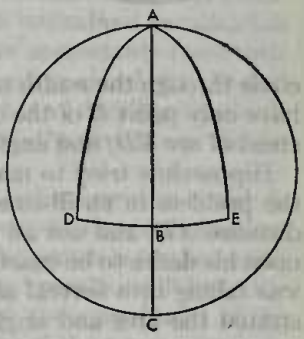
demonstrated that, when arc *ED* is given, arc *DG* is gotten (for he demonstrates this in Book I of his *Treatise on Parallaxes*), he uses arc *EF* and angle *EFC* as given for getting arc *ED*. Thus in Book II, after having calculated arc *FD*, he assumes the remainder *ED;* and he came to these conclusions because he did not notice that the point *B*, and not *F*, is given on the ecliptic; and therefore arc *EB* is given and not arc *EF*, and angle *EBC* is given and not *EFC*. Therefore, to make a partial correction, he had upset everything. For there is quite a sensible difference between arcs *ED* and *EF*, the latter being much less given than the former. But since actually *BE* is given, the difference with respect to *ED* will be at most only the magnitude of arc *BD* for each of the distances from the node.[1]

Now we can see what follows from the correction rightly made, in this way:

[1]For *BD* would always be less than *DF*, and *BE* in nearly all cases would differ from *DE* by less than *BD*.

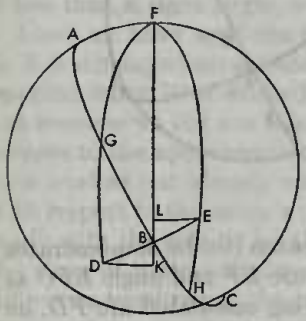
Let there be the ecliptic *ABC* and the circle *DBE* at right angles to it. And let the moon, either at *D* or at *E*, be distant from the ecliptic *ABC* by a given arc, such as *BD* and *BE*, so that the arcs from the zenith to the ecliptic point *B* and the angles at *B* are given, and the arcs and angles with respect to *D* or *E* are sought.

If, then, the ecliptic has a position such that it is at right angles to the great circle drawn through the point *B* and the point *F*, which is to be taken at the horizon's pole (for example the great circle *FB*), then, clearly, it will coincide with arc *DE*, and the angle considered at *D* and *E* will be no different from that supposed at *B*. For they are right angles and therefore relative to the ecliptic. And arc *FD* will be less than arc *FB* by arc *BD*, and arc *FE* greater by arc *BE*, with arcs *BD* and *BE* given.



Furthermore, if the ecliptic *ABC* coincides with the great circle drawn through the zenith, and if (assuming *A* to be the horizon's pole) we join the arcs *AD* and *AE*, they will differ from arc *AB*, and the angles *BAD* and *BAE* will differ from the angle which in the former case did not exist. But arcs *AD* and *AE* are given, since they are as straight lines very nearly, from arcs *AB*, and *BD* and *BE* which are given. For the sums of their squares give the squares on *AD* and *AE*. And angles *BAD* and *BAE* follow from them.

But with the ecliptic's position inclined, if from *F* the horizon's pole we join *FB*, *FGD*, and *FEH*, then both arc *FB* and angle *ABF* are given; and again, certainly, arcs *BD* and *BE*. And arcs *FD* and *FE*, and angles *AGF* and *AHF* must be given; and they are given, once *DK* and *EL* are drawn perpendicular to *FB*.



For, since angle *ABF* is given and angle *ABE* is always a right angle, the right triangles *BKD* and *BLE* are given and also the ratio of *FB* to the sides about the right angles, since it is given relative to the hypotenuses *DB* and *BE*. And so the hypotenuses *FD* and *FE* are given, and therefore also angles *DFK* and *EFL* which are the differences with the angles desired. For angle *AGF* is greater than angle *ABF* by angle *DFB;* and angle *AHF* is less than angle *ABF* by angle *EFL*.

It is clear that, with the same latitudinal distance supposed, the greatest difference between the angles occurs when the point *B* itself is the zenith. For with no angle at *B*, the arcs from the zenith to *D* and *E* make right angles with the ecliptic. The greatest difference between the arcs occurs when the position is the same; for again, with no arc at *B*, the arcs at *D* and *E* will be as great as those of the moon's latitudinal passage; and also when the great circle through the zenith is at right angles to the ecliptic. For, again, arcs *FD* and *FE* will differ from *FB* by the whole latitudinal passage. But in the other positions with *DE* inclined to *FB*, the differences of both the arcs and angles will come to less. Therefore, when

the moon is distant 5° in latitude from the ecliptic, the greatest difference of the parallaxes will be very nearly 10 sixtieths, the 5° of the greatest difference in arcs making just this many sixtieths of parallax in the case of the greatest excesses and least distances. But when the moon is at its greatest distance in the solar eclipses and the latitudinal passage is very nearly $1\frac{1}{2}°$, then the difference of parallax will be the same number of sixtieths, $1\frac{1}{2}$, which rarely happens.

Now, the method for the correction of the angles and arcs (for those who wish it for such small ratios) would be available as follows: In general, doubling the number of the angles, we take it to the Table of Chords; and multiplying the numbers corresponding to it and to its supplement by the degrees of latitude, we write down $\frac{1}{120}$ of each.[1] And we subtract the resulting number for the first angle from the given arc from the zenith when the moon is on the same side of the ecliptic as the zenith; and we shall add when it is on the opposite side of the ecliptic. And squaring the result and adding to the square from the supplement, we shall have the arc required by taking the square root of this sum.

Then multiplying the numbers of the second or supplementary angle by 120, and dividing the results by the arcs just found, we take the halves of the arcs corresponding to these numbers in the Table of Chords and add them to the arcs of the original angle if the corrected arc is greater; and we subtract, if less. And thus we have the corrected angle.

As an example, let the arc FB of the last figure be supposed 45°, the angle ABF 30°, and each of the arcs DB and BE 5° in latitude.

Now, since a chord of 60^p corresponds to double 30°, or 60°, and a chord of very nearly 104^p to the supplement or 120°, therefore

$$BL : BE :: BK : DK :: 60 : 120$$

where

$$\text{hypt.} = 120.$$

Then, multiplying each of the numbers by the 5° of the hypotenuse and taking $\frac{1}{120}$ of the results, we have

$$\text{arc } BK = \text{arc } BL = 2°30',$$
$$\text{arc } DK = \text{arc } KL = 4°20'.$$

Then first we subtract the 2°30′, if the moon is supposed at point E, from the 45° of arc FB because the moon's latitudinal distance is on the same side as the zenith—that is, because they are both either north or south of the ecliptic. And we have

$$\text{arc } FL = 42°30'.$$

And if the moon is at D, we add to it because of the contrary situation, and we have

$$\text{arc } FK = 47°30'.$$

Then, adding the square of FL to the square of LE, and the square of FK to the square of DK (that is, the square of 4°20′ to the square of 42°30′, and the square

[1]For in the last figure angle LEB equals angle AFB, angle LBE equals 90° less angle AFB. If we now conceive a circle about right triangle BLE, considering it to be rectilinear for the small arcs, we shall have the values of BL and LE in terms of the chords where BE equals 120. Let BL be s. But we know BE is so many degrees of arc, say n. Therefore, if x is the number of degrees of BL to be found,

$$x : n = s : 120.$$

Whence

$$120\,x = n \cdot s.$$

The same kind of calculation is used in the next paragraph.

of 4°20′ to the square of 47°30′) and taking the square roots of the results, we have

$$\text{arc } EF \fallingdotseq 42°46',$$
$$\text{arc } DF \fallingdotseq 47°44'.$$

Finally, multiplying 4°20′ by 120 and dividing separately by 42°46′ and by 47°44′, we have

$$\text{chord } EL \fallingdotseq 12^{p}8'$$

where

$$\text{hypt. } EF = 120^{p};$$

and

$$\text{chord } DK \fallingdotseq (10+\tfrac{1}{2}+\tfrac{1}{3})^{p}$$

where

$$\text{hypt. } DF = 120^{p}.$$

An arc of $11\frac{3}{5}°$ corresponds to the chord of $12^{p}8'$, and an arc of very nearly $10\frac{1}{3}°$ corresponds to the chord of $(10+\frac{1}{2}+\frac{1}{3})^{p}$. Taking the halves of these arcs, on the one hand we subtract the $5\frac{4}{5}°$ of angle EFL from the 30° of angle ABF, because arc EF is less than arc BF, and we have

$$\text{angle } AHF = 24\tfrac{1}{5}°.$$

On the other hand, if we add it to the same 30° because arc DF is greater than arc BF, we have

$$\text{angle } AGF = 35\tfrac{1}{6}°.$$

Which it was required to work out systematically.

BOOK SIX

1. On Conjunctions and Full Moons

The treatment of the ecliptic syzygies of the sun and moon comes next in order, and this in turn is preceded by the examination of the conjunctions and full moons truly considered. For a first understanding of such things, we think that the periodic and irregular movements already demonstrated for each of the luminaries are sufficient. By means of these it is possible, for anyone sufficiently industrious, to determine their particular positions each time, and to calculate the places and times of the future syzygies—both those taken with respect to mean movements and the true ones with the anomaly. Nevertheless, to have them at hand systematically, we first set out the times and places of the periodic conjunctions and full moons, and the moon's anomalistic and latitudinal positions at the mid-eclipses. For by means of these last the correction for the true syzygies is effected, and from these the correction for the ecliptic syzygies. And we built tables of the following kind for such an examination.

2. Construction of the Tables of the Mean Syzygies

First, to establish the epochs of the months (like the other epochs, from the year 1 of Nabonassar), we took the surplus of the noon elongation, 70°37′, already demonstrated for the first of the Egyptians' Thoth, and we found 5 days 47′ 33″ of mean movement of elongation corresponding to it; so that by just so much time had the mean conjunction preceded noon of the first day of Thoth. And the next one, therefore, took place very nearly 23 days 44′ 17″ after that same midday—that is, 44′ 17″ of a day after noon of the 24th. But in 23 days 44′ 17″ the sun's mean movement is 23°23′ 50″, and the moon moves 310° 8′ 13″ of anomaly and 314° 2′ 21″ in latitude. Furthermore, at noon of the first of Thoth, the sun's mean position was 0° 45′ within the Fishes and, for greater facility, 265°15′ from its apogee, while the moon's position in anomaly was 268°49′ from the epicycle's apogee, and 354°15′ in latitude from the oblique circle's northern limit. Therefore, at the time of the mean conjunction after the first of the month, the sun and moon were both 288°38′50″ in mean movement from the sun's apogee—that is, 5°30′ within the Twins; and the moon was 218°57′15″ of anomaly from the apogee and 308°17′21″ in latitude from the northern limit.

Now, we shall first arrange a synodic table, again in 45 rows and 5 columns. For the first row we shall place, in the first column, the first year of Nabonassar; and, in the second column, the 24 days 44′17″ of Thoth since the extra sixtieths are from noon of the 24th. In the third column we shall place the 288°38′50″ of the mean position from the sun's apogee; in the fourth, the 218°57′15″ from the apogee of the lunar anomaly; and, in the fifth, the 308°17′21″ from the northern limit of latitude.

Since, moreover, in half a mean month's time there are contained very nearly

14 days 45′55″, 14°33′12″ of solar position, 192°54′30″ of lunar anomaly, and 195°20′6″ of latitude, we shall subtract these numbers from those of the conjunction first given; and we shall arrange the remainders in a second and similar table, which will be plenilunar, in the same way as before. And there are left 9 days 58′22″, 274°5′38″ from the sun's apogee, 26°2′45″ of anomaly from the moon's apogee, and 112°57′15″ in latitude from the northern limit.

And since, in 25 Egyptian years, whole months less $2^{i}47^{ii}5^{iii}$ of a day very nearly are completed, and the sun adds on in mean movement (once and above complete circles) $353°52^{i}34^{ii}13^{iii}$, and the moon $57°21^{i}44^{ii}1^{iii}$ of anomaly and $117°12^{i}49^{ii}54^{iii}$ in latitude, we shall increase the first columns of the two tables by successive additions of 25 years, the second columns we shall successively decrease by $0°2^{i}47^{ii}5^{iii}$, and of the rest we shall successively increase the third by $353°52^{i}34^{ii}13^{iii}$, the fourth by $57°21^{i}44^{ii}1^{iii}$, and the fifth by $117°12^{i}49^{ii}54^{iii}$.

Following these, we shall arrange a table by years in 24 rows and another by months in 12 rows, each of them with as many columns as the first tables. And, in the case of the table by months, for the first row, we place in the first column the first month; in the second column the 29 days $31^{i}50^{ii}8^{iii}20^{iv}$; in the third the total $29°6^{i}23^{ii}1^{iii}$ of the sun for that much time; in the fourth the $25°49^{i}0^{ii}8^{iii}$ of the moon's anomaly; and in the fifth the $30°40^{i}14^{ii}9^{iii}$ in latitude. And we successively increase these by these same numbers of the first row.

In the case of the table by years, for the first row, we place in the first column the first year; in the second the 18 days $53^{i}52^{ii}48^{iii}$ additional in 13 months; in the third the $18°22^{i}59^{ii}18^{iii}$ of the solar surplus for that much time; in the fourth the $335°37^{i}1^{ii}51^{iii}$ of lunar anomaly; and in the fifth the $38°43^{i}3^{ii}51^{iii}$ in latitude. With a view to setting out the first syzygy following upon the whole Egyptian years, we also successively increase at one time by the thirteen-month surplus already given, and at another by the twelve-month surplus which comes to 354 days $22^{i}1^{ii}40^{iii}$, to $349°16^{i}36^{ii}16^{iii}$ in solar position, to $309°48^{i}1^{ii}42^{iii}$ in lunar anomaly, and to $8°2^{i}49^{ii}42^{iii}$ in latitude. Nevertheless it will suffice to take the tabulated numbers out to the second sixtieths. And here is the result reduced to tables:

3. Table of Conjunctions

1 *25-Tear Periods*	2 *Days of Thoth*			3 *Degrees of Solar Position*			4 *Degrees of Lunar Anomaly*			5 *Degrees of Latitude*		
				°	′	″	°	′	″	°	′	″
1	24	44	17	288	38	50	218	57	15	308	17	21
26	24	41	30	282	31	24	276	18	59	65	30	11
51	24	38	43	276	23	58	333	40	43	182	43	1
76	24	35	56	270	16	33	31	2	27	299	55	51
101	24	33	9	264	9	7	88	24	11	57	8	41
126	24	30	22	258	1	41	145	45	55	174	21	31
151	24	27	35	251	54	15	203	7	39	291	34	20
176	24	24	47	245	46	50	260	29	23	48	47	10
201	24	22	0	239	39	24	317	51	7	166	0	0
226	24	19	13	233	31	58	15	12	51	283	12	50
251	24	16	26	227	24	32	72	34	35	40	25	40
276	24	13	39	221	17	6	129	56	19	157	38	30
301	24	10	52	215	9	41	187	18	3	274	51	20
326	24	8	5	209	2	15	244	39	47	32	4	10
351	24	5	18	202	54	49	302	1	31	149	17	0
376	24	2	31	196	47	23	359	23	15	266	29	50
401	23	59	44	190	39	57	56	44	59	23	42	39
426	23	56	57	184	32	32	114	6	43	140	55	29
451	23	54	10	178	25	6	171	28	27	258	8	19
476	23	51	23	172	17	40	228	50	11	15	21	9
501	23	48	35	166	10	14	286	11	55	132	33	59
526	23	45	48	160	2	49	343	33	39	249	46	49
551	23	43	1	153	55	23	40	55	23	6	59	39
576	23	40	14	147	47	57	98	17	7	124	12	29
601	23	37	27	141	40	31	155	38	51	241	25	19
626	23	34	40	135	33	5	213	0	35	358	38	9
651	23	31	53	129	25	40	270	22	19	115	50	58
676	23	29	6	123	18	14	327	44	3	233	3	48
701	23	26	19	117	10	48	25	5	47	350	16	38
726	23	23	32	111	3	22	82	27	31	107	29	28
751	23	20	45	104	55	57	139	49	16	224	42	18
776	23	17	57	98	48	31	197	11	0	341	55	8
801	23	15	10	92	41	5	254	32	44	99	7	58
826	23	12	23	86	33	39	311	54	28	216	20	48
851	23	9	36	80	26	13	9	16	12	333	33	38
876	23	6	49	74	18	48	66	37	56	90	46	28
901	23	4	2	68	11	22	123	59	40	207	59	17
926	23	1	15	62	3	56	181	21	24	325	12	7
951	22	58	28	55	56	30	238	43	8	82	24	57
976	22	55	41	49	49	4	296	4	52	199	37	47
1001	22	52	54	43	41	39	353	26	36	316	50	37
1026	22	50	7	37	34	13	50	48	20	74	3	27
1051	22	47	20	31	26	47	108	10	4	191	16	17
1076	22	44	32	25	19	21	165	31	48	308	29	7
1101	22	41	45	19	11	56	222	53	32	65	41	57

Table of Full Moons

1 *25-Year Periods*	2 *Days of Thoth*			3 *Degrees of Solar Position*			4 *Degrees of Lunar Anomaly*			5 *Degrees of Latitude*		
				°	′	″	°	′	″	°	′	″
1	9	58	22	274	5	38	26	2	45	112	57	15
26	9	55	35	267	58	12	83	24	29	230	10	5
51	9	52	48	261	50	46	140	46	13	347	22	55
76	9	50	1	255	43	21	198	7	57	104	35	45
101	9	47	14	249	35	55	255	29	41	221	48	35
126	9	44	27	243	28	29	312	51	25	339	1	25
151	9	41	40	237	21	3	10	13	9	96	14	14
176	9	38	52	231	13	38	67	34	53	213	27	4
201	9	36	5	225	6	12	124	56	37	330	39	54
226	9	33	18	218	58	46	182	18	21	87	52	44
251	9	30	31	212	51	20	239	40	5	205	5	34
276	9	27	44	206	43	54	297	1	49	322	18	24
301	9	24	57	200	36	29	354	23	33	79	31	14
326	9	22	10	194	29	3	51	45	17	196	44	4
351	9	19	23	188	21	37	109	7	1	313	56	54
376	9	16	36	182	14	11	166	28	45	71	9	44
401	9	13	49	176	6	45	223	50	29	188	22	33
426	9	11	2	169	59	20	281	12	13	305	35	23
451	9	8	15	163	51	54	338	33	57	62	48	13
476	9	5	27	157	44	28	35	55	41	180	1	3
501	9	2	40	151	37	2	93	17	25	297	13	53
526	8	59	53	145	29	37	150	39	9	54	26	43
551	8	57	6	139	22	11	208	0	53	171	39	33
576	8	54	19	133	14	45	265	22	37	288	52	23
601	8	51	32	127	7	19	322	44	21	46	5	13
626	8	48	45	120	59	53	20	6	5	163	18	3
651	8	45	58	114	52	28	77	27	49	280	30	52
676	8	43	11	108	45	2	134	49	33	37	43	42
701	8	40	24	102	37	36	192	11	17	154	56	32
726	8	37	37	96	30	10	249	33	1	272	9	22
751	8	34	50	90	22	45	306	54	45	29	22	12
776	8	32	2	84	15	19	4	16	29	146	35	2
801	8	29	15	78	7	53	61	38	14	263	47	52
826	8	26	28	72	0	27	118	59	58	21	0	42
851	8	23	41	65	53	1	176	21	42	138	13	32
876	8	20	54	59	45	36	233	43	26	255	26	22
901	8	18	7	53	38	10	291	5	10	12	39	11
926	8	15	20	47	30	44	348	26	54	129	52	1
951	8	12	33	41	23	18	45	48	38	247	4	51
976	8	9	46	35	15	52	103	10	22	4	17	41
1001	8	6	59	29	8	27	160	32	6	121	30	31
1026	8	4	12	23	1	1	217	53	50	238	43	21
1051	8	1	25	16	53	35	275	15	34	355	56	11
1076	7	58	37	10	46	9	332	37	18	113	9	1
1101	7	55	50	4	38	44	29	59	2	230	21	51

Yearly Surpluses at Conjunctions and Full moons

1 *Single Years*	2 *Days*			3 *Degrees of Solar Position* °	′	″	4 *Degrees of Lunar Anomaly* °	′	″	5 *Degrees of Latitude* °	′	″
1	18	53	52	18	22	59	335	37	2	38	43	4
2	8	15	53	7	39	36	285	25	4	46	45	54
3	27	9	45	26	2	35	261	2	5	85	28	57
4	16	31	47	15	19	11	210	50	7	93	31	47
5	5	53	49	4	35	47	160	38	9	101	34	37
6	24	47	40	22	58	47	136	15	11	140	17	41
7	14	9	42	12	15	23	86	3	12	148	20	30
8	3	31	44	1	31	59	35	51	14	156	23	20
9	22	25	36	19	54	59	11	28	16	195	6	24
10	11	47	37	9	11	35	321	16	18	203	9	14
11	1	9	39	358	28	11	271	4	19	211	12	3
12	20	3	31	16	51	10	246	41	21	249	55	7
13	9	25	32	6	7	47	196	29	23	257	57	57
14	28	19	24	24	30	46	172	6	25	296	41	1
15	17	41	26	13	47	22	121	54	27	304	43	50
16	7	3	28	3	3	59	71	42	28	312	46	40
17	25	57	19	21	26	58	47	19	30	351	29	44
18	15	19	21	10	43	34	357	7	32	359	32	34
19	4	41	23	0	0	10	306	55	33	7	35	23
20	23	35	14	18	23	10	282	32	35	46	18	27
21	12	57	16	7	39	46	232	20	37	54	21	17
22	2	19	18	356	56	22	182	8	39	62	24	7
23	21	13	10	15	19	22	157	45	41	101	7	10
24	10	35	11	4	35	58	107	33	42	109	10	0

Mean Passages { Solar limits from 69°19′ to 101°22′ and from 258°38′ to 290°41′
Lunar limits from 74°48′ to 105°12′ and from 254°48′ to 285°12′

Months	*Days*			*Solar Position*			*Lunar Anomaly*			*Latitude*		
1	29	31	50	29	6	23	25	49	0	30	40	14
2	59	3	40	58	12	46	51	38	0	61	20	28
3	88	35	30	87	19	9	77	27	0	92	0	42
4	118	7	21	116	25	32	103	16	1	122	40	57
5	147	39	11	145	31	55	129	5	1	153	21	11
6	187	11	1	174	38	18	154	54	1	184	1	25
7	206	42	51	203	44	41	180	43	1	214	41	39
8	236	14	41	232	51	4	206	32	1	245	21	53
9	265	46	31	261	57	27	232	21	1	276	2	7
10	295	18	21	291	3	50	258	10	1	306	42	21
11	324	50	12	320	10	13	283	59	2	337	22	36
12	354	22	2	349	16	36	309	48	2	8	2	50

4. How to Locate the Periodic and True Syzygies

Now, whenever we wish to get the mean syzygies for some one of the required years, we calculate how far the supposed year is from the year 1 of Nabonassar. And looking to see which rows of either of the first two 25-year tables together with the third table's years contain that number of years, we add the corresponding numbers in each of the columns of the two rows: for the synodic syzygies those from the first table to those from the third, and, for the plenilunar syzygies, those from the second table to those from the third. From the sums in the second column we shall have the time of the syzygy from the beginning of that year: for instance, if they add up to 24 days 44′, then 44 sixtieths of a day after midday of Thoth 24; and again, if they add up to 34 days 44′, then the same sixtieths of a day after midday of Pleophi 4. From the third column we shall have the degrees from the sun's apogee; from the fourth, the degrees from the apogee of the moon's anomaly; and, from the fifth, the degrees from the northern limit of latitude.

Accordingly, we can readily calculate those that follow, whether we wish to get them all or some of them, through the proper sums in the fourth or monthly table, having changed the times, for greater facility, from the sixtieths of a day to equatorial hours. The resulting surplus of hours will be as of regular solar days, for the surplus taken seasonally is not always the same, but as of irregular solar days. And then we shall correct this by estimating the difference, subtracting it from the mean sum if the surplus of time for the irregular interval is greater, and adding it to it if it is less.

Now, once the synodic or plenilunar date considered with respect to mean passages has been gotten in this way along with the anomalies for each of the luminaries, it will be very easy to get the date and place of the true syzygy, and even the moon's latitudinal passage, from the combination of both anomalies. For in each case we examine the true passage of the sun, moon, and latitude at that periodic time by means of the addition-subtraction obtained. And if they are found to be at the same degree or directly opposite, we also have the time of the true syzygy; and if not, taking the degrees of the interval between them and adding to them a twelfth of them[1], we see in how many equatorial hours the moon will move that number of degrees. If the moon's true passage is less than the sun's, we add that number of hours to the periodic time; and if greater, we subtract it. And thus, if the moon's true passage at the periodic time is less than the sun's, we add the degrees of the interval between, together with the twelfth of them to the moon's true passage both in longitude and latitude; and if greater, we subtract them. In this way we shall have the time of the true syzygy and very nearly the moon's true passage on its oblique circles.

Now, the moon's hourly irregular movement at the syzygies is each time gotten in this way. Carrying the number of anomalistic degrees for the supposed date to the Table of the Moon's Anomaly, we take from the excess of the addition-subtractions lying opposite it the difference belonging to one degree of anom-

[1]That is, the number of degrees between them if the syzygy is synodic; between one of them and the point opposite the others, if the syzygy is plenilunar. Now while the moon moves along the ecliptic nearly 13°, the sun moves nearly 1°, or the moon moves thirteen times as fast. And while the moon moves that 1° the sun again 1/13°. Therefore, for all practical purposes, 1/13° is 1/12 of the original 13°. This approximation is even truer for the smaller numbers involved here. See Chapter 5.

aly. This difference we multiply by the hourly mean movement of anomaly which is $0°32^{i}40^{ii}0^{iii}$. If the anomalistic number is in the rows above the greatest addition-subtraction, we subtract this product from the hourly mean movement in longitude which is $0°32^{i}56^{ii}0^{iii}$; and if it is below, we add it. And we have, as a result, how much the moon moves irregularly in longitude in one equatorial hour at that particular time[1].

The time of the true syzygies for Alexandria will be gotten systematically by us in this way, because all the epochs have their hours established with respect to the meridian through Alexandria. And it is easy from the times in Alexandria to find the times which will result in any region for the same syzygy, given the number of hours of its distance from that meridian. For, if, from the difference of the localities, we consider the meridian through the place required, then, by as many degrees as it differs from that through Alexandria, by just so much time later will the appearance seem to be observed if the meridian through the place required is east of that through Alexandria; but by so much earlier if west of it, with fifteen time-degrees still making, of course, one equatorial hour.

5. On the Ecliptic Limits of the Sun and Moon

Now that these things have been worked out, the next job it to present those matters having to do with the ecliptic limits of the occultations of the sun and moon, so that, even if we do not choose to compute all the periodic syzygies but only those which can fall within the zone of ecliptic markings, we can have ready to hand a discrimination of this kind from the moon's mean latitudinal passage for each of the periodic syzygies.

Now, in the preceding Book, we have shown that the moon's diameter subtends at its greatest an arc of the great circle drawn about the ecliptic's centre equal to $0°31'20''$, calculating it to be such by two eclipses which occured at the apogee of its epicycle. Since we now wish to get the greatest limits of the ecliptic syzygies, and since these occur when the moon is about the perigee of its epicycle, we shall show by means of two eclipses observed near the perigee (for the proof of these things would be more certain by means of their appearances)

[1]This description of the method by Ptolemy is far from clear. Theo of Alexandria in his Commentary on Book V gives a better one as follows:

"For taking the anomalistic number . . . he carries it to the Table of the Moon's Simple Anomaly. And taking the difference of the numbers lying in a line with the nearest anomalistic numbers greater and less than it, he divides it by the difference of the common [*i.e.*, anomalistic] numbers. For example, if the anomalistic number is 33°, taking the 2°19′ corresponding to 30° and the 2°44′ corresponding to 36°, he finds their difference which is 0°25′. And dividing this 0°25′ by the difference between 30° and 36° (that is, by 6) he gets 0°4′24″; and this is the difference belonging to one degree of the anomaly. And multiplying 0°4′24″ by the 0°32′40″ of the mean hourly movement of anomaly, we subtract the product from the hourly mean movement in longitude of 0°32′56″ if the given anomalistic number is in the rows above the greatest addition-subtraction—that is, from 1 to 96 and from 264 to 360. But if it is from 96 to 180 and from 180 to 264, or in the rows below the greatest addition-subtraction, we shall add it to the 0°32′ 56″."

Theo of Alexandria goes on to give the reasoning behind this method. But it is clear from the method itself. One finds the average change along the ecliptic affected by a degree of anomaly for the section of the anomaly the moon happens to be at. This, being multiplied by the number of degrees of anomaly the moon moves in one hour, becomes the change produced along the ecliptic for an hour's movement in anomaly at that particular section of the epicycle. It can, of course, only be approximate. This hourly change is then added to or subtracted from, as the case may be, the moon's mean movement along the ecliptic.

how much of an arc the moon's diameter likewise intercepts at this point.

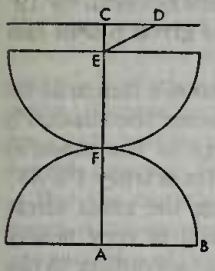
In the year 7 of Philometor then, (*i. e.*, the year 574 of Nabonassar, Egyptianwise Phamenoth 27-28 from the beginning of the eighth hour to the end of the tenth hour), in Alexandria the moon was eclipsed up to 7 digits from the north. Now, the middle occurred $2\frac{1}{2}$ seasonal hours after midnight (or $2\frac{1}{3}$ equatorial hours, because of the sun's true position's being 6°4′ within the Bull). And the time from the epoch to the middle of the eclipse comes to 573 Egyptian years, 206 days, and $14\frac{1}{3}$ equatorial hours simply considered but only 14 with respect to regular solar days. And for this much time the mean position of the moon's centre was 7°49′ within the Scorpion but its true position 6°16′; and its centre was 163°40′ from the epicycle's apogee and 98°20′ from the northern limit of the oblique circle. Therefore it is evident that, when the moon's centre, being near its least distance, is 8°20′ from the nodes along the oblique circle, and the shadow's centre is on the great circle drawn through the moon's centre at right angles to the oblique circle at which passage the greatest shadowings are effected, then $\frac{1}{2}+\frac{1}{12}$ of its diameter falls within the shadow.

Once again, in the year 37 of the Third Callippic Period (which is the year 607 of Nabonassar, Egyptianwise Tybi 2-3), the moon began to be eclipsed at the beginning of the fifth hour in Rhodes and was obscured at the most 3 digits from the south. Here once more the beginning of the eclipse occurred two seasonal hours before midnight, which were $2\frac{1}{3}$ equatorial hours both in Rhodes and in Alexandria because of the sun's true position's being 5°8′ within the Water Bearer, and the middle (when it was obscured the most) occurred very nearly $1+\frac{1}{2}+\frac{1}{3}$ equatorial hours before midnight. And here the time from the epoch to the middle of the eclipse adds up to 606 Egyptian years, 121 days, and $10\frac{1}{6}$ equatorial hours considered both simply and with respect to regular solar days. And for this much time the mean position of the moon's centre was 5°16′ within the Lion, but its true position 5°8′; and its centre was 178°46′ from the epicycle's apogee and 280°36′ from the northern limit on the oblique circle. It is therefore evident that, when the moon's centre, being near its same least distance, is 10°36′ from the nodes along the oblique circle, with the shadow's centre situated on the common section of the ecliptic, and the great circle drawn through the moon's centre at right angles to the oblique circle, then the fourth part of the moon's diameter will fall within the shadow.

But if the moon's centre is $8\frac{1}{3}$° from the nodes along the oblique circle, it is $43\frac{1}{20}$ sixtieths of one degree [*i. e.*, 43′3″] from the ecliptic along the great circle drawn through the oblique circle's poles; and, when it is $10\frac{3}{5}$° from the nodes along the oblique circle, it is $54+\frac{1}{2}+\frac{1}{3}$ sixtieths of one degree from the ecliptic along the great circle drawn through the oblique circle's poles. Now, since the difference between the two eclipses contains a third of the moon's diameter, and since the difference of the two distances of its centre along the same great circle from the same ecliptic point (that is, from the shadow's centre) is 0°11′47″, it is clear that the moon's whole diameter, at its least distance, subtends an arc of very nearly $35\frac{1}{3}$ sixtieths of one degree on the great circle drawn about the ecliptic's centre. And since also in the second of the eclipses (in which $\frac{1}{4}$ of the moon's diameter was eclipsed) the moon's centre was $54+\frac{1}{2}+\frac{1}{3}$ sixtieths from the shadow's centre, and $\frac{1}{4}$ of the moon's diameter or $8+\frac{1}{2}+\frac{1}{3}$ sixtieths from the point where the line joining their centres cuts the shadow's circumference, it is immediately evident that the shadow's radius at the moon's least distance

is the remainder, 46 sixtieths. This is hardly greater than two and three-fifths times the moon's radius, which is 0°17⅔′. But the sun's radius likewise subtends an arc of 0°15′40″ on the great circle drawn through the sun about the ecliptic's centre. For the sun and moon were shown to measure off equally on their circles in the syzygies at the greatest distance. Therefore, when the moon's apparent centre is 0°33′20″ from the sun's centre on either side of the ecliptic, it will first be possible for the moon's apparent position to be one of tangency to the sun.

For example, if we conceive arc AB of the ecliptic and arc CD of the moon's oblique circle as being sensibly parallel as far as the passages at eclipse times, and if we draw arc AEC of the great circle through the poles of the oblique circle, and conceive the sun's semicircle about point A and the moon's apparent semicircle about E so that it is just touching the sun's at point F, then arc AE, the distance of the moon's apparent centre E from the sun's centre A, can become the 0°33′20″ just given.



But in the places from Meroë, where the longest day is 13 equatorial hours, to the mouth of the Borysthenes, where the longest day is 16 equatorial hours, the moon, at its least distance in the syzygies, displays a northward parallax of at most very nearly 0°8′ (the sun's parallax being taken into account), and likewise a southward parallax of at most 0°58′. And when it displays its northward parallax of 0°8′, it displays its greatest longitudinal parallax of very nearly 0°30′ about the Lion and the Twins; but when it displays its greatest southward parallax, it displays its greatest longitudinal of very nearly 0°15′ about the Scorpion and the Fishes.

Therefore—if we suppose the moon's true centre at D and we join the DE of the whole parallax—the arc CD will be very nearly the arc of the longitudinal parallax, and CE that of the latitudinal. And so, whenever the moon is north of the sun and displays its greatest southward parallax,

$$\text{arc } CD = 0°15',$$
$$\text{arc } AEC = 1°31'.$$

Since the ratio of the arc from the node to C to the arc AC at the distance between the ecliptic limit is that which 11½ has to 1 (this being easily understood from things already demonstrated on the obliquity of the lunar circle), the arc from the node to C will be 17°26′, and added to CD it will be 17°41′. But when it is south of the sun and displays its greatest northward parallax,

$$\text{arc } CD = 0°30',$$
$$\text{arc } AEC = 0°41;$$

and for the same reasons the arc from the node to C will be 7°52′, and added to CD it will be 8°22′. Therefore, whenever the moon's centre is truly 17°41′ from either of the nodes northward along the oblique circle, first in the given places of the inhabited world can its apparent position become one of tangency to the sun.

Again, inasmuch as the greatest difference of solar anomaly was demonstrated to be 2°23′, and the greatest difference of lunar anomaly about the syzygies 5°1′, it is possible at times for the moon to be truly 7°24′ from the sun at the periodic syzygies. But while the moon is traversing that many degrees, the sun will traverse $\frac{1}{13}$ that many, or very nearly 0°34′; and again, while the moon moves these 0°34′, the sun will traverse also $\frac{1}{13}$ that many, or very nearly 0°3′; and

$\frac{1}{13}$ of this is not worthy of mention. If, therefore, we add to the 2°23′ of solar anomaly the total of 0°37′ which is $\frac{1}{12}$ of the original 7°24′, we shall then have the 3° by which the true syzygies differ very nearly the most from the mean longitudinal and latitudinal passages in the periodic syzygies. Consequently, whenever the mean passage of the moon's centre is 20°41′ from the nodes northward along the oblique circle and 11°22′ southward, then first, in the given places of the inhabited world, can its apparent position become one of tangency to the sun. Therefore, whenever the number of degrees from the northern limit of the moon's oblique circle corresponding to the periodic syzygies falls between 69°19′ and 101°22′ or between 258°38′ and 209°41′, then only in the given places can the proposed thing happen.

Yet again, respecting the moon's ecliptic limits: since the moon's radius at its least distance was shown to subtend an arc of 0°17′40″, and since the shadow's radius (being very nearly two and three fifths times as large as the moon's) comes to 45°56′, it is also clear that, whenever the moon's centre is truly 1°3′36″ from the shadow's centre on either side of the ecliptic along the great circle drawn through the centres and the oblique circle's poles, and is very nearly 12°12′ from either of the nodes along the moon's oblique circle (according to the ratio of 1 to $11\frac{1}{2}$) then first can the moon be tangent to the shadow. For the same reasons as already demonstrated concerning the anomaly, whenever the moon's centre taken at the mean passage is 15°12′ from the nodes along the oblique circle—so that, for numbers indicating distance from the northern limit, it falls between 74°48′ and 105°12′ and between 254°48′ and 285°12′, then first can the moon be tangent to the shadow.

We shall now place beside the appointed tables of the syzygies the numbers belonging to the solar and lunar limits of the moon's latitude so that we can readily judge those capable of falling within an eclipse.

6. On the Interval of Ecliptic Months

And to these things it would be useful to add how many months apart, in general, the ecliptic syzygies can occur, so that, if we take the position of one ecliptic syzygy, by an examination of the terms we may get not all the syzygies which follow but only those at month-intervals in which an eclipse can occur.

Now, it would be immediately clear that the sun and moon can be eclipsed at six-month intervals, since the moon's mean latitudinal passage comes to 184°1′25″ in six months, and since the arcs between the ecliptic limits, both for the sun and moon, embrace fewer degrees than this when they are within a semicircle and more when they are above a semicircle. For, as demonstrated, since the solar limits cut off 20°41′ from either of the nodes northward along the moon's oblique circle, and southward 11°22′, the northern anecliptic arc is 138°38′, and the southern is 157°16′. As the lunar limits cut off to either side of the ecliptic 15°12′ from the nodes along the same circle, each of the anecliptic arcs comes to 149°36′.

That by means of these hypotheses it is possible for an eclipse of the moon to occur at an interval of the greatest five months—that is, when the sun is making its greatest passage and the moon its least—we can see as follows:

Since in the mean five-month interval we find the longitudinal passage of each luminary adding on 145°32′ in terms of mean movement, and the moon 129°5′

of anomaly on the epicycle; since at its greatest passage either side of the perigee the sun's 145°32′ add 4°38′ to the mean passage; and since at its least passage either side of the apogee the 129°5′ of the moon's epicycle, subtract 8°40′ from the mean passage, it follows that in the time of the mean five-month period, whenever the sun effects its greatest and the moon its least passage, the moon will be west of the sun by the combined anomalies of both: 13°18′. Again, taking $\frac{1}{12}$ of this for reasons already demonstrated, we shall have very nearly the 1°6′ which the sun moves forward until it is overtaken by the moon.

Now, since it had added on 4°38′ from its own anomaly and another 1°6′ from the time required for overtaking at the true syzygy, the greatest five-month interval will have added on 5°44′ in longitude beyond the mean. Therefore the moon's latitudinal passage along the oblique circle will have added on just that many degrees to very nearly 153°21′ in latitude resulting for the five-month interval. And so the latitudinal passage, truly considered in the greatest five-month interval, will come to 159°5′.

But the moon's ecliptic limits either side of the ecliptic circle, at the moon's mean distance, embrace very nearly 1° on the great circle drawn through the poles of the oblique circle, since at the least distance it is 1°3′36″ and at the greatest distance 0°56′24″; and they embrace 11°30′ from the nodes along the oblique circle. Therefore the anecliptic arc between them comes to 157°, which for the greatest five-month interval is less by 2°5′ than the additional 159°5′ of the oblique circle. Now, it is clear for these reasons that it is possible for the moon, within the greatest five-month interval, to be eclipsed at the first full moon in its withdrawal from either node, and to be eclipsed again at the last full moon in its approach to the opposite node. For in both eclipses the occultation occurs on the same side of the ecliptic, and never on opposite sides.

In this way we have seen how the greatest five-month period can produce two lunar eclipses. And that it is impossible for the same thing to happen in a seven-month interval even if we suppose the least seven-month interval (that is, during which the sun effects its least passage and the moon its greatest) we can see by working in the same way as before.

For since, again, in the mean seven-month interval, the mean longitudinal passage of both luminaries takes on an additional 203°45′, that of the moon on its epicycle 180°43′; since the sun's 103°45′ at its least passage either side of the apogee subtracts 4°42′ from the mean movement; and since the 180°43′ of the moon's epicycle at its greatest passage either side of the perigee adds 9°58′ to the mean movement, it follows that in the time of a mean seven-month interval, whenever the sun effects its least passage and the moon its greatest, the moon will have passed the sun by the combined 14°40′ of both anomalies. For the same reasons, taking $\frac{1}{12}$ of this and adding it to the 4°42′ of the subtractive solar anomaly, we shall have all told very nearly 5°55′ by which the longitudinal passage in the least seven-month interval will lag behind that in the mean, and just so will the latitudinal passage fail of the total 214°42′ for the mean seven-month interval. Therefore in the least seven-month interval the moon will have added on 208°47′ in latitude along the oblique circle, whereas the whole greatest arc on the oblique circle between the moon's ecliptic limits at its mean distance, from the approach to one of the nodes to the withdrawal from the opposite node, is 203°. It is consequently not possible for the moon, in the least seven-

month interval, to be eclipsed ever so little at the first full moon and again at the last.

Then again it must be shown that it is possible for the sun to be eclipsed twice in the greatest five-month interval for the same people and in all parts of the world inhabited by us.

For we demonstrated the moon's latitudinal passage in the greatest five-month interval to be 159°5′ while the anecliptic arc for the sun, at the moon's mean distance, is 167°36′. For the ecliptic limits are 0°32′20″ from the ecliptic along the oblique circle. Therefore it is clear that, if the moon displays no parallax, the required result is impossible, because the anecliptic arc is greater than the passage in the greatest five-month interval by 8°31′ along the oblique circle, and by very nearly 0°45′ along the circle at right angles to the ecliptic, but that, wherever it can display a parallax such that the parallaxes in either of the extreme conjunctions or the parallaxes of both together exceed 0°45′, in such a place both of the extreme conjunctions can be ecliptic.

Now, since we showed that, in the time of the greatest five-month interval, whenever the moon effects its least passage and the sun effects its greatest from two-thirds within the Virgin to two-thirds within the Water Bearer, the moon falls west of the sun by the 13°18′ of both anomalies, and since the moon moves in mean terms this much and $\frac{1}{12}$ as much more in 1 day and 2¼ hours, it is clear that, while the time of the mean five-month interval is 147 days and very nearly $15+\frac{1}{2}+\frac{1}{4}$ hours, the time of the greatest five-month interval is 148 days and 18 hours. And, for this reason, if the first conjunction is nearly two-thirds within the Virgin, the last one nearly two-thirds within the Water Bearer will be earlier by the six hours lacking to fill out the whole days. Therefore one must seek where and when the moon can display a parallax of more than these 0°45′, either in one or the other of these two dodecatemories or in both, at a position 6 hours earlier in the Water Bearer than in the Virgin.

Now nowhere in the world inhabited by us is the moon found to display such a parallax northward, as we have said. Whence it is impossible for the sun to be twice eclipsed in the greatest five-month interval with the moon's passage south of the ecliptic—that is, whenever the moon at the first conjunction is withdrawing from the descending node and at the last conjunction is approaching the ascending node. But it can display just such a parallax southward, for those living below the equator northward, in both these dodecatemories at a position six hours earlier whenever two-thirds the Virgin is supposed setting at the first conjunction and two-thirds the Water Bearer is culminating at the second. For in such positions we find the moon, in its mean distance, displaying a southward parallax (the sun's parallax being accounted for) of very nearly 0°22′ at the equator when it is in the Virgin, and of 0°14′ when it is in the Water Bearer; but where the longest day is 12½ hours, 0°27′ when it is in the Virgin, and 0°22′ when it is in the Water Bearer; so that in this case the two parallaxes together exceed the 0°45′ by 4 sixtieths. And since the southward parallax becomes greater for the still more northern places, therefore it is evident that it will be even more possible for the sun to appear twice eclipsed in the least five-month interval for those inhabiting these places, but only at the moon's passage north of the ecliptic—that is, when the moon in the first eclipse withdraws from the ascending node and, in the second, approaches the descending node.

Thus I say again that, also in the least seven-month interval, it is possible for the sun to be twice eclipsed for the same people. For in the least seven-month interval we demonstrated the moon's latitudinal passage to be 208°47′. And since the greatest arc of the oblique circle cut off between the ecliptic limits is that from the limit at the approach to one node to the limit at the withdrawal from the other, therefore, in the case of the sun this distance will come to 192°24′ at the moon's mean distance. Therefore it is clear that, if the moon displays no parallax, the required appearance will be impossible, because the oblique circle's arc of the least seven-month interval is greater than the greatest arc cut off by the sun's ecliptic limits which is 16°23′ along the oblique circle and 1°25′ along the circle drawn through the ecliptic's poles. But wherever it can display a parallax such that the parallaxes in either of the extreme conjunctions or both parallaxes together exceed this 1°25′, in such a place can both extreme conjunctions be ecliptic.

Now since we showed that, in the time of the mean seven-month interval, whenever the moon effects its greatest passage, and the sun its least passage from the last part of the Water Bearer to the middle of the Virgin, the moon has already truly passed the sun by 14°40′. Since the moon moves in mean terms just that many degrees together with $\frac{1}{12}$ of them in 1 day and 5 hours, therefore it is evident that, while the time of the mean seven-month interval embraces 206 days and very nearly 17 hours, the time of the least seven-month interval will be 205 days and 12 hours. For this reason the time of the last conjunction near the middle of the Virgin will be 12 hours later than the first near the end of the Water Bearer. Therefore one must seek where and when the moon can display a parallax of more than 1°25′, either in one or the other of the given dodecatemories or in both at positions 12 hours apart—that is, when the one is setting and the other is rising. For not otherwise could both eclipses occur above the earth.

Now again, in the world inhabited by us, the moon is nowhere found in any position displaying such a parallax northward; and even to those living under the equator a parallax of not more than the 23 sixtieths of the latitudinal parallax at the greatest distance. Whence it is impossible for the sun to be twice eclipsed in the least seven-month interval with the moon's passage south of the ecliptic; that is, whenever it approaches the ascending node at the first conjunction and withdraws from the descending node at the last. But we find such a parallax effected southward from around the parallel through Rhodes whenever the last of the Water Bearer is rising and the middle of the Virgin setting. For at its mean distance in Rhodes and in places under the same parallel, the moon displays in either position a southward parallax (the solar parallax being subtracted) up to very nearly 0°46′, so that the parallaxes at both conjunctions are together greater than the 1°25′. Now, since the southward parallax is even greater in those places north of this parallel, it is evident that, for those inhabiting these places, it is possible for an eclipse of the sun to appear twice in the least seven-month interval, but again only at the moon's passage north of the ecliptic—that is, when the moon in the first eclipse approaches the descending node and in the second withdraws from the ascending node.

It remains further to be shown, moreover, that at an interval of one month it is not possible for the sun to be twice eclipsed in the world inhabited by us, either in the same parallel or in different parallels, even if one supposes all to-

gether those things which cannot concur, but as combined to make the required appearance possible; I mean, even if we suppose the moon at its least distance so that it may display a parallax (the month the least so that the monthly latitudinal passage may be greater, by as little as possible) that is greater than that embraced by the sun's ecliptic limits, using indifferently hours and dodecatemories in which it appears to make the greatest parallaxes.

Since, then, in the mean month the longitudinal passage of each of the luminaries takes on 29°6′ in mean terms, and the passage on the moon's epicycle takes on 25°49′; and since of these the sun's 29°6′ at its least passage either side of the apogee subtract 1°8′ from the mean, and the 25°49′ of the moon's epicycle at its greatest passage either side of the perigee add 2°28′ to the mean—if, according to what has already been demonstrated, we combine the addition-subtractions of both anomalies which come to 3°36′ and add on a twelfth or 0°18′ to what the sun lacked, we shall have 1°26′. And by just this amount we shall have the least month's passage less than the longitudinal and latitudinal in the mean month. And so, since the mean month's latitudinal passage is 30°40′, the least month's passage is 29°14′, which makes very nearly 2°33′ along the great circle at right angles to the ecliptic. But the whole passage of the sun's ecliptic limits comes to 1°6′ when the moon is at its least distance, so that the least month's passage is greater by 1°27′.

Now, if the sun were to be twice eclipsed in one month, it would be absolutely necessary, either that the moon display no parallax at one of the conjunctions and one greater than 1°27′ at the other; or that it display parallaxes in the same direction at both conjunctions with the difference between the parallaxes greater than 1°27′; or that both parallaxes come to something greater when the parallax at one conjunction is northward and at the other southward. But nowhere on the earth in the syzygies—not even at its least distance—will the moon display (the sun's parallax being accounted for) a parallax greater than one degree. Therefore it is not possible in the least month for the sun to be twice eclipsed, either when the moon displays no parallax at one of the conjunctions, or when it displays one in the same direction at both, since their difference is never greater than one degree although it needs to be greater than 1°27′.

The required appearance could happen only if the total number of degrees from both parallaxes, one being in an opposite direction to the other, should total more than 1°27′. And this is possible in different regions because the moon can display, to those north of the equator in the regions inhabited by us, a southward parallax, and to those south of the equator in what is called the antichthones, a northward parallax of from 0°25′ to 1° beyond the sun's parallax. But this could never happen for the same region. Because the moon displays in like manner a greatest parallax northward and southward of not more than 0°25′ for those under the equator; and to the most northern and most southern not more than the stated 1° in the opposite directions, so that in this way both parallaxes still total less than 1°27′. Inasmuch as each of the opposite parallaxes would always be much less in the case of regions between the equator and one of the limits, the impossibility would be even greater for them.

Nowhere on the earth for the same latitude, therefore, can the sun be twice eclipsed in one month; and for different latitudes nowhere in the same inhabited region. Which things we were required to show.

7. Construction of Ecliptic Tables

It has now become clear to us what intervals of syzygies we must take for a research into the eclipses. In order that, when the mean times at them have been distinguished and the moon's passages for those times calculated both in the case of the apparent conjunctive syzygies and in the case of the true plenilunar ones we might (by means of the moon's latitudinal positions) be able easily to review those of the syzygies certain to be ecliptic, their magnitudes, and the times of the obscurities, we constructed tables for so distinguishing them: two for solar eclipses and two for lunar ones, both at the moon's greatest and least distance, supposing the increase of the obscurations through twelve parts of the obscured diameter of each of the luminaries.

Now, the first table of the solar eclipses, which contains the ecliptic limits at the moon's greatest distance, we have arranged in 25 rows and 4 columns. The first two columns contain the moon's apparent latitudinal passage along the oblique circle for each of the obscurations. Since the sun's diameter is 0°31′20″, and the moon's was shown to be also 0°31′20″ at its greatest distance—and since, therefore, whenever the moon's apparent centre was 0°31′20″ from the sun's centre along the great circle through both centres, and 6° from the node along the oblique circle according to the ratio previously set out of 11½ to 1, then first will the moon be in a position of tangency to the sun, therefore, in the first rows of the columns we shall place 84° for the first, and 206° for the second; and, in the last rows, 96° for the first, and 264° for the second. And since very nearly 30 sixtieths of one degree along the oblique circle correspond to one-twelfth of the sun's diameter, we shall continually increase and decrease these two columns by just so much, beginning from the extremes and going to the middle. At the middle rows we shall put 90° and 270°. The third column will contain the magnitudes of the obscurations: in the case of the end rows, the corresponding zero of tangency; and in those following them one digit for each $\frac{1}{12}$ of the diameter; and thus for the rest a continual increase of one digit to the middle row, to which the number of twelve digits will correspond. And the fourth column will contain the resulting passages of the moon's centre for each obscuration, without any calculations of the added movements of the sun or of the added parallaxes of the moon.

The second table of solar eclipses, which contains the ecliptic limits at the moon's least distance, we shall arrange as to other things like the first, but in 27 rows and 4 columns. Because the moon's radius at its least distance has been shown to be 17°40′ to the 15°40′ of the sun's radius; and whenever the moon first becomes tangent to the sun, then its apparent centre is 0°33′20″ from the sun's centre and 6°24′ from the nodes along the oblique circle. The numbers of apparent latitude in the end rows are 83°36′ and 206°24′ and again 96°24′ and 263°36′, and for the middle row of the digits, the number is twelve digits and, because of a like excess, four fifths of a digit for the delay of occultation.

Each of the two lunar tables is arranged in 45 rows and 5 columns, the first table being furnished with latitudinal numbers as of the moon at its greatest distance. For, since the moon's radius was shown to be 0°15′40″ at its greatest distance and the shadow's radius 0°40′44″ (so that, whenever the moon is first tangent to the shadow, its centre is 0°56′24″ from the shadow's centre along the great circle through both centres, and 1°48′ from the nodes along the oblique

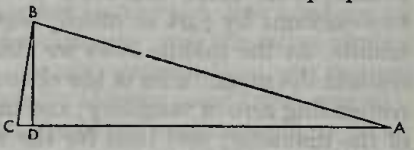
circle), therefore, in the first rows we shall place the numbers 79°12′ and 280°48′, and, in the last rows, the numbers 100°48′ and 259°12′. For the same reasons as the first we shall continually increase and decrease them by the 30 sixtieths corresponding to $\frac{1}{12}$ of the lunar diameter at that time.

The second table we shall furnish with latitudinal numbers as of the moon at its least distance, at which distance its radius was shown to be 0°17′40″ and the shadow's radius 0°45′56″ so that, whenever the moon first touches the shadow, then its centre is likewise 1°3′36″ from the shadow's centre and 12°12′ from the node along the oblique circle. Now, therefore, putting the numbers 77°48′ and 282°12′ in the first rows, and the numbers 202°12′ and 257°48′ in the last, we shall again continually increase and decrease them by the 34 sixtieths corresponding to $\frac{1}{12}$ of the lunar diameter at that time.

And they will contain third columns of digits like the solar tables; and likewise succeeding columns containing the moon's passages for each obscuration, those of immersion and those of emersion, and further those of half the delay of occultation.

We calculated geometrically for each obscuration the given passages of the moon. But we treated the proofs as if in one plane and in straight lines, because arcs of that magnitude differ insensibly from their chords, and the moon's passage along the oblique circle differs hardly at all from that considered along the ecliptic. Let no one suppose that we do not know that, in general, with respect to the moon's latitudinal passage, there is a difference in using the arcs of the oblique circle for those of the ecliptic; furthermore it does not follow that the times of the syzygies are exactly the same as the mid-eclipses.

For if we cut off from the node A two equal arcs AB and AC of the proposed circles, and, joining BC, draw BD from B at right angles to AC, then it will be immediately evident that, with the moon supposed at B, if we use arc AC of the ecliptic instead of the AD used because of considering the passages relative to it with respect to circles through the ecliptic's poles, then the difference due to the inclination of the lunar circle will be CD. Again, if the sun or the shadow's centre is conceived at B, the time of the syzygy, in accordance with the little difference between the circles, will be when the moon is at C; and the mid-eclipse will be when the moon is at D, because the mid-times of the obscurations are considered with respect to the circles through the poles of the lunar circle. And the time of the syzygy will differ from the mid-eclipse by arc CD.

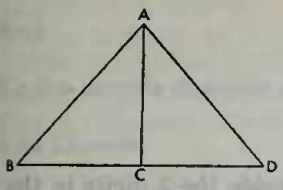


The cause of our not calculating these arcs in detail is that their differences are small and insensible, and that (while not knowing this is absurd) yet, because of the difficulty in working out each case, this willing neglect of something which can be overlooked both in the hypotheses and the observations makes the greatest difference in the usefulness of a simple method, and no difference or very little in the discrepancy with appearances. Now, we find the arc similar to CD to be, in general, not greater than 5 sixtieths of one degree. This is proved by the same theorem as that by which we calculated the differences of the equatorial arcs with respect to those of the ecliptic for circles drawn through the equator's poles. But in the case of the eclipses we find it to be not greater than 2 sixtieths, since BD is almost 1° to the 12° of each of the arcs AB and AC. For the moon's passages at eclipses reach almost that far. Therefore arc AD is very

nearly 11°58′, and that leaves CD equal to 2 sixtieths of a degree, which does not make one sixteenth of an equatorial hour. Concern over such a fractional degree is more vanity than love of truth.

Now, by these means, we also worked out the required passages of the moon's obscurations as if the circles differed insensibly. And our calculation for one or two examples is as follows:

Let there be the sun's or the shadow's centre A, and the straight line BCD in place of the lunar circle's arc. And let B be supposed the moon's centre when it first touches the sun or shadow as it approaches; and D as it withdraws. And with AB and AD joined, let AC be drawn from A perpendicular to BD.



Now, that the mid-time of the eclipse and the greatest obscuration occurs when the moon's centre is at C is evident from AB's being equal to AD; and, therefore, the passage BC's being equal to CD, and from AC's being less than all the lines joining the two centres to BD. It is clear also that either of the lines AB and AD contains both the radius of the moon and of the sun or shadow, and that line AC is less than either of them by that part of the diameter of the body being eclipsed which is cut off by the obscuration.

With these things so, let the obscuration be 3 digits as an example, and let A be first supposed the sun's centre. Now, therefore, when the moon is at its greatest distance,

$$AB = 31'20'',$$

and

$$\text{sq. } AB = 981'47''.$$

And

$$AC = 23'30'',$$

for it is less than AB by the 3 twelfths of the sun's diameter, or 7ᵖ50′. And

$$\text{sq. } AC = 552'15''.$$

And so

$$\text{sq. } BC = 429'32'',$$

and, in length,

$$BC = 20'43'',$$

which we shall place in the first table of the solar eclipses beside the 3 digits in the fourth column.

Again, in the case of the moon's least distance,

$$AB = 33'20'',$$

and

$$\text{sq. } AB = 1111'7'';$$

and

$$AC = 25'30'',$$
$$\text{sq. } AC = 650'15'';$$

and, by subtraction,

$$\text{sq. } BC = 460'52'',$$

and therefore, in length,

$$BC = 21'28''$$

which we shall place in the second table of the solar eclipses beside the 3 digits in the fourth column.

Yet again, let A be supposed the shadow's centre and the obscuration the

same ¼ of the moon's diameter. Therefore, for the moon's greatest distance,

$$AB = 56'24''$$

and

$$\text{sq. } AB = 3180'58''.$$

And

$$AC = 48'34'',$$

for it is less than AB by ¼ of the lunar diameter or, at the greatest distance, by 7′50″. And

$$\text{sq. } AC = 2358'43''.$$

And so, by subtraction,

$$\text{sq. } BC = 822'15'',$$

and, in length,

$$BC = 28'41'',$$

which we shall place in the first of the lunar tables beside the 3 digits in the fourth column and which contain the passage of immersion, sensibly the same as that of emersion.

And, in the case of the least distance,

$$AB = 63'36''$$

and

$$\text{sq. } AB = 4044'58''.$$

And

$$AC = 54'46'',$$

for the 8′50″ difference is again ¼ of the lunar diameter at the least distance. And

$$\text{sq. } AC = 2999'23''.$$

And so, by subtraction,

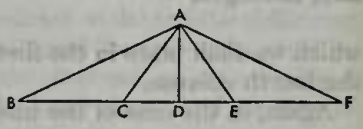
$$\text{sq. } BC = 1045'35'',$$

and, in length,

$$BC = 32'20'',$$

which we shall place likewise beside the 3 digits in the fourth column in the second of the lunar tables.

Once more, because the lunar obscurations have a time of delay, let the point A be the shadow's centre, and let the straight line $BCDEF$ be in place of the arc of the moon's oblique circle. Let B be supposed the point where the moon's centre will be when the moon, as approaching, first touches the shadow from without; and C where the moon's centre will be when the totally eclipsed moon will first touch the shadow's circle from within. And let E be the point where the moon's centre will be again when the moon, as withdrawing, first touches the shadow's circle from within; and F where the moon's centre will be when, taking its departure, it touches the shadow from without for the last time. And again let AD be drawn from A perpendicular to BF.



Now, given here those things already demonstrated as remaining true, it is also evident that either of the straight lines AC and AE bounds the difference by which the shadow's radius exceeds the moon's radius, so that the passage CD equals the passage DE, and each contains half the delay, and the remainder BC of immersion is equal to the remainder EF of emersion.

So let there be supposed an eclipse in which 15 digits of the moon are available

—that is, in which its centre D is within the bound of the ecliptic limits by one lunar diameter plus a fourth of it; in other words when AD is less than either AB or AF by the given one lunar diameter plus a fourth of it, and less than either AC or AE by a fourth of one lunar diameter.

Therefore, when the moon is at its greatest distance,

$$AB = 56'24''$$

and

$$\text{sq. } AB = 3180'58''.$$

And

$$AC = 25'4'',$$

for the moon's diameter at its greatest distance is 31′20″. And

$$\text{sq. } AC = 628'20''.$$

And likewise

$$AD = 17'14''$$

and

$$\text{sq. } AD = 396'59''.$$

And so, by subtraction,

$$\text{sq. } BD = 2883'59''$$

and, in length,

$$BD = 53'42''.$$

And, by subtraction,

$$\text{sq. } CD = 331'21''$$

and, in length,

$$CD = 18'12'';$$

and, by subtraction,

$$BC = 35'30''.$$

Now we shall place these 35′30″ of immersion, equal to those of emersion, beside the number of 15 digits, in the fourth column of the first table of lunar eclipses; and the 18′12″ of the half-time of delay, in the fifth column.

When the moon is at its least distance, then again

$$AB = 63'36''$$

and

$$\text{sq. } AB = 4044'56''.$$

And

$$AC = 28'16'',$$

for the moon's diameter at the least distance was shown to be 35′20″. And

$$\text{sq. } AC = 799'.$$

And likewise

$$AD = 19'26''$$

and

$$\text{sq. } AD = 377'39''.$$

And so also, by subtraction,

$$\text{sq. } BD = 3667'19''$$

and, in length,

$$BD = 60'34''.$$

And, by subtraction,

$$CD = 421'21''$$

and, in length,

$$CD = 20'32''.$$

And, by subtraction,

$$BC = 40'2''.$$

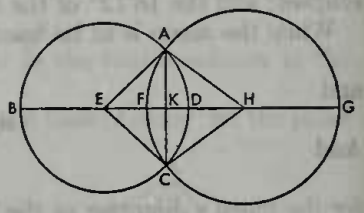
Therefore we shall also place these 40′2″ of immersion, again equal to those of emersion, beside the number of 15 digits, in the fourth column of the second table of lunar eclipses; and the 20′32″ of the half-time of delay in the fifth column.

But, for the moon's passages on its epicycle between the greatest and least distances, in order that we might easily obtain by the method of sixtieths the corresponding parts of the whole difference for each passage, we placed under the foregoing tables another short table containing the numbers of the passage on the epicycle and the corresponding sixtieths for each of the apparent excesses from the first and second tables of the eclipses. And the quantity of these sixtieths has been worked out by us and placed in the seventh column of the moon's Parallactic Table, for the epicycle is assumed to be at the eccentric's apogee throughout the syzygies.

Since most of those who observe the ecliptic markings do not measure the magnitudes of the obscurations by the circles' diameters but in general by the whole plane figures (the eye comparing in a rough way the whole apparent part with that not appearing), we have added to these also another short table of 12 rows and 3 columns. In the first column we placed the 12 digits, each digit containing $\frac{1}{12}$ of the diameter of each luminary as in the ecliptic tables; and in the next columns in turn the twelfths of the whole areas corresponding to them—in the second the sun's, in the third the moon's. These impositions we calculated only for the magnitudes of the moon's mean distance (for very nearly the same ratio is established for such increases and decreases of the diameters) and with the ratio of the circumferences to the diameters being as $3^p8'30''$ to 1^p. For this ratio is between (very nearly) a triple plus a seventh part and a triple plus ten seventy-firsts, both of which Archimedes used for greater simplicity.

Then first, for the solar eclipses, let there be the sun's circle $ABCD$ about centre E; and the moon's circle, as its mean distance, $AFCG$ about centre H cutting the sun's circle at points A and C. And with $BEHG$ joined, let it be supposed that $\frac{1}{4}$ of the sun's diameter has been eclipsed, so that

$$FD = 3^p$$



where

$$\text{diam. } BD = 12^p$$

and

$$\text{moon's diam. } FG \fallingdotseq 12^p20'$$

in the ratio of $15^p40'$ to $16^p40'$. And therefore

$$EH = 9^p10',$$

With the perimeters in the ratio of 1^p to $3^p8'30''$, that of the sun's circle is $37^p42'$, and that of the moon's $38^p46'$. Likewise of the whole areas, since the radius multiplied by the perimeter gives twice the circle's area, that of the sun's circle will come to $113^p6'$, and that of the moon's to $119^p32'$.

Then, these things being so, it is required to find how great is the area contained by $ADCF$ where the whole area of the sun's circle is 12^p.

Let AE and AH, and CE and CH be joined, and further the perpendicular AKC.

Now, since it is supposed

$$AE = CE = 6^p,$$
$$AH = CH = 6^p10'$$

where

$$EH = 9^{p}10',$$

and the angle at K is right, therefore, if we apply the excess by which the square on AH exceeds the square on AE, or $2^{p}2'$, to the line EH, we shall have the difference between EK and HK of $13'3''$[1]. And

$$EK = 4^{p}28',$$
$$HK = 4^{p}42';$$

and therefore

$$AK = CK \fallingdotseq 4^{p}.$$

And consequently we shall have

$$\text{trgl. } AEC = 17^{p}52',$$

and

$$\text{trgl. } AHC = 18^{p}48'.$$

Again, since

$$AC = 8^{p}$$

where

$$\text{diam. } BD = 12^{p}$$

and likewise

$$\text{diam. } FG = 12^{p}20',$$

and hence

$$AC = 80^{p}$$

where

$$\text{diam. } BD = 120^{p},$$

and

$$AC = 77^{p}50'$$

where

$$\text{diam. } FG = 120^{p},$$

therefore

$$\text{arc } ADC = 83°37'$$

where circle $ABCD$ is 360°, and

$$\text{arc } AFC = 80°52'$$

where circle $AFCG$ is 360°.

And so, since the ratio of circles to arcs is the same as the circles' areas to the areas of the sectors of the arcs, therefore

$$\text{sect. } AECD = 26^{p}16'$$

where it was shown

$$\text{circle } ABCD = 113^{p}6';$$

and

$$\text{sect. } AHCF = 26^{p}51'$$

for

$$\text{circle } AFCG = 119^{p}32'.$$

But it was shown also that

$$\text{trgl. } AEC = 17^{p}52'$$

and likewise

$$\text{trgl. } AHC = 18^{p}48'.$$

And therefore, by subtraction, we shall have

$$\text{seg. } ADCK = 8^{p}24',$$
$$\text{seg. } AFCK = 8^{p}3'.$$

And therefore the whole area contained by arcs AF, CD is $16^{p}27'$ where the area

[1]Application is here equivalent to division. The geometric proof of this statement is simple.

of circle $ABCD$ is supposed $113^p6'$. Thus, where the area of the sun's circle is 12, the area contained by the eclipsed segment is very nearly $1\frac{3}{4}$ which we shall place in the table just mentioned, along the 3-digits row, in the second column.

Again, in the same figure for the lunar eclipses, let $ABCD$ be supposed the moon's circle, and $AFCG$ the shadow's circle at mean distance. And likewise let there be an eclipse to $\frac{1}{4}$ of the moon's diameter so that, where diameter BD is 12^p, diameter DF of the eclipse is 3^p and diameter FG of the shadow is $31^p12'$ according to the ratio of 1^p to $2^p36'$. And therefore

$$EKH = 18^p36'.$$

And therefore again, of the perimeters, that of the lunar circle is $37^p42'$ and that of the shadow's $98^p1'$; and, of the areas, that of the lunar circle $113^p6'$ and that of the shadow's $764^p32'$.

Now, since here it is supposed

$$AE = CE = 6^p,$$
$$AH = CH = 15^p36'$$

where

$$EH = 18^p36',$$

therefore, if likewise we apply to EH the excess by which AH exceeds AE, we shall have

$$HK - EK = 11^p8'.$$

And so

$$EK = 3^p44',$$
$$HK = 14^p52',$$

and therefore

$$AK = CK = 4^p42'.$$

And consequently we shall have

$$\text{trgl. } AEC = 17^p33'$$

and

$$\text{trgl. } AHC = 69^p52'.$$

Again, since

$$AC = 9^p24'$$

where

$$\text{diam. } BD = 12^p$$

and likewise

$$\text{diam. } FG = 31^p12',$$

and

$$AC = 94^p$$

where

$$\text{diam. } BD = 120^p,$$

and

$$AC = 36^p9'$$

where

$$\text{diam. } FG = 120^p,$$

therefore

$$\text{arc } ADC = 103°8'$$

where circle $ABCD$ is 360°, and

$$\text{arc } AFC = 35°4'$$

where circle $AFCG$ is 360°.

And so, by means of things already said, we shall have

$$\text{sect. } AECD = 32^p24'$$

where it was shown

circle $ABCD = 113^{p}6'$;

and

sect. $ACHF = 74^{p}28'$

for

circle $AFCG = 764^{p}32'$.

But it was shown also

trgl. $AEC = 17^{p}33'$

and likewise

trgl. $AHC = 69^{p}52'$.

And therefore, by subtraction, we shall have

seg. $ADCK = 14^{p}51'$,

seg. $AFCK = 4^{p}36'$.

Consequently, the whole area contained by arcs AF, CD is $19^{p}27'$ where the area of circle $ABCD$ is supposed $113^{p}6'$. Where the area of the moon's circle is 12, therefore, that contained by the eclipsed segment will be very nearly $2\frac{1}{15}$, which we shall place in the same table, along the 3-digits row, in the third column belonging to the moon. And here is the exposition of the tables:

8. Table of Solar Eclipses

At Greatest Distance							*At Least Distance*						
Numbers of Latitude				*Digits*	*Fraction of Immersion*		*Numbers of Latitude*				*Digits*	*Fraction of Immersion*	
1		2		3	4		1		2		3	4	
84	0	276	0	0	0	0	83	36	276	24	0	0	0
84	30	275	30	1	12	32	84	6	275	54	1	12	57
85	0	275	0	2	17	19	84	36	275	24	2	17	54
85	30	274	30	3	20	43	85	6	274	54	3	21	28
86	0	274	0	4	23	27	85	36	274	24	4	24	14
86	30	273	30	5	25	38	86	6	273	54	5	26	27
87	0	273	0	6	27	8	86	36	273	24	6	28	16
87	30	272	30	7	28	29	87	6	272	54	7	29	45
88	0	272	0	8	29	32	87	36	272	24	8	30	55
88	30	271	30	9	30	20	88	6	271	54	9	31	51
89	0	271	0	10	30	54	88	36	271	24	10	32	33
89	30	270	30	11	31	13	89	6	270	54	11	33	1
90	0	270	0	12	31	20	89	36	270	24	12	33	16
90	30	269	30	11	31	13	90	0	270	0	12	33	22
91	0	269	0	10	30	54	90	24	269	36	12	33	16
91	30	268	30	9	30	20	90	54	269	6	11	33	1
92	0	268	0	8	29	32	91	24	268	36	10	32	33
92	30	267	30	7	28	29	91	54	268	6	9	31	51
93	0	267	0	6	27	8	92	24	267	36	8	30	55
93	30	266	30	5	25	38	92	54	267	6	7	29	45
94	0	266	0	4	23	27	93	24	266	36	6	28	16
94	30	265	30	3	20	43	93	54	266	6	5	26	27
95	0	265	0	2	17	19	94	24	265	36	4	24	14
95	30	264	30	1	12	32	94	54	265	6	3	21	28
96	0	264	0	0	0	0	95	24	264	36	2	17	54
							95	54	264	6	1	12	57
							96	24	263	36	0	0	0

Table of Lunar Eclipses

At Greatest Distance								*At Least Distance*							
Numbers of Latitude				*Digits*	*Fraction of Immersion*		*Half Delay*	*Numbers of Latitude*				*Digits*	*Fraction of Immersion*		*Half Delay*
1		2		3	4		5	1		2		3	4		5
79	12	280	48	0	0	0		77	48	282	12	0	0	0	
79	42	280	18	1	16	59		78	22	281	38	1	19	9	
80	12	279	48	2	23	43		78	56	281	4	2	26	45	
80	42	279	18	3	28	41		79	30	280	30	3	32	20	
81	12	278	48	4	32	42		80	4	279	56	4	36	53	
81	42	278	18	5	36	6		80	38	279	22	5	40	42	
82	12	277	48	6	39	1		81	12	278	48	6	43	59	
82	42	277	18	7	41	34		81	46	278	14	7	46	53	
83	12	276	48	8	43	50		82	20	277	40	8	49	25	
83	42	276	18	9	45	48		82	54	277	6	9	51	40	
84	12	275	48	10	47	35		83	28	276	32	10	53	39	
84	42	275	18	11	49	9		84	2	275	58	11	55	25	
85	12	274	48	12	50	31		84	36	275	24	12	57	59	
85	42	274	18	13	40	35	11 9	85	10	274	50	13	45	47	12 34
86	12	273	48	14	37	28	15 20	85	44	274	16	14	42	15	17 17
86	42	273	18	15	35	30	18 12	86	18	273	42	15	40	2	20 32
87	12	272	48	16	34	6	20 22	86	52	273	8	16	38	28	22 58
87	42	272	18	17	33	7	22 0	87	26	272	34	17	37	20	24 49
88	12	271	48	18	32	23	23 14	88	0	272	0	18	36	37	26 1
88	42	271	18	19	31	51	24 8	88	34	271	26	19	35	55	27 13
89	12	270	48	20	31	32	24 43	89	8	270	52	20	35	34	27 42
89	42	270	18	21	31	22	23 1	89	42	270	18	21	35	22	28 12
90	0	270	0	Complete	31	20	23 4	90	0	270	0	Complete	35	20	28 6
90	18	269	42	21	31	22	23 1	90	18	269	42	21	35	22	28 12
90	48	269	12	20	31	32	24 43	90	52	269	8	20	35	34	27 42
91	18	268	42	19	31	51	24 8	91	26	268	34	19	35	55	27 13
91	48	268	12	18	32	23	23 14	92	0	268	0	18	36	37	26 1
92	18	267	42	17	33	7	22 0	92	34	267	26	17	37	20	24 49
92	48	267	12	16	34	6	20 22	93	8	266	52	16	38	28	22 58
93	18	266	42	15	35	30	18 12	93	42	266	18	15	40	2	20 32
93	48	266	12	14	37	28	15 20	94	16	265	44	14	42	15	17 17
94	18	265	42	13	40	35	11 9	94	50	265	10	13	45	47	12 34
94	48	265	12	12	50	31		95	24	264	36	12	57	59	
95	18	264	42	11	49	9		95	58	264	2	11	55	25	
95	48	264	12	10	47	35		96	32	263	28	10	53	39	
96	18	263	42	9	45	48		97	6	262	54	9	51	40	
96	48	263	12	8	43	50		97	40	262	20	8	49	25	
97	18	262	42	7	41	34		98	14	261	46	7	46	53	
97	48	262	12	6	39	1		98	48	261	12	6	43	59	
98	18	261	42	5	36	6		99	22	260	38	5	40	42	
98	48	261	12	4	32	42		99	56	260	4	4	36	53	
99	18	260	42	3	28	41		100	30	259	30	3	32	20	
99	48	260	12	2	23	43		101	4	258	56	2	26	45	
100	18	259	42	1	16	59		101	38	258	22	1	19	9	
100	48	259	12	0	0	0		102	12	257	48	0	0	0	

TABLE OF CORRECTION

Numbers of Anomaly 1	*Numbers of Anomaly* 2	*Sixtieths of Differences* 3	
6	354	0	21
12	348	0	42
18	342	1	42
24	336	2	42
30	330	4	1
36	324	5	21
42	318	7	18
48	312	9	15
54	306	11	37
60	300	14	0
66	294	16	48
72	288	19	36
78	282	22	36
84	276	25	36
90	270	28	42
96	264	31	48
102	258	34	54
108	252	38	0
114	246	41	0
120	240	44	0
126	234	46	45
132	228	49	30
138	222	51	39
144	216	53	48
150	210	55	32
156	204	57	15
162	198	58	18
168	192	59	21
174	186	59	41
180	180	60	0

TABLE OF SUN'S AND MOON'S MAGNITUDE

Digits	*Digits of Sun*	*Digits of Moon*
1	$0\frac{1}{3}$	$0\frac{1}{2}$
2	1	$1\frac{1}{6}$
3	$1\frac{3}{4}$	$2\frac{1}{15}$
4	$2\frac{2}{3}$	$3\frac{1}{6}$
5	$3\frac{2}{3}$	$4\frac{1}{3}$
6	$4\frac{2}{3}$	$5\frac{1}{2}$
7	$5\frac{5}{6}$	$6\frac{3}{4}$
8	7	8
9	$8\frac{1}{3}$	$9\frac{1}{6}$
10	$9\frac{2}{3}$	$10\frac{1}{3}$
11	$10\frac{5}{6}$	$11\frac{1}{3}$
12	12	12

9. DETERMINATION OF LUNAR ECLIPSES

Now, with these things set out beforehand, we shall study the lunar eclipses in this way. For we set out the resulting number of degrees of the required full moon for the hour of the mid-syzygy in Alexandria, both the number of degrees from the so-called anomalistic epicycle's apogee and the number of degrees in latitude from the northern limit after determination of the addition-subtraction. We first carry the latitudinal number to the Tables of Lunar Eclipses, and, if it falls in the numbers of the first two columns, we take down separately each of those lying in the row with the latitudinal number in either table, both those in the columns of passages and those in the columns of digits. Then we carry the anomalistic number to the Table of Correction. And as many sixtieths as lie in the same row with it, just so many do we take of the difference between the digits and sixtieths taken down for either table, and we add this to the numbers gotten from the first Table. If it happens that the latitudinal number falls only

in the second Table, we set out that number of sixtieths of the digits and fractions lying in a row with them.[1]

Whatever number of digits we get from this correction, we say the obscuration of the moon's diameter at mid-eclipse embraces just so many twelfths. And to the resulting sixtieths in the same correction we always add $\frac{1}{12}$ of them for the sun's added movement. Dividing them by the moon's hourly irregular movement at that time, to whatever the division comes, just so many equatorial hours shall we have for each passage-time of the eclipse: those calculated from the fourth table giving separately the time of immersion and the time of emersion, those from the fifth the time of half the delay of occultation. The hour-positions at the beginnings and ends of incidence and restoration are immediately evident from the addition-subtraction of particular passages with respect to the middle of the delay—that is, very nearly the time of the true full moon. And immediately, if we carry the twelfths of the diameter to the shortest table, we shall also find the twelfths of the whole areas of the moon from the corresponding numbers in the third column, and likewise those of the sun's in the second column.

Now, reason proves that the time from the beginning of the eclipse to the middle is not always equal to the time from the middle to the end, because of the irregularity of the sun and moon. Equal passages are effected for that reason in unequal times. But supposing these times not unequal would work no discrepancy perceptible to sense in the appearances. For even if they were near the mean courses where the differences of the increases are greater, a passage for the number of hours of a whole eclipse would make the difference of the excess in no way sensible.

But we would think that probably we find the moon's latitudinal period shown by Hipparchus to be mistaken, since, by his hypothesis, the surplus is less than the surplus between the given eclipses but greater taken according to our calculations, although we start from the same things.

For he takes for this demonstration two lunar eclipses which occurred at an interval of 7,160 months, in both of which the quarter of the moon's diameter was eclipsed at the same passage from the ascending node: the first having been observed in the year 2 of Mardokempad, and the second in the year 37 of the Third Callippic Period. Hipparchus avails himself of the same latitudinal passage's being contained consistently in either eclipse with respect to the demonstration of return, because of the fact that the first eclipse occurred with the moon at the epicycle's apogee and the second with it at the perigee, and therefore of no anomalistic difference's having resulted as he thought. But he is mistaken first to this extent that, since an appreciable difference results from the irregularity, the regular passage is found greater than the true by an amount not

[1]The obscurity here is cleared up by Theo's *Commentary*. If the latitudinal number falls only in the second Table, we carry the anomalistic number to the Table of Correction as before; but we take the number of sixtieths, not of the difference which does not exist in this second case, but of the digits and fraction of immersion found in the second Table. In the first case, we added the result to the digits and fraction of the first Table; here we simply take that fraction of the second Table's numbers. This method is valid here because, for a latitudinal number to fall only in the second Table, the anomalistic number must lie between 150 and 210. The corresponding fraction for such numbers must lie between 55/60 and 1. Then if we let a represent the digits or immersion at the least distance and b those at the greatest, by the first method we should have $55/60\ (a - b) + b$ or $55/60\ a + 5/60\ b$; and by the second method $55/60a$. But b does not exist, therefore the two methods give the same result

equal in the two eclipses. But in the first it is by very nearly 1° and in the second $\frac{1}{8}$°, so that according to this the latitudinal period lacks $\frac{7}{8}$° of whole returns where the moon's oblique circle is 360°.

Next he did not calculate the difference occurring in the magnitudes of the obscurations because of the moon's distances. And the greatest possible of these differences occurred in the case of these eclipses because the first took place with the moon at its greatest distance, and the second with it at its least. For necessarily the obscuration of the same quarter part followed at a less distance from the ascending node at the first eclipse and at a greater at the second. We demonstrated that the difference between them amounts to $1\frac{1}{5}$° [Tables of Lunar Eclipses], so that in this case the latitudinal period added just that much to complete returns. Now the periodic return in latitude would have erred by the approximately 2° of both mistakes together if both had happened to involve a difference towards the greater or towards the less. But since, by happy chance, the one makes the return fail of completion and the other makes it exceed, the result obtained appeared greater than the return by only the $\frac{1}{3}$° of the excess of one error over the other.

10. Determination of Solar Eclipses

Now, the study of lunar eclipses would proceed soundly, therefore, only in the ways we have described, with the calculations made accurate. But next we shall make a determination of the solar eclipses—a much harder thing to do because of the moon's parallaxes.

First we look up the time of the true conjunction in Alexandria to see how many equatorial hours before or after midday it occurred. Then, if the supposed longitude of the required region is different—that is, if its meridian is not the same as that through Alexandria—we add or subtract the longitudinal difference in equatorial hours between the two meridians. And since we know how many equatorial hours before or after this the time of the true conjunction fell, we determine first the time of the apparent conjunction for the required longitude, which will be very nearly the same as the middle of the eclipse, by the method of parallaxes already explained. For we take out of the Table of Angles and the Table of Parallaxes its parallax first along the great circle drawn through the zenith and the moon's centre and proper to the latitude, to the hour-interval from the meridian, and further to the conjunctive part of the ecliptic, and in addition to the moon's distance. Always subtracting from this the sun's parallax lying in the same row, we determine within the remainder, as we have shown, by means of the angle obtained at the intersection of the ecliptic and the great circle through the zenith, the parallax computed with respect to the longitudinal passage alone. And we always add to this the difference of added parallax corresponding to the equatorial time embraced by the first, that is of the excess, obtained in the same table, between the two parallaxes—one corresponding to the first interval of the zenith, and the other to the interval with the addition of the equatorial time, again using only the parts belonging to the longitudinal parallax alone, together with as great a part of them (if it is sensible) as they are of the first parallax.[1] And to the degrees of the whole longitudinal parallax thus

[1]First the moon's longitudinal parallax at the true conjunction is computed. The problem is to find the length of time to the apparent conjunction. The moon must move the distance of this parallax plus the difference between this parallax and the different parallax it will have

combined we shall again add $\frac{1}{12}$ of them for the sun's movement, and we shall reduce the result to equatorial hours by dividing by the degrees of the moon's irregular hourly courses about the conjunction (for we have shown above how we obtain this determination). If the longitudinal parallax is eastward, we subtract the degrees we reduced to equatorial hours from the moon's degrees already determined separately for longitude, latitude, and anomaly at the true time of the conjunction, and we shall have the moon's true passages at the time of the apparent conjunction. And we shall have found the number of hours by which the apparent conjunction precedes the true. But if the longitudinal parallax is found to be westward, we shall then add the degrees to the passages already determined separately for longitude, latitude, and anomaly at the true time of the conjunction; and we shall have the hours by which the apparent conjunction will follow the true.

Now again, by the same methods, we look to see first how great a parallax the moon displays along the great circle drawn through itself and the zenith, at the apparent conjunction's interval of equatorial hours from the meridian. And subtracting from the number of degrees found the sun's parallax lying in the row with that same number, we determine within the remainder (likewise by means of the angle found for that time at the intersection of the circles) the latitudinal parallax lying along the circle at right angles to the ecliptic. The resulting number of degrees we change to the corresponding number along the oblique circle—that is, we multiply them by 12. And if the latitudinal parallax is effected to the north of the ecliptic, we shall add the product to the latitudinal passage already determined for the time of the apparent conjunction, when the moon is about the ascending node; but we subtract it likewise when it is about the descending node. And if the latitudinal parallax is effected to the south of the ecliptic, contrariwise we subtract the degrees resulting from the parallax from the degrees in latitude already determined at the time of the apparent conjunction, when the moon is about the ascending node; but we likewise add when it is about the descending node. Thus we shall have the number of apparent latitude at the time of the apparent conjunction. And carrying this to the Tables of Solar Eclipses, if it falls in the numbers of the first two columns, we shall say there will be an eclipse of the sun, whose mid-time very nearly coincides with the apparent conjunction. Now we set out the quantity of digits lying in the row with that number of apparent latitude, and also the degrees of immersion and emersion separately from each table. And we carry the number representing the degrees from the apogee of lunar anomaly to the Table of Correction. And taking as many sixtieths of each excess noted down as lie in the row with this number, we always add that much to the numbers taken from the first Table. By this correction we shall have the resulting digits, and in turn for that many twelfths of the sun's diameter there will be an obscuration, at very nearly the mid-eclipse. Again, adding to the degrees of each passage $\frac{1}{12}$ of them for the sun's movement, and changing this sum into equatorial hours by means of the moon's irregular movement, we shall have just so much time for immersion and emersion each, supposing no difference occurs for that time because of the parallaxes.

But since there is a sensible inequality about these times, because of the

when it has moved the first parallax; and so on in an unending series. The approximation is good enough after the first or second time. To this, of course, must be added also the movement of the sun during this time.

moon's parallaxes and not because of the luminaries' irregularity, according to which both the immersions and emersions predicted turn out to be greater and in general unequal to each other, we shall not know this, even if it happens to be small. Now this circumstance arises from there always being in the moon's apparent passage certain regressive appearances due to the parallaxes, as if it were not shifted by its proper movement forward to the east. For if it appears to be passing before the meridian, rising little by little and displaying a continually shorter parallax eastward, it appears to move more slowly forward to the east. And if it be passing beyond the meridian, descending now little by little and displaying a continually longer parallax westward, likewise it will again appear to move more slowly forward to the east. Now, because of this, the aforesaid times will always be greater than those taken simply. And since the difference in the excesses of the parallaxes is always greater in the passages nearer the meridian, the times of the eclipses near the meridian are fulfilled more slowly. For this reason, whenever the mid-eclipse falls at midday, then only is the time of immersion very nearly equal to that of emersion, since the regressive appearance due to the parallaxes happens then to be very nearly equal on either side. But, when it precedes midday, then the time of emersion being nearer the meridian becomes greater; and when it follows midday, then the time of immersion being nearer the meridian becomes greater.

Now, in order to make this correction of the times, we shall look, in the way we demonstrated, for the time worked out before this correction for each of the passages set out, and for the distance from the zenith at the mid-eclipse.

For example, let either time be one equatorial hour and the distance from the zenith 75°. Now we shall look in the Parallactic Table for the sixtieths of parallax corresponding to the number of 75°, with the moon, for instance, at its greatest distance, at which distance those in the third column are taken. And we find 52 sixtieths corresponding to 75°. Since both the time of immersion and the time of emersion are supposed one equatorial hour considered in mean terms, or 15° in time, therefore subtracting these from the 75° of the distance, we find, in the same column, 47 sixtieths of parallax corresponding to the remaining 60°, so that at the mid-passage near the meridian there results from the parallax a regression of 5 sixtieths. Now adding these to the 75° we find in the same column, corresponding to the sum of 90°, the 53½ sixtieths of the whole parallax, so that here the regression of the passage near the horizon comes to 1½ sixtieths. Now taking from these differences the correspondences in longitude, and reducing each in turn to a part of an equatorial hour by means of the moon's irregular movement, as shown, we shall add the result from each, properly to each of the times taken in mean and simple terms, the greater to the time at the passage nearer the meridian and the less to the time at the passage nearer the horizon. And it is clear that the difference of these times was 3½ sixtieths or very nearly ⅛ of one equatorial hour, for the moon will move just that many sixtieths of mean movement in that much time. There is then left the easy task, if we wish, of reducing the equatorial hours for each interval to the particular seasonal hours in the way we have already shown above.

11. On the Directions in the Eclipses

The examination of the directions of the obscurations comes next. And this study is established both from the direction of the obscurations with respect to

the ecliptic, and from the inclination of the ecliptic to the horizon. Either of these in each of the ecliptic times would suffer the greatest indefinite variation at the shifts if one wished to work out the inclinations which would take place during the whole time; but such prediction is neither necessary nor useful. For the ecliptic's configuration relative to the horizon is considered from the position of its rising and setting points at the horizon; and necessarily throughout the time of the eclipse, as different parts of the ecliptic become continuously rising and setting, the sections of the horizon bounded by them become continuously different. Likewise the inclination of the obscurations relative to the ecliptic is considered along the great circle drawn through both centres, the moon's and the sun's or shadow's; and again necessarily, throughout the passage of the moon's centre at the time of the eclipse, the circle drawn through both centres also takes a position continuously other relative to the ecliptic and makes the angles contained by their intersection continuously unequal.

Now, since this examination will be sufficient if it is taken for only the obscurations having some significance and more generally for the arcs considered relative to the horizon, it will be possible for those appraising the situation with their eyes to estimate immediately the particular directions by attending to both inclinations. For, as we said, in these matters a general judgment is sufficient. Yet, not to pass over this subject entirely, we shall try to expound ways as easy as possible for getting at this.

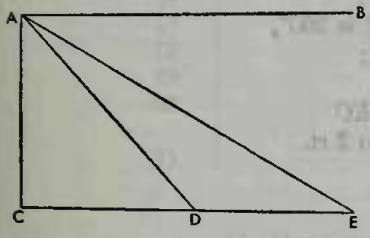
Of the obscurations, we have chosen as significant markings that of the first part eclipsed, which occurs at the beginning of the whole time of eclipse; that of the last part eclipsed, which occurs at the beginning of the time of delay; that of the greatest part eclipsed, which occurs at the mid-eclipse without the delay; that of the first part restored, which occurs at the end of the whole time of delay; and that of the last part restored which occurs at the end of the whole time of eclipse. And of the directions we have chosen again, as the more reasonable and expressible, those defined by the meridian and by the risings and settings of the ecliptic's equinoxial and tropic points. For the principle of directions could be differently understood by many people at many times, and could, if one wished, be designated by the given angles of the horizon. Now of the intersections of the meridian with the horizon, let us understand by ἄρκτοι the northern, and by μεσημβρίος the southern; and of the rising and setting intersections, let us understand by equatorial rising and setting those intersections made with the horizon by the beginning of the Ram and of the Balance and which are always an equal quadrant from those produced at the meridian; by summer rising and setting, those made by the beginning of the Crab; and, by winter rising and setting, those made by the beginning of the Goat. These intervals are different according to latitude, and the determination of the directions is sufficient when they have been shown to be either at or between these limits.

Therefore, to have every configuration of the ecliptic with the horizon, we calculated (in the first part of the Composition as already shown) the intervals produced on the horizon at risings and settings by the beginning of each one of the dodecatemories either side of the intersections produced by the equator and for each latitude from the Borysthenes to Meroë for which we laid out the angles. And, for an easy understanding, we have drawn, instead of a table, 8 concentric circles conceived in the horizon's plane and containing the intervals and names of the 7 latitudes. Then, drawing two straight lines through all the

circles at right angles to each other—the horizontal one as the common section of the planes of the horizon and equator, and the vertical one as the common section of the planes of the horizon and meridian—we have designated the extremities of the horizontal line at the outside circles as equatorial rising and equatorial setting; and of the vertical line as north and south. Likewise we have drawn straight lines at equal intervals on either side of the equatorial line and again through all the circles, and we have placed on the lines within the seven intervals the distances, found for each latitude, of the tropic points from the equator along the horizon, a quadrant being 90°. On those at the inner extremities of the circles we have placed, to the south, winter rising and winter setting; and to the north, summer rising and summer setting. For the intervening dodecatemories (further inserting, within each of the four intervals, two other lines) we have placed on these also the distances along the horizon from the equator for the particular dodecatemories, with the name of each written at the outside circle. And about the meridian line we have indicated the names of the parallels, the hourly magnitudes, and the heights of the pole, making the list in order of northerliness from the greater and encompassing circle.

In order that we may also have set out the apparent directions of the obscurations with respect to the ecliptic (that is, the angles produced at each of the aforesaid markings by the intersection of the ecliptic and the great circle through both the centres indicated), we calculated them for each of the moon's passages for differences of 1 digit of obscuration, and only for those at the mean distance as being sufficient. And it is supposed the arcs of the ecliptic and of the moon's oblique circle are sensibly parallel in the obscurations.

For, again as an example, let there be the straight line AB in place of the ecliptic's arc, and on it let A be supposed the centre of the sun or shadow. And let there be the straight line CDE in place of the arc of the moon's oblique circle. And let C be the point where the moon's centre lies at mid-eclipse; D the point where again its centre will be when the whole moon is first eclipsed or first begins to be restored (that is, when it touches the shadow's circle from within); and E the point where its centre lies when either the sun or moon first begins to be eclipsed or finishes being restored—that is, when the circles touch each other from without. And let AC, AD, and AE be joined.



Now, it is evident that the angles BAC and ACE embracing the mid-eclipses are sensibly right, that angle BAE contains that at the first part eclipsed and the last part restored, and that angle BAD contains that at the last part eclipsed and the first part restored. It is also immediately clear that the line AE embraces the radii of both circles, and AD their difference.

Now as an example, let the eclipse be supposed one in which at the middle the half of the sun's diameter will be obscured. And let A be the sun's centre, so that, by supposing the moon at its mean distance,

$$AE = 32^{p}20';$$

and, since it is less by half the sun's diameter,

$$AC = 16^{p}40'.$$

Since, then, according to the given magnitude of the obscuration,

$$AC = 16^{p}40'$$

where

$$\text{hypt. } AE = 32^{p}20',$$

therefore

$$AC = 61^{p}51'$$

where

$$\text{hypt. } AE = 120^{p};$$

and, where the circle about right triangle ACE is 360°,

$$\text{arc } AC = 62°2'.$$

And so also

$$\begin{aligned}\text{angle } BAE &= \text{angle } AEC \\ &= 62°2' \text{ to 2 rt.} \\ &= 31°1'.\end{aligned}$$

Again, for the lunar eclipses, let A be the shadow's centre, so that, since the moon's mean's distance is likewise supposed, always

$$AE = 60^{p},$$

and likewise

$$AD = 26^{p}40'.$$

And let the moon be eclipsed to 18 digits, so that again AC is less than AD by half the diameter, and by subtraction

$$AC = 10^{p}.$$

Since, then,

$$AC = 20^{p}$$

where

$$\text{hypt. } AE = 120^{p},$$

and, where the circle about right triangle ACE is 360°,

$$\text{arc } AC = 19°12';$$

therefore

$$\begin{aligned}\text{angle } BAE &= \text{angle } AEC \\ &= 19°12' \text{ to 2 rt.} \\ &= 9°36'.\end{aligned}$$

And, likewise, since

$$AC = 45^{p}$$

where

$$\text{hypt. } AD = 120^{p},$$

and, where the circle about right triangle ACD is 360°,

$$\text{arc } AC = 44°2';$$

therefore

$$\begin{aligned}\text{angle } BAD &= \text{angle } ADC \\ &= 44°2' \text{ to 2 rt.} \\ &= 22°1'.\end{aligned}$$

Then, in the same way, in the case of the other digits taking the sizes of the angles less than a right angle which is 90° (and which is also what a quadrant of the horizon is taken to be), we arranged a table in 22 rows and 4 columns. The first column contains the digits of the diameter's obscuration found at the mid-eclipse; the second contains the angles produced in solar eclipses at the time of the first part eclipsed and the last part restored; the third contains the angles produced in lunar eclipses at the time of the first part eclipsed and the last part

restored; and the fourth contains the angles produced, again in lunar eclipses at the time of the last part eclipsed and the first part restored. And here are the diagrams of the table and of the circles:

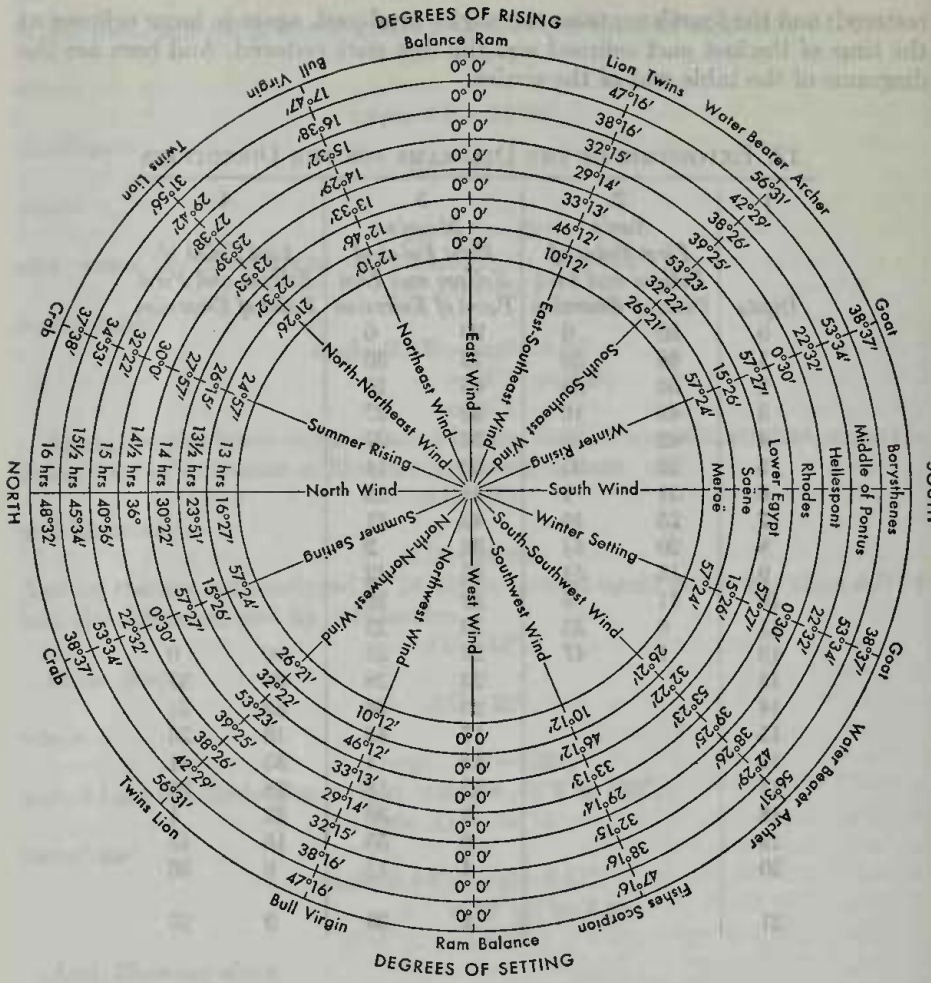
12. Exposition of the Diagrams for the Directions

1	2		3		4	
	Sun's First Point of Eclipse and Last Point of Emersion		*Moon's First Point of Eclipse and Last Point of Emersion*		*Last Point of Eclipse and First Point of Emersion*	
Digits						
0	90	0	90	0		
1	66	50	72	30		
2	56	59	65	10		
3	49	16	59	27		
4	42	36	54	27		
5	36	35	50	14		
6	31	1	46	15		
7	25	46	42	31		
8	20	44	39	2		
9	15	51	35	42		
10	11	6	32	29		
11	6	25	29	23		
12	1	47	26	23	90	0
13			23	28	63	37
14			20	36	52	24
15			17	48	43	26
16			15	1	35	41
17			12	18	28	38
18			9	36	22	1
19			6	55	15	43
20			4	15	9	36
21			1	36	3	35

13. Determination of Directions

We now have in our hands, already determined in the way we showed, the times of each of the given markings; from the times, of course, the ecliptic's rising and setting parts at those times; and from the figure, their positions on the horizon. And whenever the moon's centre (either the apparent as for the solar eclipses, or the true as for the lunar) is on the ecliptic, we shall have, from the horizon-position of the part then setting, the direction of the sun's first part eclipsed and, further, the direction at the moon's last part eclipsed and restored. Moreover, from the horizon-position of the part then rising, we shall have the direction of the sun's last part restored and, further, the direction of the moon's first part eclipsed and restored.

But whenever the moon's centre is not on the ecliptic, we take the numbers of the angles lying in the same row and proper to the number of digits, counting them from the common sections of the horizon and ecliptic. If the moon's centre is north of the ecliptic, for the sun's first part eclipsed and for the moon's last



part eclipsed, we count them northward from the setting intersection; for the sun's last part restored and the moon's first part restored, northward from the rising intersection; and again, for the moon's first part eclipsed, southward from the rising intersection, and, for the moon's last part restored, southward from the setting intersection. And if the moon's centre is south of the ecliptic, for the sun's first part eclipsed and the moon's last part eclipsed, we count them southward from the setting intersection; for the sun's last part restored and the moon's first part restored, southward from the rising intersection; and again, for the moon's first part eclipsed, northward from the rising intersection, and, for the moon's last part restored, northward from the setting intersection. We shall thus have, established from this correction, that part of the horizon in general to which are directed the parts of the luminaries receiving the first and last markings of the eclipses and restorations.

BOOK SEVEN

1. That the Fixed Stars Always Keep the Same Position with Respect to Each Other

So far in this treatise, Syrus, we have gone through the accidents relative to the right and oblique sphere, and then those relative to the hypotheses of the movements of the sun and moon and of the configurations considered according to those movements. Now, for the theory to follow, we shall begin a discussion of the stars—and first, naturally, of those called fixed.

This must first of all be understood: that, as far as this term is concerned, because the stars always appear to keep similar figures and equal distances with respect to each other, we would do well to call them "fixed." But, because the whole sphere of them, on which they hang (as it were) and revolve, appears itself to move eastward toward the prime movement's rising with a proper ordered motion, it would not be right to call that sphere "fixed." For we find both of these views to be true, judging from the results of a great stretch of time. And Hipparchus first, from the appearances he had, suspected both of these things; but for the most part conjectured rather than affirmed them because of the very few observations on the fixed stars that had been made before him, which were only those recorded by Aristyllus and Timocharis and which were uncertain and not worked over. And we, from a comparison of what is observed now with what was observed then, have come to the same conclusion, but more sure because the examination has taken place over a longer time, and because Hipparchus' own laborious records concerning the fixed stars (with respect to which we, for the most part, made the comparisons) have been furnished us.

If the present appearances agree with the records of Hipparchus, then it would be clear to anyone wishing to pursue the investigation and truthfully examine, that no change has hitherto taken place in their position with respect to each other, but the configurations observed by Hipparchus are seen to be absolutely the same now. This is true not only of those inside the zodiac with respect to each other or those outside (which would be the case if, as Hipparchus first supposed, only those stars around the zodiac made an eastward change in the order of the signs) but also of those inside with respect to those outside and far off.

And we shall now set out here, for an easy test, a few of the records which can be most readily grasped and which can bring to view the complete comparison by which it is shown that the observed configurations contained by those outside are the same with respect to each other and those inside the zodiac.

In the case of the stars in the Crab, he writes that the star in the southern claw of the Crab, the bright star west of this and of the head of the Water Snake [Hydra], and the bright star among those in the Little Dog [Procyon] are very nearly on the same straight line. For the middle one lies to the north and east

of the straight line between the ends of 1½ digit, and the distances between are equal.

In the case of the stars in the Lion, he writes that, of the four in the Lion's head, the eastern two and the one at the beginning of the Water Snake's neck are on the same straight line; and again, that the straight line drawn through the Lion's tail and the star at the tip of the Bear's tail leaves the bright star under the Bear's tail 1 digit to the west; likewise that the straight line through the star under the Bear's tail and through the Lion's tail joins the western stars in Berenice's Hair.

In the case of the stars in the Virgin, he says that, between the Virgin's northern foot and the Ploughman's [Boötes] right foot, lie two stars of which the southern and bright one (like that in the Ploughman's foot) lies west of the straight line through the feet, and the northern and half bright one is on a straight line with the feet. He further says that, west of the half bright one of these two, there are two bright ones making with it an isosceles triangle whose vertex is the half bright one; and they are on a straight line with Arcturus and the Virgin's southern foot. Again, he says that between Spica and the second from the end of the Water Snake's tail lie three stars in a straight line with one another; and the middle one of these is in a straight line with Spica and the second star from the end of the Water Snake's tail.

In the case of the stars in the Balance [*i.e.*, the claws of the Scorpion], he says that very nearly in a straight line with the bright stars of the Balance, to the north, is a bright triple star. For on each side of it lies a small one.

In the case of the stars of the Scorpion, he says that the straight line drawn through the eastern one in the center of the Scorpion and through the right knee of Serpentarius bisects the distance between the western stars in the right foot of Serpentarius. He also says that the fifth and seventh joints are in a straight line with the bright star in the middle of the Censer [Ara]; moreover, that the more northern of those in the Censer's base is almost in a straight line with the fifth joint and the star in the middle of the Censer, being nearly equidistant from both.

In the case of the stars in the Archer, he says that in the Circle [Southern Crown] under the Archer toward the east lie two bright stars about three cubits from each other; moreover, that the more southern and brighter, in the Archer's foot, is very nearly in a straight line with the middle star of those three bright ones in the Circle lying eastward and with the eastern star of the opposite bright ones in the Quadrilateral; and the two distances between them are equal. The northern one of them is off the straight line to the east, but is in a straight line with the bright and opposite ones in the Quadrilateral.

In the case of the stars in the Water Bearer, he says that the two consecutive stars in the Horse's head and the eastern shoulder of the Water Bearer are very nearly in a straight line, to which is parallel the straight line drawn from the western shoulder of the Water Bearer to the star in the Horse's jaws. And again he says that the Water Bearer's western shoulder and the bright star of the two in the Horse's [Pegasus] throat and the star in the horse's navel are in a straight line, and the distances equal; and that the straight line through the Horse's muzzle and through the eastern star of the four in the Urn very nearly bisects at right angles the straight line between the two consecutive stars in the Horse's head.

In the case of the stars in the Fishes, he says that the star in the mouth of the southern Fish, the bright star in the Horse's shoulders, and the bright star in his breast are in a straight line.

In the case of the stars in the Ram, he says that the western star of the Triangle's base lies one digit east of the straight line drawn through the star in the Ram's jaw and the left foot of Andromeda; and also that the western stars in the Ram's head and the midpoint of the Triangle's base are in a straight line.

In the case of the stars in the Bull, he says that the eastern stars of the Hyades and the sixth star, counting from the south, of the hide which Orion holds in his left hand are in a straight line; and that the straight line drawn through the Bull's western eye and the seventh from the south of those in the hide leaves the bright star of the Hyades a digit to the north.

In the case of the stars in the Twins, he says that in a straight line with the Twin's heads there is a star east of the eastern head by a distance three times that between the heads, and that this same star is in a straight line with the more southern of those about the little nebula.

Now, in these and similar configurations affording a comparison for nearly the whole sphere we see nothing changed up to now. And such a change would have been certainly sensible in the intervening two hundred and sixty years if only those stars about the ecliptic had made a movement to the east.

In order that those after us may make a comparison over a longer time from more configurations of the same kind, beginning with the stars in the Ram, we shall also set out those configurations most easily grasped of all those not given in the older record but obtained by us.

Now, of the three stars in the Ram's head the more northern two, the bright one in the southern knee of Perseus and the star called Capella are in a straight line. Again the straight line joining Capella and the bright star of the Hyades [Aldebaran] leaves a little to the east the star in the Charioteer's [Auriga] western foot. And Capella, and the star common to the Charioteer's eastern foot and the northern end of the Bull's horns, and the star in Orion's western shoulder are in a straight line. Again, the bright stars in the Twins' heads and the bright star in the Water Snake's throat are very nearly in a straight line. Yet again, the two consecutive stars in the Bear's forefoot, the star at the top of the Scorpion's northern claw, and the more southern of the Asses are in a straight line. Likewise the southern Ass, the bright star in the Little Dog [Procyon], and the bright star between them, west of the Water Snake's head, are very nearly in a straight line. Furthermore, the straight line drawn from the middle star of the bright ones in the Lion's throat to the bright star in the Water Snake leaves the star of the Lion's heart a little to the east. The straight line from the bright star in the Lion's loin to the bright star in the back of the Bear's thigh, which is south of the eastern side of the quadrilateral, leaves a little to the west the two consecutive stars in the Bear's eastern foot. Again, the straight line from the star in the back of the Virgin's thigh to the second star from the end of the Water Snake's tail leaves a bit to the west the star called Spica. The straight line from Spica to the star in the Ploughman's head leaves Arcturus a bit to the east. Spica and the stars on the Raven's wings are in a straight line. Spica, the star in the back of the Virgin's thigh, and the bright northern star in the Ploughman's western calf are in a straight line. Again, the bright stars in the Balance and the star at the tip of the Water Snake's tail are very nearly in a straight

line. The bright star in the southern Claw, Arcturus, and the middle star of the three in the Big Bear's tail are in a straight line. The bright star in the northern Claw, Arcturus, and the star in the back of the Bear's thigh are in a straight line. Again the star on the eastern shin of Serpentarius, the star in the Scorpion's fifth joint, and the western star of the two consecutive ones in its centre are in a straight line. The western star of the three in the Scorpion's breast and the two in Serpentarius' knees makes an isosceles triangle, whose vertex is the western star of the three in the breast. Again, the star on the Archer's southern and forward ankle and of second magnitude, the star on the barb, and the star in Serpentarius' eastern knee are in a straight line. The star in the knee of the Archer's same foot, near the Crown, the star on the barb, and the star in Serpentarius' western knee are in a straight line. Again, the straight line joined from the bright star [Vega] in the Lyre to the star in the Goat's [Capricorn] horns leaves a little to the east the bright star in the Eagle [Aquila]. The straight line from the bright star in the Eagle to the star of the first magnitude in the mouth of the southern Fish nearly bisects the distance between the two bright stars in the Goat's tail. Moreover, the straight line from the star of the first magnitude in the mouth of the southern Fish to the star in the Horse's muzzle leaves a little to the east the bright star in the Water Bearer's eastern shoulder. Again, the stars in the mouths of the two southern Fishes and the western stars of the square in the Horse are in a straight line.

And finally, if one should compare these configurations with those of the constellations on Hipparchus' solid sphere, he would find the positions gotten by the observations of that time and recorded on the sphere to be very nearly the same as those gotten now.

2. That the Sphere of the Fixed Stars Makes a Movement Eastward on the Ecliptic

By means of the foregoing and similar comparisons we can show that the relation and movement of the so-called fixed stars is one and the same. And that their sphere makes its own proper movement in the direction contrary to the motion of the whole—that is, eastward in the order of the signs from the great circle drawn through both sets of poles, those of the equator and those of the ecliptic—becomes evident to us especially through the fact that the same stars do not keep the same distances formerly and in our time with respect to the tropic and equinoctial points, but are always found in later times to be a greater distance eastward than the first from those same points.

For Hipparchus, in his treatise *On the Precession of the Tropic and Equinoctial Points,* compares the lunar eclipses from those carefully observed by him and from those observed earlier by Timocharis, and he calculates that Spica is 6° west of the autumn equinox in his time, but very nearly 8° in the time of Timocharis. Finally he says this: "If then, for example, Spica was formerly 8° in longitude west of the fall equinox, and is now 6° west" and so forth. And similarly, in the case of the other planets of which he made a comparison, he shows just about the same eastward shift accomplished. And we, comparing the apparent distances of the fixed stars relative to the tropic and equinoctial points in our time with those observed and recorded by Hipparchus, find their shift eastward along the ecliptic the same proportionately to the foregoing one. This research we have made by means of the instrument we prepared for the obser-

vation of the moon's particular elongations from the sun, placing one circle of the astrolabe at the moon's apparent passage falling at the hour of the observation, and carrying the other circle to the star being sighted so that the moon and star may be sighted together in their proper places. And thus, from its distance to the moon, we get the position of each one of the bright stars.

For one example, in the year 2 of Antonine, Egyptianwise Pharmouthi 9, as the sun was about to set in Alexandria and the last section of the Bull was culminating (that is, $5\frac{1}{2}$ equatorial hours after noon of the ninth) we observed the apparent moon to be $92\frac{1}{8}°$ from the sun which was sighted about 3° within the Fishes. And a half hour after the sun had set, with the first quarter of the Twins culminating and the apparent moon sighted in the same position as before, the star in the Lion's heart [Regulus] appeared, with the aid of the other circle of the astrolabe, to be $57\frac{1}{6}°$ from the moon eastward along the ecliptic. But in the first observation the sun's true position was very nearly $3\frac{1}{20}°$ within the Fishes, so that the apparent moon was then a distance of $92\frac{1}{8}°$ eastward or very nearly $5\frac{1}{6}°$ within the Twins, where, according to our hypotheses, it should have been. A half hour later, the moon must have moved eastward very nearly $\frac{1}{4}°$ and have displayed a westward parallax of very nearly $\frac{1}{12}°$ at the first position. Therefore the half hour after the apparent moon was $5\frac{1}{3}°$ within the Twins, so that the star in the heart (since it appeared to be $57\frac{1}{6}°$ eastward from it) was situated $2\frac{1}{2}°$ within the Lion, and was a distance of $32\frac{1}{2}°$ from the summer tropic.

But in the year 50 of the Third Callippic Period, as Hipparchus observes and records, it [Regulus] was then $(29+\frac{1}{2}+\frac{1}{3})°$ eastward from the same summer tropic point. Therefore, the star in the Lion's heart had shifted eastward along the ecliptic $2\frac{2}{3}°$ in the years from Hipparchus' observation to the beginning of Antonine's reign when we observed most of the passages of the fixed stars, a period which comes to about 265 years. And so from this it is found that there is an eastward shift of 1° in very nearly 100 years, as Hipparchus seems to have guessed when he says his treatise *On the Magnitude of the Solar Year:* "If for this cause the tropics and equinoxes shifted westward not less than $\frac{1}{100}°$ in a year, then they would have to shift not less than 3° in 300 years." And having sighted Spica and the brightest stars about the ecliptic in their distance from the moon in this same way, then finally from these more accessible ones the other also, we find their distances with respect to each other very nearly the same as those observed by Hipparchus but their distances with respect to the tropic and equinoctial points to have shifted eastward very nearly $2\frac{1}{2}°$ compared to the record of Hipparchus.

3. That the Eastward Movement of the Sphere of the Fixed Stars Takes Place About the Pole of the Ecliptic[1]

It has now become clear to us that the sphere of the fixed stars makes an eastward motion of very nearly this amount. And the next thing is to find out the direction of this movement—that is, whether it takes place about the poles of equator or of the ecliptic. And this would become evident to us from the

[1]The history of the interpretation of the appearance called the precession of the equinoxes is one of the most beautiful in the development of astronomy and physics.

In the Copernican theory, it is explained by having the restoration of the annual declination of the earth's axis with respect to the plane through the earth and the sun anticipate the

longitudinal shift. For the great circles drawn through the poles of one of these cut off unequal arcs on the other unless the difference due to this cause should be altogether indistinguishable to the senses because of such a small shift's taking place in this amount of time. But it would be very easily gotten from their latitudinal shift then and now. For with respect to whichever of the circles, the equator or ecliptic, they always appear to keep their latitudinal distances, it is clear that the movement of the sphere will be effected about that circle.

Now Hipparchus also agrees to the movement's being about the ecliptic's poles. For in his treatise *On the Precession of the Tropic and Equinoctial Points* he in turn concludes, from the things observed by Timocharis and by himself, that Spica keeps unchanged; not relative to the equator, but relative to the ecliptic, the magnitude of its latitudinal distance of 2° south of the ecliptic. Therefore in his treatise *On the Magnitude of the Solar Year* he supposes only the movement about the poles of the ecliptic. And yet he still has doubts, as he himself says, because the trustworthy observations of Timocharis have not been taken in a general enough way and because the intervening time has not been long enough for a conclusive observation.

However, since we find over a longer time and for nearly all the fixed stars just this thing, we can reasonably suppose their movement about the poles of the ecliptic to be fairly sure. For, observing the latitudinal distance of each on the great circle drawn through its poles, we find nearly the same distances contained as were recorded and brought together by Hipparchus, or else disagreeing so little that the discrepancy could be neglected as the result of the observations themselves. But in the case of the distances observed with respect to the equator along the great circle drawn through its poles, neither do we find those gotten by us agreeing with those recorded in the same way by Hipparchus, nor these with those gotten still earlier by Timocharis. And from these themselves we find even more thoroughly confirmed the identity of the latitude with respect to the ecliptic. For the stars in the hemisphere from the winter tropic through the spring equinox to the summer tropic are always found farther north than the earlier distance with respect to the equator; but those in the opposite hemisphere farther south, those near the equinoctial points having greater differences, and those near the tropic points having less—almost in proportion as the ecliptic's eastward sections in their longitudinal progression get farther north or farther south of the equator.

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restoration of the earth's centre in its revolution about the sun. Copernicus thought there was an irregularity in the rate of the precession. Neither Ptolemy nor the moderns find this appearance.

In Newton's *Principles*, the precession is deduced by an extraordinary analogy with the three body problem in the famous Proposition 66 of Book I, particularly in the first eleven corollaries and in Corollaries 20, 21, and 22.

It is in this last corollary also that it is brought out that from Newtonian principles it must follow that the rotation of a body on its axis is a regular motion and no centripetal force will change the velocity of this rotation. Thus the Ptolemaic assumption of the regularity of the two motions which made up the apparent motion of the fixed stars is, at least for the first one, in effect the same as the Newtonian. The astronomers before Hipparchus had simply considered the revolution of the fixed stars as regular.

The choice of regular motion is one that is arbitrary. But the very fact that one can and must choose a regular motion, and that the apparent motions are so ambiguous as to allow any one of them to be chosen as such is the sign for St. Augustine that points to a non-sensible theory of time and of memory. See the *Confessions*, Books X and XI.

In order to state this more clearly in a few easily understood cases, we shall set out, for either hemisphere, their recorded latitudinal distances from the equator along the great circle drawn through its poles, those gotten by Timocharis, those by Hipparchus, and those gotten in the same way by ourselves.

Now, Timocharis records the bright star in the Eagle [Altair] as being $5\frac{4}{5}°$ north of the equator, and Hipparchus the same; but we find it to be $(5+\frac{1}{2}+\frac{1}{3})°$. And Timocharis records the centre of the Pleiades as being $14\frac{1}{2}°$ north of the equator, Hipparchus $15\frac{1}{6}°$, and we find it to be $16\frac{1}{4}°$. Again, Timocharis records the bright star of the Hyades [Aldebaran] as being $(8+\frac{1}{2}+\frac{1}{4})°$ north of the equator, and Hipparchus $(9+\frac{1}{2}+\frac{1}{4})°$, and we find it to be 11°. Aristyllus records the brightest star in the Charioteer, called the Goat [Capella] as being 40° north of the equator, Hipparchus $40\frac{2}{5}°$, and we find it to be $41\frac{1}{6}°$. Timocharis records the star in the western shoulder of Orion as being $1\frac{1}{5}°$ north of the equator, Hipparchus $1\frac{1}{5}°$, and we find it to be $2\frac{1}{2}°$. Timocharis records the star in the eastern shoulder of Orion [Betelgeuse] as being $(3+\frac{1}{2}+\frac{1}{3})°$ north of the equator, Hipparchus $4\frac{1}{3}°$, and we find it to be $5\frac{1}{4}°$. Timocharis records the bright star in the Dog's mouth [Sirius] as being $16\frac{1}{3}°$ south of the equator, Hipparchus 16°, and we find it to be $(15+\frac{1}{2}+\frac{1}{4})°$. Aristyllus records the western star of the bright ones in the Twins' heads [Castor] to be 33° north of the equator, Hipparchus $33\frac{1}{6}°$, and we find it to be $33\frac{2}{5}°$. And Aristyllus records the eastern one [Pollux] as being 30° north of the equator, Hipparchus the same, and we find it to be $30\frac{1}{6}°$.

Of all these distances intercepted longitudinally within the hemisphere containing the spring equinox, the later ones have become all more northern in latitude with respect to the equator than the earlier ones, those near the tropic sections by a very small amount and those near the equinoctial sections by a considerable amount. This is consequent upon the precession eastward about the ecliptic's poles, because the eastward sections of this semicircle always get more northern than the western, those near the equinoctial points with greater differences and those near the tropic points with smaller.

In the opposite hemisphere, Timocharis records the star in the Lion's heart [Regulus] as being $21\frac{1}{3}°$ north of the equator, Hipparchus $20\frac{2}{3}°$, and we find it to be $(19+\frac{1}{2}+\frac{1}{3})°$. Timocharis records Spica as being $1\frac{2}{5}°$ north of the equator, Hipparchus only $\frac{3}{5}°$, and we find it to be $\frac{1}{2}°$ south of the equator. Aristyllus records that of the three stars in the Big Bear's tail the one at the tip is $61\frac{1}{2}°$ north of the equator, Hipparchus $(60+\frac{1}{2}+\frac{1}{4})°$, and we find it to be $59\frac{2}{3}°$. Aristyllus records the second from the end, in the middle of the tail, as being $67\frac{1}{4}°$ north of the equator, Hipparchus $66\frac{1}{2}°$, and we find it to be 65°. Aristyllus records the third from the end, at the beginning of the tail, as being $68\frac{1}{2}°$ north of the equator, Hipparchus $67\frac{3}{5}°$, and we find it to be $66\frac{1}{4}°$. Timocharis records Arcturus as being $31\frac{1}{2}°$ north of the equator, Hipparchus 31°, and we find it to be $(29+\frac{1}{2}+\frac{1}{3})°$. Timocharis records that, of the bright stars in the Scorpion's claws, the one in the southern tip is 5° south of the equator, Hipparchus $5\frac{3}{5}°$, and we find it to be $7\frac{1}{6}°$. Timocharis records the star in the tip of the northern claw as being $1\frac{1}{5}°$ north of the equator, Hipparchus only $\frac{2}{5}°$, and we find it to be 1° south of the equator. And Timocharis records the bright star in the Scorpion's breast, called Antares, as being $18\frac{1}{3}°$ south of the equator, Hipparchus 19°, and we find it to be $20\frac{1}{4}°$.

Now, of all these passages, in the opposite way, the later have become propor-

tionately more southern in latitude with respect to the equator than the earlier. Therefore it could be concluded that the eastward longitudinal shift of the sphere of the fixed stars is 1°, as we said before, in very nearly 100 years, and 2⅔° in the 265 years between the observation of Hipparchus and our own. And this conclusion is reached through the latitudinal difference found for those stars about the equinoctial points.

For the centre star of the Pleiades, found by Hipparchus to be 15⅙° north of the equator and by us 16¼°, has become more northern in the intervening time by 1 1/12°. And the 2⅔° on the ecliptic for stars at the end of the Ram, resulting from the eastward longitudinal shift over the same amount of time, gives just that difference in latitude with respect to the equator. Again, the star called Goat [Capella], found by Hipparchus to be 40⅔° north of the equator and by us 41⅙°, has become more northern by ⅘°, the 2⅔° on the ecliptic for stars at the middle of the Bull again giving just that difference in latitude with respect to the equator. In the western shoulder of Orion, the star found by Hipparchus to be 1⅘° north of the equator and by us 2½° has become more northern by very nearly ⅔°, the 2⅔° on the ecliptic for stars two-thirds within the Bull giving just that difference in latitude with respect to the equator.

Likewise for the opposite hemisphere, Spica, found by Hipparchus to be ⅜° north of the equator and by us ½° south, has become more southern by 1 1/10°. Again, the 2⅔° on the ecliptic for stars at the end of the Virgin give just that difference in latitude with respect to the equator. The star at the tip of the Big Bear's tail, found by Hipparchus to be 60¾° north of the equator and by us 59⅔°, has become more southern by 1 1/12°; and the 2⅔° on the ecliptic for stars at the beginning of the dodecatemory of the Balance give just this difference in latitude with respect to the equator. And Arcturus, found by Hipparchus to be 31° north of the equator and by us (29+½+⅓)°, has become more southern by 1⅙°, the 2⅔° on the ecliptic for stars at the beginning of the Balance likewise giving very nearly this difference in latitude with respect to the equator.

And what is required will be even clearer to us from the following observations.

For Timocharis, having observed these things in Alexandria, records that in the year 47 of the first Callippic Period of 76 years (Anthesterion 8, Egyptianwise Athyr 29), at the end of the third hour, the southern half of the moon appeared to have advanced eastward one-third or one-half the way through the Pleiades, in true position. And this date is the year 465 of Nabonassar, Egyptianwise Athyr 29-30, 3 seasonal hours before midnight and 3⅓ equatorial hours. For the sun was 7° within the Water Bearer, and with respect to regular solar days the amount of time before midnight adds up to that many hours. According to hypotheses already demonstrated by us, the moon at that hour had its true position 0°20′ within the Bull—that is, it was 30°20′ from the spring equinox and 3°45′ north of the ecliptic. In Alexandria it appeared to be 29°20′ within the Ram in longitude, and 3°35′ north of the ecliptic, since the second third of the Twins was culminating. Therefore, the eastern limit of the Pleiades was then very nearly 29½° east of the spring equinox, since the moon's centre was still west of it and very nearly 3⅔° north of the ecliptic. For this limit was, in turn, a little more north than the moon's centre.

And Agrippa who observed in Bithynia records that, in the year 12 of Domitian (in their fashion Metroius 7), at the beginning of the third hour of the

night, the moon occulted with its southern horn the southeastern part of the Pleiades. This date is the year 840 of Nabonassar, Egyptianwise Tybi 2-3, 4 seasonal hours before midnight, and 5 equatorial hours. Because the sun was 6° within the Archer. Therefore, with respect to the meridian of Alexandria, the observation was 5⅓ equatorial hours before midnight, and relative to regular solar days 5¾ hours, at which time the true position of the moon's centre was 3°7′ within the Bull and (4+½+⅓)° north of the ecliptic. In Bithynia it appeared to be 3°15′ in longitude within the Bull and 4° north of the ecliptic because the point two-thirds within the Fishes was culminating. Therefore the eastern part of the Pleiades was then 33¼° east of the spring equinox and 3⅔° north of the ecliptic. So it is evident that, in latitude, the eastern part of the Pleiades was then and now 3⅔° north of the ecliptic along the great circle drawn through its poles, and, in longitude, has moved 3°45′ eastward from the spring equinox. Because at the first observation it was 29½° from it, and at the second 33¼°; and the time between the two observations embraces 365 years. And, therefore, the eastern part of the Pleiades has moved eastward 1° in 100 years.

Again having observed in Alexandria, Timocharis records that in the year 36 of the First Callippic Period (Elaphebolion 15 or Tybi 5), at the beginning of the third hour, the moon reached Spica with the midpoint of its disc toward the equatorial rising, and Spica went behind it cutting off exactly one-third of the diameter towards the north. And this date is the year 454 of Nabonassar, Egyptianwise Tybi 5-6, very nearly 4 seasonal and equatorial hours before midnight, because the sun was about 15° within the Fishes. With respect to regular solar days, the determination comes to just about that many hours before. And at that hour the true position of the moon's centre was again 21°21′ in longitude within the Virgin—that is, it was 81°21′ east of the summer tropic and (1+½+⅓)° south of the ecliptic. But it appeared to be 82°12′ in longitude from the summer tropic and very nearly 2° south of the ecliptic. For the middle of the Crab was culminating. And therefore, for these reasons, Spica was at that time 82⅓° from the summer tropic and 2° south of the ecliptic.

Timocharis likewise says that in the year 48 of the same period (on the sixth of Pyanepsion waning, or Thoth 7), at half-past the ninth hour, the moon having risen above the horizon, Spica appeared to be exactly touching the northern part of it. And this date is the year 466 of Nabonassar, Egyptianwise Thoth 7-8, as he himself says, 3½ seasonal hours after midnight, and very nearly 3⅛ equatorial hours, since the sun was near the middle of the Scorpion. Consequently, it was 2½ hours after midnight. For at just that many equatorial hours after midnight the point 22½° within the Twins was culminating, and about the same point within the Virgin was rising. And the moon was in this position when it rose, as he says. With respect to regular solar days we find it to be only 2 equatorial hours after midnight. Again, at this time the true position of the moon's centre was 81°30′ from the summer equinoxes and 2⅙° south of the ecliptic; but it appeared to be 82°30′ in longitude away from it and 2¼° south of the ecliptic. And therefore Spica, by this observation, was in turn very nearly 2° south of the ecliptic and 82½° from the summer tropic. Consequently, in the 12 years between the two observations, it has moved very nearly ⅙° eastward from the summer tropic.

And Menelaus, the geometer, says that he observed in Rome, in the year 1 of Trajan (Mechir 15-16), at the end of the tenth hour, Spica occulted by the

moon; for it could not be seen; but that at the end of the eleventh hour he observed it west of the moon's centre equally distant from the horns by less than a diameter. And this date is the year 845 of Nabonassar, Egyptianwise Mechir 15-16, 4 seasonal hours after midnight when the moon's centre overtook Spica, but 5 equatorial hours because the sun was 20° within the Goat; with respect to the meridian through Alexandria, it was 6⅓ hours after, and with respect to regular solar days was 6¼ hours after or a little more. At this hour the true position of the moon's centre was 85¾° from the summer tropic and very nearly 1⅓° south of the ecliptic, but it appeared to be 86¼° in longitude from it and 2° south. For the first quarter of the Balance was culminating. Therefore Spica had that position at that time.

It is clear that, according both to Timocharis and to ourselves, Spica was the same distance south of the ecliptic (that is, 2°) but that since the observation in the year 36 it has shifted 3°55′ during the 391 intervening years; and, since the observation in the year 48, 3°45′ during the 379 intervening years. And so, from this the eastward shift of Spica in 100 years comes to very nearly 1°.

Again, Timocharis says he observed in Alexandria that in the year 36 of the First Callippic Period (Poseidon 25, or Phaophi 16), exactly at the beginning of the tenth hour, the moon appeared to overtake with its northern arc the northern star of those in the Scorpion's forehead. And this date is the year 454 of Nabonassar, Egyptianwise Phaophi 16-17, 3 seasonal hours after midnight and 3⅖ equatorial hours, because the sun was 26° within the Archer, but 3⅙ hours with respect to regular solar days. At that hour the true position of the moon's centre was 31¼° from the autumn equinox and 1⅓° north of the ecliptic, but it appeared to be 32° in longitude and 1°12′ north of the ecliptic. For the middle of the Lion was culminating. And, therefore, the northernmost star in the Scorpion's forehead was at that time 32° in longitude from the autumn equinox and very nearly 1⅓° north of the ecliptic.

Likewise Menelaus says he observed in Rome that in the year 1 of Trajan, Mechir 18-19, at the end of the eleventh hour, the moon's southern horn appeared in a straight line with the middle and southern stars of those in the Scorpion's forehead. But its centre lay east of this line and was as far from the middle star as the middle star was from the southern star; and its centre seemed to have overtaken the northern star of those in the forehead. For it made no appearance whatsoever. This date is again the year 845 of Nabonassar, Egyptianwise Mechir 18-19, 5 seasonal hours after midnight and 6½ equatorial hours, because the sun was 23° within the Goat; but 7½ hours after with respect to the meridian through Alexandria, and very nearly the same with respect to regular solar days. At this hour the true position of the moon's centre was 35⅓° from the autum equinox and 2⅙° north of the ecliptic. But it appeared to be 35°55′ in longitude and 1⅓° north. For the end of the Balance was culminating. Therefore, the northernmost star in the Scorpion's forehead, at that time, had very nearly that same position.

It is thus evident that also in the case of this star the same latitudinal distance has been observed then and now, but that the longitudinal distance has shifted 3°55′ eastward from the autumn equinox for the total of 391 years between the observations. Again it follows from this that the eastward shift of the star is 1° in 100 years.

4. On the Manner of Recording the Fixed Stars

And now, from the similar observation and comparison of these and other bright stars and from the constant distance of the rest with respect to the stars observed, we find it certain, to the extent that the time elapsed allows, that the sphere of the fixed stars makes a shift of this amount eastward from the tropic and equinoctial points, and moreover that this movement is effected about the poles of the ecliptic and not about the poles of the equator—that is, those of the prime movement. We therefore thought it proper, for each of these and of the other fixed stars, to make observations and records of their epochs observed at the present time, considered in longitude and latitude, not with respect to the equator but defined with respect to the ecliptic by the great circles drawn through its poles and each one of the stars. In accord with the given hypothesis on the movement, it is necessary that their latitudinal passages with respect to the ecliptic remain always the same, and that the longitudinal shifts eastward cut off equal arcs in equal times. And so again, using the same instrument, by the turning of the armillary circles in it about the ecliptic's poles we observed as many stars as we could, up to those of the sixth magnitude. We always directed one of these armillary circles towards one of the bright stars gotten by means of the moon, according to its proper section on the ecliptic; another armillary circle, graduated and capable of turning on the ecliptic's poles, we directed towards the desired star until, like the first one, it also was sighted through the hole of its own proper circle. When this took place, both passages of the stars desired were immediately evident to us on the armillary circle, since the longitudinal position was defined by its intersection with the ecliptic and the latitudinal positions by the arc on it cut off between this intersection and the upper hole.

Now, to have the solid sphere's arrangement of constellations expounded in this way, we set it up tablewise in 4 parts, placing in the first part the configurations for each one of the stars by constellation. In the second we placed the positions according to longitude within the dodecatemories as they are gotten from observations at the start of Antonine's reign, the beginning of the quadrants again being taken at the tropic and equinoctial points. In the third we placed the latitudinal distances from the ecliptic for both the northern and southern sides. And in the fourth we placed the order of magnitude. Now, the latitudinal distances are always the same. But the longitudinal positions can easily provide the passage for other dates, if we subtract the degrees corresponding to the time between the date of the position and the date sought, at the rate of 1° in a hundred years, from the degrees of the position in the case of an earlier date; and if we add in the case of a later date.

The designations by configurations, therefore, must be understood in accordance with the convention of constellations and by the determinations of the ecliptic's poles. For by "stars preceding certain stars" and by "stars following certain stars" we mean those having their positions in the preceding western or following eastern sections of the ecliptic; and by "more southern" or "more northern" we mean those nearer the corresponding pole of the ecliptic.

We have not used for each of the stars altogether the same formations as our predecessors, just as they did not use the same as their predecessors. But often we use others according to the greater propriety and fittingness of the configura-

tions—as, for example, when those stars which Hipparchus places in the shoulders of the Virgin, we call her sides because their distance from the stars in the head appears greater than that from the hands, and thus they better fit the sides and are quite different from the shoulders. But the correspondence of the differently designated stars would be easy to ascertain by the comparison of their recorded positions. And here is the exposition of the records:

5. Tabular Exposition of the Constellations of the Northern Hemisphere

Configurations	*Longitude*	*Latitude*	*Magn.*
Constellation of the Little Bear			
The star on the tip of the tail	Twins 0°⅙′	N66°	3
The next one in the tail	Twins 2½°	N70°	4
The next one, before the beginning of the tail	Twins 16°	N74⅓°	4
The southern one on the western side of the rectangle	Twins 29⅔°	N75⅔°	4
The northern one on the same side	Crab 3⅔°	N77⅔°	4
The southern one of those on the eastern side	Crab 17½°	N72⅚°	2
The northern one on the same side	Crab 26⅙°	N74⅚°	2
In all, 7 stars of which 2 are of 2nd magnitude, 1 of 3rd, 4 of 4th.			
The unfigured star near it, that is the more southern star of first magnitude in a straight line with those on the eastern side	Crab 13°	N71⅙°	4
Constellation of the Big Bear			
The star at the tip of the muzzle	Twins 25⅓°	N39⅚°	4
The western star of those in the two eyes	Twins 25⅚°	N43°	5
The eastern one of these	Twins 26⅓°	N43°	5
The western star of the two in the forehead	Twins 26⅙°	N47⅙°	5
The eastern one of these	Twins 26⅔°	N47°	5
The star at the end of the western ear	Twins 28⅙°	N50½°	5
The western star of the 2 in the neck	Crab ½°	N43⅚°	4
The eastern one of these	Crab 2½°	N44⅓°	4
The northern star of the 2 in the breast	Crab 9°	N42°	4
The southern one of these	Crab 11°	N44°	4−
The star in the left knee	Crab 10⅔°	N35°	3
The northern star at the end of the left forefoot	Crab 5½°	N29⅓°	3
The southern one of these	Crab 6⅓°	N28⅓°	3
The star above the right knee	Crab 5⅔°	N36°	4
The star below the right knee	Crab 5⅚°	N33°	4
Of those in the quadrilateral, the star on the back	Crab 17⅔°	N49°	2
Of these, the star on the flank	Crab 22⅙°	N44½°	2
The star at the beginning of the tail	Lion 3⅙°	N51°	3
The remaining star in the left thigh	Lion 3°	N46½°	2
The western star of those at the end of the left hindfoot	Crab 22⅔°	N29⅓°	3
The star east of this one	Crab 24⅙°	N28¼°	3
The star in the left ham	Lion 1⅔°	N35¼°	4+
The northern star of those at the end of the right hindfoot	Lion 9⅚°	N25⅚°	3
The southern star of these	Lion 10⅓°	N25°	3
The first star of the 3 in the tail after the beginning	Lion 12⅙°	N53½°	2
The middle one of these	Lion 18°	N55⅔°	2
The third one at the end of the tail	Lion 29⅚°	N25°	3
In all, 27 stars of which 6 are of 2nd magnitude, 8 of 3rd, 8 of 4th, 5 of 5th.			

Configurations	*Longitude*		*Latitude*	*Magn.*
Of the Unfigured Stars beneath it				
The star under the tail far to the south	Lion	27⅚°	N39¾°	3
The dimmer star west of it	Lion	20⅙°	N41⅓°	5
The more southern star of those between the forefeet of the Bear and the head of the Lion	Crab	15°	N17¼°	4
The star north of this	Crab	13⅓°	N19⅙°	4
The star east of the other 3 dim ones	Crab	16⅙°	N20°	dim
The star west of this last	Crab	12⅙°	N22½°	dim
The star still farther west than this last	Crab	11⅙°	N23°	dim
The star between the forefeet and the Twins	Crab	0°	N22¼°	dim
In all, 8 unfigured stars of which 1 is of 3rd magnitude, 2 of 4th, 1 of 5th, and 4 dim.				
Constellation of the Dragon				
The star on the tongue	Balance	26⅔°	N76½°	4
The star in the mouth	Scorpion	11⅚°	N78½°	4+
The star above the eye	Scorpion	13⅙°	N75⅔°	3
The star in the jaw	Scorpion	27⅓°	N80⅓°	4
The star above the head	Scorpion	29⅔°	N75½°	3
The northern star of the 3 in a straight line in the first fold of the neck	Archer	24⅔°	N82⅓°	4
The southern one of these	Goat	2⅓°	N78¼°	4
The middle star of these	Archer	28⅚°	N80⅓°	4
The star east of this one	Goat	19½°	N81½°	4
The southern star of the western side of the square in the next turn	Fishes	8°	N81⅔°	4
The northern star of the western side	Fishes	20½°	N83°	4
The northern star of the eastern side	Ram	7⅔°	N78⅚°	4
The southern star of the eastern side	Fishes	22⅚°	N77⅚°	4
The southern star of the triangle in the next turn	Ram	10⅔°	N80½°	5
The western star of the remaining 2 of the triangle	Ram	21⅔°	N81⅓°	5
The eastern star of these	Ram	26⅙°	N80¼°	5
The eastern star of those in the next triangle west of this	Twins	13⅓°	N84½°	4
The southern star of the remaining 2 of the triangle	Bull	20⅓°	N87½°	4
The northern star of the remaining two	Bull	11⅚°	N84⅚°	4
The eastern star of the 2 small stars west of the triangle	Crab	28⅔°	N87½°	6
The western one of these	Crab	21⅔°	N86⅚°	6
The more northern of the next 3 in a straight line	Virgin	9°	N81¼°	5
The middle one of the 3	Virgin	9⅓°	N80⅓°	5
The northern one of these	Virgin	8⅓°	N84⅚°	3
The northern one of the next 2 to the west	Virgin	10°	N78°	3
The southern one of these	Virgin	10⅓°	N74⅔°	4+
The western star of those to the west in the turn near the tail	Virgin	12⅔°	N70°	3
The western star of the 2 rather distant from the last one	Lion	7⅓°	N64⅔°	4
The eastern star of these	Lion	11⅙°	N65½°	3
The star near them by the tail	Crab	19⅙°	N61¼°	3
The remaining star at the end of the tail	Crab	13⅙°	N56¼°	3
In all, 31 stars of which 8 are of 3rd magnitude, 16 of 4th, 5 of 5th, 2 of 6th.				
Constellation of Cepheus				
The star in the right foot	Bull	5°	N75⅔°	4
The star in the left foot	Bull	3°	N64¼°	4
The star under the belt on the right side	Ram	7⅓°	N71⅙°	4

Configurations	*Longitude*	*Latitude*	*Magn.*
The star touching the right shoulder from above	Fishes 16⅔°	N69°	3
The star touching the right elbow from above	Fishes 9⅓°	N72°	4
The star touching the same elbow from below	Fishes 10°	N74°	4
The star in the chest	Fishes 28½°	N65½°	5
The star in the left arm	Ram 7½°	N62½°	4+
The southern star of the 3 in the tiara	Fishes 16⅓°	N60¼°	5
The middle one of the 3	Fishes 17⅓°	N61¼°	4
The northern one of the 3	Fishes 19°	N61⅓°	5
In all, 11 stars of which 1 is of 3rd magnitude, 7 of 4th, 3 of 5th.			
Of the Unfigured Stars About Cepheus			
The star west of the tiara	Fishes 13⅔°	N64°	5
The star east of the tiara	Fishes 21⅓°	N59½°	4
In all, 2 unfigured stars of which 1 is of 4th magnitude, 1 of 5th.			
Constellation of the Ploughman			
The star west of the 3 in the left hand	Virgin 2⅓°	N58⅔°	5
The middle and southern one of the 3	Virgin 4⅙°	N58⅓°	5
The eastern one of the 3	Virgin 5⅓°	N60⅙°	5
The star in the left elbow	Virgin 9⅔°	N54⅔°	5
The star in the left shoulder	Virgin 19⅔°	N49°	3
The star in the head	Virgin 26⅔°	N53⅚°	4+
The star in the right shoulder	Balance 5⅔°	N48⅔°	4+
The star north of these and in the crook	Balance 5⅔°	N53¼°	4
The star north of this at the tip of the crook	Balance 5°	N57½°	4
The northern one of the 2 below the shoulder in the cudgel	Balance 7⅔°	N46½°	4+
The southern one of these	Balance 8½°	N45½°	5
The star at the tip of the right hand	Balance 8⅙°	N41⅓°	5
The western one of the 2 in the wrist	Balance 6⅔°	N41⅔°	5
The eastern one of these	Balance 7°	N42½°	5
The star at the end of the crook's haft	Balance 7⅓°	N40⅓°	5
The star in the right thigh in the girdle	Balance 0°	N40¼°	3
The eastern one of the two in the girdle	Virgin 25⅔°	N41⅔°	4
The western one of these	Virgin 25°	N42⅙°	4+
The star in the right heel	Balance 5⅓°	N28°	3
The northern one of the 3 in the left shank	Virgin 21⅓°	N28°	3
The middle one of the 3	Virgin 20½°	N26½°	4
The southern one of these	Virgin 21⅓°	N25°	4
In all, 22 stars of which 4 are of 3rd magnitude, 9 of 4th, 9 of 5th			
One Unfigured Star Under This of 1st Mag.			
The fiery star called Arcturus between the thighs	Virgin 27°	N31½°	1
Constellation of the Northern Crown			
The bright star in the Crown	Balance 14⅔°	N44½°	2+
The most western of all	Balance 11⅔°	N46½°	4+
The one east and north of this one	Balance 11⅚°	N48°	5
The one again east and north of this last	Balance 13⅔°	N50½°	6
The star east of the bright one southwards	Balance 17⅙°	N44¾°	4
The star east of this last and nearby	Balance 19⅙°	N44⅚°	4
The star still east of these	Balance 21⅓°	N46⅙°	4
The star east of all those in the Crown	Balance 21⅔°	N49⅓°	4
In all, 8 stars of which 1 is of 2nd magnitude, 5 of 4th, 1 of 5th, 1 of 6th.			

Configurations	*Longitude*	*Latitude*	*Magn.*
Constellation of Man Kneeling			
The star in the head	Scorpion 17⅔°	N37½°	3
The star in the right shoulder beside the armpit	Scorpion 3⅔°	N43°	3
The star in the right arm	Scorpion 1⅔°	N40⅙°	3
The star in the right elbow	Balance 28°	N37⅙°	4
The star in the left shoulder	Scorpion 16⅔°	N48°	3
The star in the left arm	Scorpion 22°	N49½°	4+
The star in the left elbow	Scorpion 27⅔°	N52°	4+
The eastern one of the 3 in the left wrist	Archer 5½°	N52⅚°	4+
The northern one of the remaining 2	Archer 1⅔°	N54°	4+
The southern one of these	Archer 1½°	N53°	4
The star in the right side	Scorpion 3⅚°	N50⅔°	3
The star in the left side	Scorpion 10⅙°	N53½°	5
The one north of this last in the left buttock	Scorpion 10°	N56½°	5
The star at the beginning of the same thigh	Scorpion 11⅙°	N58½°	3
The western star of the 3 in the left thigh	Scorpion 14°	N59⅚°	4
The one east of this last	Scorpion 15⅓°	N60⅓°	4
The star again east of this one	Scorpion 16⅓°	N61¼°	4+
The star in the left knee	Archer ⅚°	N61°	4
The star in the left shin	Scorpion 22⅙°	N69⅓°	4
The western one of the 3 in the foot	Scorpion 15⅓°	N70¼°	6
The middle one of the 3	Scorpion 16⅚°	N71¼°	6
The eastern one of these	Scorpion 19⅔°	N72¼°	6
The star at the beginning of the right thigh	Scorpion ⅔°	N60¼°	4+
The one north of this and in the same thigh	Balance 25⅓°	N63°	4
The star in the right knee	Balance 15⅔°	N65½°	4+
The southern star of the 2 under the right knee	Balance 13⅔°	N63⅔°	4
The northern one of these	Balance 10⅙°	N64¼°	4
The star in the right shank	Balance 11⅙°	N60°	4
The star at the end of the right foot which is the same as that at the tip of the crook			
Except for this last, in all 28 stars of which 6 are of 3rd magnitude, 17 of 4th, 2 of 5th, 3 of 6th.			
The Unfigured Star Beyond This			
The star south of that in the right arm	Scorpion 2⅔°	N38⅙°	5
In all 1 star of 5th magnitude.			
Constellation of the Lyre			
The bright star on the shell called the Lyre	Archer 17⅓°	N62°	1
The northern one of the 2 lying next it	Archer 20⅓°	N62⅔°	4+
The southern one of these	Archer 20⅓°	N61°	4+
The star east of these and at the beginning of the horns	Archer 23⅔°	N60°	4
The northern one of the two near the eastern side of the shell	Goat 2°	N61⅓°	4
The southern one of these	Goat 1⅔°	N60⅓°	4
The northern one of the two western stars in the crossbar	Archer 21°	N56⅙°	3
The southern one of these	Archer 20⅚°	N55°	4−
The northern one of the 2 in the cross-bar	Archer 24⅙°	N55⅓°	3
The southern one of these	Archer 24°	N54¾°	4−
In all, 10 stars of which 1 is of 1st magnitude, 2 of 3rd, 7 of 4th.			
Constellation of the Bird			
The star in the beak	Goat 4½°	N49°	3
The one east of this and in the head	Goat 9°	N50½°	5

Configurations	*Longitude*	*Latitude*	*Magn.*
The star in the middle of the throat	Goat 16⅓°	N54½°	4+
The star in the breast	Goat 28½°	N57⅓°	3
The bright star in the tail	Water Bearer 9⅙°	N60°	2
The star in the elbow of the right wing	Goat 19⅓°	N64⅔°	3
The southern star of the 3 in the right wing-spread	Goat 22½°	N69⅔°	4
The middle one of the 3	Goat 21⅙°	N71½°	4+
The northern one at the edge of the wing-spread	Goat 16⅔°	N74°	4+
The star in the elbow of the left wing	Water Bearer 5⅙°	N49½°	3
The star north of these and in the middle of the same wing	Water Bearer 3⅚°	N52⅙°	4+
The star at the edge of the left wing-spread	Water Bearer 6⅔°	N44°	3
The star in the left foot	Water Bearer 10°	N55⅙°	4+
The star in the left knee	Water Bearer 14½°	N57°	4+
The western one of the 2 in the right foot	Water Bearer 1⅙°	N64°	4
The eastern one of these	Water Bearer 2⅔°	N64½°	4
The star in the nebula of the right knee	Water Bearer 12⅙°	N64¾°	5
In all, 17 stars of which 1 is of 2nd magnitude, 5 of 3rd, 9 of 4th, 2 of 5th.			
The Unfigured Stars About It			
The southern star of the 2 under the left wing	Water Bearer 10⅔°	N49⅔°	4+
The northern one of these	Water Bearer 13⅚°	N51⅔°	4+
In all, 2 stars of 4th magnitude.			
Constellation of Cassiopeia			
The star in the head	Ram 7⅚°	N45⅓°	4+
The star in the breast	Ram 10⅚°	N46¾°	3
The star north of this and in the girdle	Ram 13°	N47⅚°	4
The star above the chair along the thighs	Ram 16⅔°	N49°	3+
The star in the knees	Ram 20⅔°	N45½°	3
The star in the shank	Ram 27°	N47¾°	4
The star at the tip of the foot	Bull 1⅔°	N47⅓°	4
The star in the left arm	Ram 14⅔°	N44⅓°	4
The star below the left elbow	Ram 17⅔°	N45°	5
The star in the right forearm	Ram 2⅓°	N50°	6
The star above the foot of the throne	Ram 15°	N52⅔°	4−
The star in the middle of the back of the chair	Ram 7⅚°	N51⅔°	3
The star at the end of the back	Ram 3⅔°	N51⅔°	6
In all, 13 stars, of which 4 are of 3rd magnitude, 6 of 4th, 1 of 5th, 2 of 6th.			
Constellation of Perseus			
The nebula in the right hand	Ram 26⅔°	N40½°	neb.
The star in the right elbow	Bull 1⅙°	N37½°	4
The star in the right shoulder	Bull 2⅔°	N34½°	3−
The star in the left shoulder	Ram 27½°	N32⅓°	4
The star in the head	Bull ⅔°	N34½°	4
The star in the broad of the back	Bull 1½°	N31⅙°	4
The bright star in the right side	Bull 4⅚°	N30°	2
The western one of the 3 after the one in the side	Bull 5⅓°	N27⅚°	4
The middle one of the 3	Bull 7°	N27⅔°	4
The eastern one of these	Bull 7⅔°	N27⅓°	3
The star in the left elbow	Bull ½°	N27°	4
The bright star in the Gorgon's head	Ram 29⅔°	N23°	2
The star east of this	Ram 29⅙°	N21°	4
The star west of the bright one	Ram 27⅔°	N21°	4
The star left farther west than this	Ram 26⅚°	N22¼°	4
The star in the right knee	Bull 14⅚°	N28°	4

Configurations	*Longitude*		*Latitude*	*Magn.*
The star west of this and above the knee	Bull	13°	N28⅙°	4
The western star of the 2 above the joint	Bull	12⅓°	N25°	4
The eastern one in the joint itself	Bull	14°	N26¼°	4
The star in the right calf	Bull	14⅙°	N24½°	5
The star in the right ankle	Bull	16⅓°	N18¾°	5
The star in the left thigh	Bull	6⅚°	N21⅚°	4+
The star in the left knee	Bull	8⅔°	N19¼°	3
The star in the left shank	Bull	8⅓°	N14¾°	4
The star in the left heel	Bull	4⅙°	N12°	3−
The star east of it in the left foot	Bull	6⅓°	N11°	3+
In all, 26 stars, of which 2 are of 2nd magnitude, 5 of 3rd, 16 of 4th, 2 of 5th, a nebula.				
The Unfigured Stars about Perseus				
The star east of the one in the knee	Bull	11⅚°	N18°	5
The star north of those in the right knee	Bull	15°	N31°	5
The western star of those in the Gorgon's Head	Ram	24⅔°	N20⅔°	dim
In all, 3 stars, of which 2 are of 5th magnitude, 1 indistinct.				
Constellation of the Charioteer				
The southern one of the 2 at the head	Twins	2½°	N30°	4
The northern one and above the head	Twins	2⅓°	N31⅚°	4
The star in the left shoulder called Capella	Bull	25°	N22½°	1
The star in the right shoulder	Twins	2⅚°	N20°	2
The star in the right elbow	Twins	1⅙°	N15¼°	4
The star in the right wrist	Twins	2⅚°	N13⅓°	4+
The star in the left elbow	Bull	22°	N20⅔°	4+
The eastern one of the 2 in the left wrist called the Kids	Bull	22⅙°	N18°	4+
The western one of these	Bull	22°	N18°	4
The star in the left ankle	Bull	19⅚°	N10⅙°	3−
The star common to the right ankle and the horn	Bull	25⅔°	N5°	3+
The star north of this near the foot	Bull	26°	N8½°	5
The star still north of this in the buttock	Bull	26⅓°	N12⅙°	5
The little star above the left foot	Bull	20⅔°	N16°	6
In all, 14 stars, of which 1 is of 1st magnitude, 1 of 2nd, 2 of 3rd, 7 of 4th, 2 of 5th, 1 of 6th.				
Constellation of Serpentarius				
The star in the head	Scorpion	24⅚°	N36°	3+
The western one of the 2 in the right shoulder	Scorpion	28°	N27¼°	4+
The eastern one of these	Scorpion	29°	N26½°	4
The western star of the 2 in the left shoulder	Scorpion	13⅓°	N33°	4
The eastern one of these	Scorpion	14⅔°	N31⅚°	4
The star in the left elbow	Scorpion	8⅓°	N24½°	4
The western star of the 2 in the left hand	Scorpion	5°	N17°	3
The eastern one of these	Scorpion	6°	N16½°	3
The star in the right elbow	Scorpion	26⅔°	N15°	4
The western star of the 2 in the right hand	Archer	2⅓°	N13⅔°	4−
The eastern one of these	Archer	3⅓°	N14⅓°	4
The star in the right knee	Scorpion	21⅙°	N7½°	3
The star in the right shank	Scorpion	26⅔°	N2¼°	4+
The western star of the 4 in the right foot	Scorpion	23°	S2¼°	4
The one east of this one	Scorpion	24⅓°	S1½°	4+
The star still east of this last	Scorpion	25°	S⅓°	4
The remaining and easternmost of the 4	Scorpion	25⅚°	S¼°	5
The one east of these and touching the heel	Scorpion	27⅙°	N1°	5
The star in the left knee	Scorpion	12⅙°	N11⅚°	3

Configurations	*Longitude*	*Latitude*	*Magn.*
The northern one of the 3 in a straight line in the left shank	Scorpion 11⅔°	N5⅓°	5+
The middle one of these	Scorpion 10⅔°	N3⅙°	5
The southern one of the 3	Scorpion 9⅚°	N1⅔°	5+
The star in the left heel	Scorpion 12⅓°	N⅔°	5
The star touching the hollow of the left foot	Scorpion 10⅔°	S¾°	4
In all, 24 stars, of which 5 are of 3rd magnitude, 13 of 4th, 6 of 5th.			
The Unfigured Stars about the Serpentarius			
The northern one of the 3 east of the right shoulder	Archer 2°	N28⅙°	4
The middle one of the 3	Archer 2⅔°	N26⅓°	4
The southern one of these	Archer 3°	N25°	4
The star east of the 3 above the middle one	Archer 3⅔°	N27°	4
The lone star north of the 4	Archer 4⅔°	N33°	4
In all, 5 stars of 4th magnitude.			
Constellation of the Serpent of Serpentarius			
Of the square in the head, the star at the end of the jaw	Balance 18⅚°	N38°	4
The star touching the nostrils	Balance 21⅔°	N40°	4
The star in the temple	Balance 24⅓°	N36°	3
The star at the beginning of the throat	Balance 22°	N34¼°	3
The star in the middle of the square and in the jaw	Balance 21⅓°	N37¼°	4
The star outside and north of the head	Balance 26⅙°	N42½°	4
The star after the first curve of the neck	Balance 21⅔°	N29¼°	3
The northern one of the 3 following this	Balance 24⅚°	N26½°	4
The middle one of the 3	Balance 24⅓°	N25⅓°	3
The southern one of these	Balance 26⅓°	N24°	3
The star after the next curve, west of the Serpentarius' left hand	Balance 28⅚°	N16½°	4
The star east of those in the hand	Scorpion 8⅙°	N13¼°	5
The star after the Serpentarius' right thigh	Scorpion 23⅔°	N10½°	4
The southern one of the 2 east of this	Scorpion 27°	N8½°	4+
The northern one of these	Scorpion 27⅚°	N10⅚°	4
The star after the right hand in the tail's curve	Archer 3⅔°	N20°	4
The star east of this likewise in the tail	Archer 8⅔°	N21⅙°	4+
The star at the tip of the tail	Archer 18⅓°	N27°	4
In all, 18 stars, of which 5 are of 3rd magnitude, 12 of 4th, 1 of 5th.			
Constellation of the Arrow			
The lone star in the point	Goat 10⅙°	N39⅓°	4
The eastern one of the 3 in the shaft	Goat 6⅔°	N39⅙°	6
The middle one of these	Goat 5⅚°	N39½°	5
The western one of the 3	Goat 4⅔°	N39°	5
The star at the extremity of the notched end	Goat 3⅓°	N38⅔°	5
In all, 5 stars, of which 1 is of 4th magnitude, 3 of 5th, 1 of 6th.			
Constellation of the Eagle			
The star in the middle of the head	Goat 7⅙°	N26⅚°	4
The star west of this and in the neck	Goat 4⅚°	N27⅙°	3
The bright one in the broad of the back called the Eagle	Goat 3⅚°	N29⅙°	2+
The one near this to the north	Goat 4⅔°	N30°	3−
The western one of the 2 in the left shoulder	Goat 3⅙°	N31½°	3
The eastern one of these	Goat 6°	N31½°	5

Configurations	*Longitude*	*Latitude*	*Magn.*
The western one of the 2 in the right shoulder	Archer 29⅔°	N28⅔°	5
The eastern one of these	Goat 1⅙°	N26⅔°	5+
The star farther off under the Eagle's tail touching the Milky Way	Archer 22⅙°	N36⅔°	3
In all, 9 stars, of which 1 is of 2nd magnitude, 4 of 3rd, 1 of 4th, 3 of 5th.			
The Unfigured Stars about the Eagle			
The western star of the 2 south of the Eagle's head	Goat 3⅔°	N21⅔°	3
The eastern one of these	Goat 8⅚°	N19⅙°	3
The star southwest of the Eagle's right shoulder	Archer 26°	N25°	4+
The star south of this	Archer 28½°	N20°	3
The star still south of this	Archer 29⅔°	N15½°	5
The star west of all these	Archer 21⅙°	N18⅙°	3
In all, 6 stars, of which 4 are of 3rd magnitude, 1 of 4th, 1 of 5th.			
Constellation of the Dolphin			
The western star of the 3 in the tail	Goat 17⅔°	N29⅙°	3−
The northern one of the remaining 2	Goat 18⅔°	N29°	4−
The southern one of these	Goat 18⅔°	N27¼°	4
The southern one of the western side of those stars in the rhomboidal figure of 4 sides	Goat 18½°	N32°	3−
The northern one of the western side	Goat 20⅙°	N33⅚°	3−
The southern one of the eastern side of the rhombus	Goat 21⅓°	N32°	3−
The northern one of the eastern side	Goat 23⅙°	N33⅙°	3−
The southern one of the 3 between the tail and the rhombus	Goat 17½°	N30¼°	6
The western one of the remaining 2 northern ones	Goat 17½°	N31⅚°	6
The remaining eastern one of these	Goat 19°	N31½°	6
In all, 10 stars, of which 5 are of 3rd magnitude, 2 of 4th, 3 of 6th.			
Constellation of Forepart of Horse			
The western one of the 2 in the head	Goat 26⅓°	N20½°	dim
The eastern one of these	Goat 28°	N20⅔°	dim
The western one of the 2 in the jaw	Goat 26⅓°	N25½°	dim
The eastern one of these	Goat 27⅔°	N25°	dim
In all, 4 dim stars.			
Constellation of the Horse			
The star common to the Horse's navel and Andromeda's head	Fishes 17⅚°	N26°	2−
The star in the loin and at the end of the wing	Fishes 12⅙°	N12½°	2−
The star in the right shoulder and at the beginning of the foot	Fishes 2⅙°	N31°	2−
The star in the broad of the back, and shoulder of the wing	Water Bearer 26⅔°	N19⅔°	2−
The northern one of the 2 in the body under the wing	Fishes 4½°	N25½°	4
The southern one of these	Fishes 5°	N25°	4
The northern one of the 2 in the right knee	Water Bearer 29°	N35°	3
The southern one of these	Water Bearer 28½°	N34½°	5
The western one of the 2 close together in the chest	Water Bearer 26⅙°	N29°	4
The eastern one of these	Water Bearer 27°	N29½°	4
The western one of the 2 close together in the neck	Water Bearer 18⅚°	N18°	3
The eastern one of these	Water Bearer 20½°	N19°	4
The southern one of the 2 in the mane	Water Bearer 21⅓°	N15°	5
The northern one of these	Water Bearer 20½°	N16°	5
The northern one of the 2 close together in the head	Water Bearer 9⅓°	N16½°	3

Configurations	*Longitude*	*Latitude*	*Magn.*
The southern one of these	Water Bearer 8°	N16°	4
The star in the muzzle	Water Bearer 5⅓°	N22½°	3+
The star in the right ankle	Water Bearer 23⅓°	N41⅙°	4+
The star in the left knee	Water Bearer 17⅓°	N34¼°	4+
The star in the left ankle	Water Bearer 12⅓°	N36⅚°	4+
In all, 20 stars, of which 4 are of 2nd magnitude, 4 of 3rd, 9 of 4th, 3 of 5th.			
Constellation of Andromeda			
The star in the broad of the back	Fishes 25⅓°	N24½°	3
The star in the right shoulder	Fishes 26⅓°	N27°	4
The star in the left shoulder	Fishes 24⅓°	N23°	4
The southern star of the 3 in the right arm	Fishes 23⅔°	N32°	4
The northern one of these	Fishes 24⅔°	N33½°	4
The middle one of the 3	Fishes 25°	N32⅓°	5
The southern star of the 3 at the end of the right hand	Fishes 19⅔°	N41°	4
The middle one of these	Fishes 20⅔°	N42°	4
The northern one of the three	Fishes 22⅙°	N44°	4
The star in the left arm	Fishes 24⅙°	N17½°	4
The star in the left elbow	Fishes 25⅔°	N15⅚°	4
The southern one of the 3 above the girdle	Ram 3⅚°	N26⅓°	3
The middle one of these	Ram 1⅚°	N30°	4
The northern one of the 3	Ram 2°	N32½°	4
The star above the left foot	Ram 16⅚°	N28°	3
The star in the right foot	Ram 17⅙°	N37⅓°	4−
The star south of this one	Ram 15⅙°	N35⅔°	4+
The northern one of the 2 in the left bend of the knee	Ram 12⅓°	N29°	4
The southern one of these	Ram 12°	N28°	4
The star in the right knee	Ram 10⅙°	N35½°	5
The northern one of the 2 in the train	Ram 12⅔°	N34½°	5
The southern one of these	Ram 14⅙°	N32½°	5
The star outside and west of the 3 in the right hand	Fishes 11⅔°	N44°	3
In all, 23 stars, of which 4 are of 3rd magnitude, 15 of 4th, 4 of 5th.			
Constellation of the Triangle			
The star at the vertex of the Triangle	Ram 11°	N16½°	3
The western one of the 3 in the base	Ram 16°	N20⅔°	3
The middle one of these	Ram 16⅓°	N19⅔°	4
The eastern one of the 3	Ram 16⅚°	N19°	3
In all, 4 stars, of which 3 of 3rd magnitude, 1 of 4th.			
—Thus, in all, 360 stars for the northern part, of which 3 are of 1st magnitude, 18 of 2nd, 81 of 3rd, 177 of 4th, 58 of 5th, 13 of 6th, 9 dim, 1 nebular.			

Constellations of the Stars in the Zodiac

Constellation of the Ram			
The western star of the 2 in the horn	Ram 6⅔°	N7⅓°	3−
The eastern one of these	Ram 7⅔°	N8⅓°	3
The northern one of the 2 in the muzzle	Ram 11°	N7⅔°	5
The southern one of these	Ram 11½°	N6°	5
The star in the neck	Ram 6½°	N5½°	5
The star in the loins	Ram 17⅔°	N6°	6
The star at the beginning of the tail	Ram 21⅓°	N4⅚°	5
The western one of the 3 in the tail	Ram 23⅚°	N1⅔°	4
The middle one of the 3	Ram 25⅓°	N2½°	4

Configurations	*Longitude*	*Latitude*	*Magn.*
The eastern one of these	Ram 27°	N1⅚°	4
The star in the calf	Ram 19⅔°	N1½°	5
The star under the bend of the knee	Ram 18°	S1½°	5
The star in the hind-foot	Ram 15°	S5¼°	4+
In all, 13 stars, of which 2 are of 3rd magnitude, 4 of 4th, 6 of 5th, 1 of 6th.			
The Unfigured Stars about the Ram			
The star above the head which Hipparchus placed in the muzzle	Ram 10⅔°	N10½°	3+
The eastern and brightest one of the 4 above the loins	Ram 21⅔°	N10⅙°	4
The northern one of the remaining fainter 3	Ram 21⅓°	N12⅔°	5
The middle one of the 3	Ram 19⅔°	N11⅙°	5
The southern one of these	Ram 19⅙°	N10⅔°	5
In all, 5 stars, of which 1 is of 3rd magnitude, 1 of 4th, 3 of 5th.			
Constellation of the Bull			
The northern star of the 4 in the section	Ram 26⅙°	S6°	4
The one next it	Ram 26°	S7¼°	4
The one again next to this last	Ram 24⅓°	S8½°	4
The southernmost one of the 4	Ram 24⅓°	S9¼°	4
The one east of these in the right shoulder blade	Ram 29⅔°	S9½°	5
The star in the chest	Bull 3⅔°	S8°	3
The star in the right knee	Bull 6⅔°	S12⅔°	4
The star in the right ankle	Bull 3°	S14⅚°	4
The star in the left knee	Bull 12⅙°	S10°	4
The star in the left forearm	Bull 13°	S13°	4
Of those in the face called the Hyades, the one on the nostrils	Bull 9°	S5¾°	3–
The star between this last and the northern eye	Bull 10⅓°	S4¼°	3–
The star between this last one and the southern eye	Bull 10⅚°	S5⅚°	3–
The bright red star of the Hyades in the southern eye	Bull 12⅔°	S5⅙°	1
The other one in the northern eye	Bull 11⅚°	S3°	3–
The star at the beginning of the southern horn and the ear	Bull 17½°	S4°	4
The southern one of the 2 in the southern horn	Bull 20⅓°	S5°	5
The northern one of these	Bull 20°	S3½°	5
The star at the tip of the southern horn	Bull 27⅔°	S2½°	3
The star at the beginning of the northern horn	Bull 15⅔°	S¼°	4
The star at the tip of the northern horn the same as the one in the Charioteer's right foot	Bull 25⅔°	N5°	3
The northern one of the 2 close together in the northern ear	Bull 12°	N½°	5
The southern one of these	Bull 11⅔°	N¼°	5
The western one of the 2 small ones in the neck	Bull 7°	N⅔°	5
The eastern one of these	Bull 9°	S1°	6
The southern one of the western side of the square in the neck	Bull 8°	N5°	5
The northern one of the western side	Bull 8½°	N7⅓°	5
The southern one of the eastern side	Bull 12°	N3°	5
The northern one of the eastern side	Bull 11⅔°	N5°	5
The northern limit of the eastern side of the Pleiads	Bull 2⅙°	N4½°	5
The southern limit of the eastern side	Bull 2½°	N3⅔°	5
The eastern and narrowest limit of the Pleiads	Bull 3⅔°	N3⅓°	5
The small star outside and north of the Pleiads	Bull 3⅔°	N5°	4
In all, 33 stars, of which 1 of 1st magnitude, 7 of 3rd, 11 of 4th, 13 of 5th, 1 of 6th.			

Configurations	*Longitude*	*Latitude*	*Magn.*
The Unfigured Stars about the Bull			
The star below the right foot and the shoulder-blade	Ram 25°	S17½°	4
The western one of the 3 above the southern horn	Bull 20°	S2°	5
The middle one of the 3	Bull 21°	S1¾°	5
The eastern one of these	Bull 26°	S2°	5
The northern one of the 2 below the tip of the southern horn	Bull 29°	S6⅓°	5
The southern one of these	Bull 29°	S7⅔°	5
The western star of the 5 eastern ones under the northern horn	Bull 27°	N⅔°	5
The one east of this	Bull 29°	N1°	5
The one again east of this	Twins 1°	N1⅓°	5
The northern one of the 2 remaining eastern ones	Twins 2⅓°	N3⅓°	5
The southern one of these	Twins 3⅓°	N1¼°	5
In all, 11 stars, of which 10 of 4th magnitude, 1 of 5th.			
Constellation of the Twins			
The star in the head of the western Twin	Twins 23⅓°	N9½°	2
The red star in the head of the eastern Twin	Twins 26⅔°	N6¼°	2
The star in the left forearm of the western Twin	Twins 16⅔°	N10°	4
The star in the same arm	Twins 18⅔°	N7⅓°	4
The star east of this one and in the broad of the back	Twins 22°	N5½°	4
The one east of this in the right shoulder of the same Twin	Twins 24°	N4⅚°	4
The star in the eastern shoulder of the eastern Twin	Twins 26⅔°	N2⅔°	4
The star in the right side of the western Twin	Twins 21⅔°	N2⅔°	5
The star in the left side of the eastern Twin	Twins 23⅙°	N⅓°	5
The star in the left knee of the western Twin	Twins 13°	N1½°	3
The star under the left knee of the eastern Twin	Twins 18¼°	S2½°	3
The star in the left testicle of the eastern Twin	Twins 21⅔°	S½°	3
The star under the bend of the right knee of the same Twin	Twins 21⅔°	S6°	3
The star in the forward foot of the western Twin	Twins 6½°	S1½°	4+
The star east of this in the same foot	Twins 8½°	S1¼°	4+
The star at the end of the right foot of the western Twin	Twins 10⅙°	S3½°	4+
The star at the end of the left foot of the eastern Twin	Twins 12°	S7½°	3
The star in the right foot of the eastern Twin	Twins 14⅔°	S10½°	4
In all, 18 stars, of which 2 are of 2nd magnitude, 5 of 3rd, 9 of 4th, 2 of 5th.			
The Unfigured Stars about the Twins			
The western star of the forward foot of the western Twin	Twins 4⅙°	S⅔°	4
The bright western star of the western knee	Twins 6½°	N5⅚°	4+
The western star of the left knee of the eastern Twin	Twins 15⅙°	S2¼°	5
The northern star of the 3 in a straight line east of the right hand of the eastern Twin	Twins 28⅓°	S1⅓°	5
The middle one of the 3	Twins 26⅓°	S3⅓°	5
The southern one of these between the forearm and hand	Twins 26°	S4½°	5
The bright one east of these 3	Crab ⅔°	S2⅔°	4
In all, 7 stars, of which 3 are of 4th magnitude, 4 of 5th.			

Configurations	*Longitude*	*Latitude*	*Magn.*
Constellation of the Crab			
The middle of the nebula called the Crab, in the breast	Crab 10⅓°	N⅓°	neb.
The northern star of the 2 western ones of the square about the nebula	Crab 7⅔°	N1¼°	4–
The southern star of the 2 western ones	Crab 8°	S1⅙°	4–
The northern star of the 2 eastern ones of the square called the Asses	Crab 10⅓°	N2⅔°	4+
The southern one of these 2	Crab 11⅓°	S⅙°	4+
The star in the southern claw	Crab 16½°	S5½°	4
The star in the northern claw	Crab 8⅓°	N11⅚°	4
The star in the northern hind-foot	Crab 2⅔°	N1°	5
The star in the southern hind-foot	Crab 7⅙°	S7½°	4+
In all, 9 stars, of which 7 are of 4th magnitude, 1 of 5th, 1 nebula.			
The Unfigured Stars about the Crab			
The star above the joint of the southern claw	Crab 19⅔°	S2⅓°	4–
The star east of the tip of the southern claw	Crab 21⅙°	S5⅔°	4–
The western star of the 2 above the nebula	Crab 14°	N4⅚°	5
The eastern one of these	Crab 17°	N7¼°	5
In all, 4 stars, of which 2 are of 4th magnitude, 2 of 5th.			
Constellation of the Lion			
The star at the tip of the nostril	Crab 18⅓°	N10°	4
The star in the open mouth	Crab 21⅙°	N7½°	4
The northern one of the 2 in the head	Crab 24⅓°	N12°	3
The southern one of these	Crab 24⅙°	N9½°	3+
The northern one of the 3 in the throat	Lion ⅙°	N11°	3
The middle one of the 3, nearby	Lion 2⅙°	N8½°	2
The southern one of these	Lion ⅔°	N4½°	3
The one at the heart called Regulus	Lion 2½°	N⅙°	1
The one south of this, in the chest	Lion 3½°	S1⅚°	4
The star a little west of the one in the heart	Lion 0°	S4°	5
The star in the right knee	Crab 27⅓°	0°	5
The star in the right foreclaw	Crab 24⅙°	S3⅔°	5
The star in the left foreclaw	Crab 27⅓°	S4⅙°	4
The star in the left knee	Lion 2½°	S4¼°	4
The star in the left armpit	Lion 9⅙°	S⅙°	4
The western star of the 3 in the belly	Lion 7°	N4°	6
The northern one of the other eastern 2	Lion 10⅓°	N5⅓°	6
The southern one of these	Lion 12⅙°	N2⅓°	6
The western one of the 2 in the loin	Lion 11⅓°	N12¼°	6
The eastern one of these	Lion 14⅙°	N13⅔°	2–
The northern one of the 2 in the buttocks	Lion 14⅓°	N11⅙°	5
The southern one of these	Lion 16⅓°	N9⅔°	3
The star in the calfs of the legs	Lion 20⅓°	N5⅚°	3
The star in the hind-leg joints	Lion 21⅔°	N1¼°	4
The star south of this one, in the forelegs	Lion 24⅔°	S⅚°	4
The star in the hind-claws	Lion 27½°	S3⅕°	5
The star at the tip of the tail	Lion 24½°	N11⅚°	1–
In all, 27 stars, of which 2 are of 1st magnitude, 2 of 2nd, 6 of 3rd, 8 of 4th, 5 of 5th, 4 of 6th.			
The Unfigured Stars about the Lion			
The western star of the 2 above the back	Lion 6°	N13⅓°	5
The eastern one of these	Lion 8⅙°	N15½°	5

Configurations	*Longitude*		*Latitude*	*Magn.*
The northern one of the 3 above the flank	Lion	17½°	N1⅙°	4−
The middle one of these	Lion	17⅙°	S½°	5
The southern one of these	Lion	18°	S2⅔°	5
The northernmost part of the nebula called the Hair, lying between the extremities of the Lion and the Bear	Lion	24⅚°	N30°	dim
The western one of the eminent southern stars of the Hair	Lion	24⅓°	N25°	dim
The eastern one of those in the figure of the ivy leaf	Lion	28½°	N25½°	dim
In all, 5 stars, of which 1 is of 4th magnitude, 4 of 5th, and the Hair.				
Constellation of the Virgin				
The southern star of the 2 in the tip of the skull	Lion	26⅓°	N4¼°	5
The northern one of these	Lion	27°	N5⅔°	5
The northern one of the 2 in the face east of these	Virgin	⅔°	N8°	5
The southern one of these	Virgin	½°	N5½°	5
The star at the tip of the southern and left wing	Lion	29°	N⅓°	3
The western star of the 4 in the left wing	Virgin	8¼°	N1½°	3
The one east of this	Virgin	13⅙°	N2⅚°	3
The star east of this again	Virgin	17½°	N2½°	5
The last and eastern one of these 4	Virgin	21°	N1⅔°	4
The star in the right side under the girdle	Virgin	14⅓°	N8½°	3
The western star of the 3 in the right and northern wing	Virgin	8⅙°	N13½°	5
The southern of the 2 remaining ones	Virgin	10⅙°	N11⅔°	6
The northern one of these called Vindemiatrix	Virgin	12⅙°	N15⅙°	3+
The star in the left hand called Spica	Virgin	26⅔°	S2°	1
The star under the girdle on the right buttock	Virgin	24⅚°	N8⅔°	3
The northern star of the western side of the square in the left thigh	Virgin	26⅓°	N3⅓°	5
The southern star of the western side	Virgin	27¼°	N⅙°	6
The northern star of the 2 of the eastern side	Balance	0°	N1½°	4−
The southern one of the eastern side	Virgin	28°	S3°	5
The star in the left knee	Balance	1⅔°	S1½°	5
The star in the hinder part of the right thigh	Virgin	28°	N8½°	5
The middle one of the 3 in the train about the feet	Balance	6⅔°	N7⅙°	4
The southern one of these	Balance	7⅓°	N2⅔°	4
The northern one of the 3	Balance	8⅓°	N11⅔°	4
The star in the left and southern foot	Balance	10°	N½°	4
The star in the right and northern foot	Balance	12⅔°	N9⅚°	4
In all, 26 stars, of which 1 is of 1st magnitude, 6 of 3rd, 7 of 4th, 10 of 5th, 2 of 6th.				
The Unfigured Stars about the Virgin				
The western star of the 3 in a straight line under the left forearm	Virgin	14⅔°	S3½°	5
The middle one of these	Virgin	19°	S3½°	5
The eastern one of the 3	Virgin	22¼°	S3⅓°	5
The western one of the 3 in a straight line under Spica	Virgin	27⅙°	S7⅙°	6
The middle one of these which is double	Virgin	28⅙°	S8⅓°	5
The eastern one of these	Balance	5°	S7⅚°	6
In all, 6 stars, of which 4 are of 5th magnitude, 2 of 6th.				

BOOK EIGHT

1. Tabular Exposition of the Constellations of the Southern Hemisphere

Configurations	*Longitude*	*Latitude*	*Magn.*
Constellation of the Balance			
The bright star at the end of the southern claw (of the Scorpion)	Balance 18°	N⅔°	2
The one north of and dimmer than this last	Balance 17°	N2½°	5
The bright star of those at the end of the northern claw	Balance 22⅙°	N8⅚°	2
The dim one west of this	Balance 17⅔°	N8½°	5
The star in the middle of the southern claw	Balance 24°	S1⅔°	4
The star west of this in the same claw	Balance 21⅓°	N1¼°	4
The star in the middle of the northern claw	Balance 27⅚°	N4¾°	4
The one east of this in the same claw	Scorpion 3°	N3½°	4—
In all, 8 stars, of which 2 are of 2nd magnitude, 4 of 4th, 2 of 5th.			
The Unfigured Stars about the Balance			
The western star of the 3 northern ones in the northern claw	Balance 26⅙°	N9°	5
The southern star of the 2 eastern ones	Scorpion 3⅔°	N6⅔°	4—
The northern one of these	Scorpion 4⅓°	N9¼°	4—
The eastern one of the 3 between the claws	Scorpion 3½°	N½°	6
The northern one of the 2 remaining eastern ones	Scorpion ⅔°	N⅓°	5
The southern one of these	Scorpion 1⅙°	S1½°	4
The western star of the more southern 3 of the southern claw	Balance 23°	S7½°	3
The northern one of the 2 remaining eastern ones	Scorpion 1⅙°	S8½°	4
The southern one of these	Scorpion 2°	S9⅔°	4
In all, 9 stars, of which 1 is of 3rd magnitude, 5 of 4th, 2 of 5th, 1 of 6th.			
Constellation of the Scorpion			
The northern one of the 3 bright ones in the forehead	Scorpion 6⅓°	N1⅓°	3
The middle one of these	Scorpion 5⅔°	S1⅔°	3
The southern of the 3	Scorpion 5⅔°	S5°	3
The star south of this again, in one of the feet	Scorpion 6°	S7⅚°	3
The northern one of the 2 lying beside the northernmost of the bright ones	Scorpion 7°	N1⅔°	4
The southern one of these	Scorpion 6⅓°	N½°	4
The western star of the 3 bright ones in the body	Scorpion 10⅔°	S3¾°	3
The red middle one of these called Antares	Scorpion 12⅔°	S4°	2
The eastern one of the 3	Scorpion 14½°	S5½°	3
The western one of the 2 beneath these in the farthest foot	Scorpion 9⅓°	S6½°	5
The eastern one of these	Scorpion 10⅔°	S6⅔°	5
The star in the first joint from the body	Scorpion 18½°	S11°	3
The star after this one in the 2nd joint	Scorpion 18⅚°	S15°	3
The northern one of the double star in the 3rd joint	Scorpion 20°	S18⅔°	4

Configurations	*Longitude*	*Latitude*	*Magn.*
The southern one of the double	Scorpion 20⅙°	S18°	4
The next one in the 4th joint	Scorpion 23⅙°	S19½°	3
The star after this one in the 5th joint	Scorpion 28⅙°	S18⅚°	3
The star next to this last in the 6th joint	Archer ½°	S16⅔°	3
The star in the 7th joint near the center	Scorpion 29°	S15⅙°	3
The eastern one of the 2 in the center	Scorpion 27½°	S13⅓°	3
The western one of these	Scorpion 27°	S13½°	4
In all, 21 stars, of which 1 is of 2nd magnitude, 13 of 3rd, 5 of 4th, 2 of 5th.			
The Unfigured Stars about the Scorpion			
The nebula east of the center	Archer 1⅙°	S13¼°	neb.
The western star of the 2 north of the center	Scorpion 25½°	S6⅙°	5+
The eastern one of these	Scorpion 25½°	S4⅙°	5
In all, 3 stars, of which 2 are of 5th magnitude, a nebula.			
Constellation of the Archer			
The star at the tip of the arrow	Archer 4½°	S6½°	3
The star in the grip of the left hand	Archer 7⅔°	S6½°	3
The star in the southern part of the bow	Archer 8°	S10⅚°	3
The southern star of those in the northern part of the bow	Archer 9°	S1½°	3
The northern of these, at the tip of the bow	Archer 6⅔°	N2⅚°	4
The star in the left shoulder	Archer 15⅓°	S3⅙°	3
The star west of this in the arrow	Archer 13°	S3½°	4
The nebular and double star in the eye	Archer 15⅙°	N¾°	neb.
The western star of the 3 in the head	Archer 15⅔°	N2⅙°	4
The middle one of these	Archer 17⅔°	N1½°	4
The eastern one of the 3	Archer 19⅙°	N2°	4
The southern one of the 3 in the northern part of cloak	Archer 21⅓°	N2⅚°	5
The middle one of these	Archer 22⅓°	N4½°	4
The northern one of the 3	Archer 22⅚°	N6½°	4
The dim one east of these 3	Archer 25⅔°	N5½°	6
The northern one of the 2 in the southern part of cloak	Archer 29⅓°	N5⅚°	5
The southern one of these	Archer 27⅔°	N2°	6
The star in the right shoulder	Archer 22⅔°	S1⅚°	5
The star in the right elbow	Archer 24⅚°	S2⅚°	4
Of the 3 in the back, the star in the broad of the back	Archer 20°	S2½°	5
The middle one of these in the shoulder blade	Archer 17⅔°	S4½°	4+
The remaining one below the armpit	Archer 16⅓°	S6¾°	3
The star in the left fore-ankle	Archer 17⅔°	S23°	2
The star in the knee of the same foot	Archer 17°	S18°	2−
The star in the right fore-ankle	Archer 6⅔°	S13°	3
The star in the left thigh	Archer 27⅓°	S13½°	3
The star in the right forearm behind	Archer 23⅚°	S20⅙°	3
The western star of the northern side of the 4 at the beginning of the tail	Archer 27⅔°	S4⅚°	5
The eastern star of the northern side	Archer 28⅚°	S4⅚°	5
The western star of the southern side	Archer 28⅚°	S5⅚°	5
The eastern star of the southern side	Archer 29⅔°	S6½°	5
In all, 31 stars, of which 2 are of 2nd magnitude, 9 of 3rd, 9 of 4th, 8 of 5th, 2 of 6th, a nebula.			
Constellation of the Goat			
The northern star of 3 in the eastern horn	Goat 7⅓°	N7⅓°	3
The middle star of these	Goat 7⅔°	N6⅔°	6

Configurations	*Longitude*	*Latitude*	*Magn.*
The southern one of the 3	Goat 7⅓°	N5°	3
The star at the tip of the western horn	Goat 5°	N8°	6
The southern star of the 3 in the muzzle	Goat 9°	N¾°	6
The western one of the remaining 2	Goat 8⅔°	N1¾°	6
The eastern one of these	Goat 8⅚°	N1½°	6
The western one of the 3 under the right eye	Goat 6⅙°	N⅔°	5
The northern one of the 2 in the neck	Goat 11⅔°	N3⅚°	6
The southern one of these	Goat 11⅚°	N⅚°	5
The star in the left bent knee	Goat 11⅔°	S8⅔°	4
The star under the right knee	Goat 10⅚°	S6½°	4
The star in the left shoulder	Goat 16⅔°	S7⅔°	4
The western star of the 2 close together under the belly	Goat 20⅙°	S6⅚°	4
The eastern one of these	Goat 20⅓°	S6°	5
The eastern one of the 3 in the middle of the body	Goat 18½°	S4¼°	5
The southern one of the 2 remaining western stars	Goat 16⅔°	S4°	5
The northern one of these	Goat 16⅔°	S2⅚°	5
The western one of the 2 in the back	Goat 16⅔°	0°	4
The eastern one of these	Goat 21°	S⅚°	4
The western one of the 2 in the southern part of the thorn	Goat 23⅓°	S4¾°	4
The eastern one of these	Goat 25°	S4¼°	4
The western star of the 2 near the tail	Goat 24⅚°	S2⅙°	3
The eastern one of these	Goat 26⅓°	S2°	3
The western one of the 4 in the northern part of the tail	Goat 26⅚°	N⅓°	4
The southern one of the remaining 3	Goat 28⅔°	0°	5
The middle one of these	Goat 27⅞°	N2⅚°	5
The northern one of these at the tail end	Goat 28⅔°	N4⅓°	5
In all, 28 stars, of which 4 of 3rd magnitude, 9 of 4th, 9 of 5th, 6 of 6th.			
Constellation of the Water Bearer			
The star in the head of the Water Bearer	Water Bearer ⅓°	N15¾°	5
The brighter of the 2 in the right shoulder	Water Bearer 6⅓°	N11°	3
The dimmer one beneath this	Water Bearer 5⅙°	N9⅔°	5
The star in the left shoulder	Goat 26½°	N8⅚°	3
The star under it in the back, as if under the armpit	Goat 27⅓°	N6¼°	5
The eastern one of the 3 in the left hand in the strap	Goat 17⅔°	N5½°	3
The middle one of these	Goat 16⅙°	N8°	4
The western one of the 3	Goat 14⅔°	N8⅔°	3
The star in the right forearm	Water Bearer 9½°	N8¾°	3
The northern one of the 3 in the end of the right hand	Water Bearer 11⅔°	N10¾°	3
The western one of the remaining 2 northern ones	Water Bearer 12°	N9°	3
The eastern one of these	Water Bearer 13⅓°	N8½°	3
The western star of the 2 close together in the right socket	Water Bearer 6⅙°	N3°	4
The eastern one of these	Water Bearer 7°	N3⅙°	5
The star in the right buttock	Water Bearer 8⅔°	S⅚°	4
The southern star of the 2 in the left buttock	Water Bearer 1⅔°	S1⅔°	4
The northern one of these	Water Bearer 3⅙°	N¼°	6
The southern star of the 2 in the right shin	Water Bearer 11⅔°	S7½°	3
The northern one of these below the bend of the knee	Water Bearer 11⅓°	S5°	4
The star in the left calf	Water Bearer 4⅔°	S5⅔°	5
The southern one of the 2 in the left shin	Water Bearer 8⅓°	S10°	5
The northern one of these under the knee	Water Bearer 7⅚°	S9°	5
The western one of those in the flow of water from the hand	Water Bearer 15°	N2°	4

Configurations	*Longitude*	*Latitude*	*Magn*
The star near this last southwards	Water Bearer 15⅚°	N1⅙°	4
The star near this last after the curve	Water Bearer 17⅔°	S1⅙°	4
The star still east of this	Water Bearer 20°	S½°	4
The star in the curve south of this last	Water Bearer 20½°	S1⅔°	4
The northern one of the 2 south of this	Water Bearer 19°	S3½°	4
The southern one of the 2	Water Bearer 19⅚°	S4⅙°	4
The lone star distant from these towards the south	Water Bearer 20⅚°	S8¼°	5
The western star of the 2 close together after this one	Water Bearer 22⅔°	S11°	5
The eastern one of these	Water Bearer 23⅙°	S10⅚°	5
The northern one of the 3 in the nearby stream	Water Bearer 21⅔°	S14°	5
The middle one of the 3	Water Bearer 22⅙°	S14¾°	5
The eastern one of these	Water Bearer 23⅙°	S15⅔°	5
The northern one of the next 3, in like manner	Water Bearer 17°	S14⅙°	4
The southern one of the 3	Water Bearer 18⅓°	S15¾°	4
The middle one of these	Water Bearer 17½°	S15°	4
The western one of the 3 in the remaining stream	Water Bearer 11⅚°	S14¾°	4
The southern one of the remaining 2	Water Bearer 12⅓°	S15⅛°	4
The northern one of these	Water Bearer 13⅙°	S14°	4
The last star in the water and in the southern Fish's mouth	Water Bearer 7°	S20⅛°	1
In all, 42 stars, of which 1 is of 1st magnitude, 9 of 3rd, 18 of 4th, 13 of 5th, 1 of 6th.			
The Unfigured Stars about the Water Bearer			
The western star of the 3 stars east of the water's curve	Water Bearer 26⅔°	S15½°	4+
The northern one of the remaining 2	Water Bearer 29⅔°	S14⅔°	4+
The southern one of these	Water Bearer 29°	S18¼°	4+
In all, 3 stars of 4th magnitude.			
Constellation of the Fishes			
The star in the mouth of the western Fish	Water Bearer 21⅔°	N9¼°	4
The southern one of the 2 in the top of his head	Water Bearer 24⅙°	N7½°	4
The northern one of these	Water Bearer 26°	N9⅓°	4
The western one of the 2 in the back	Water Bearer 28⅙°	N9½°	4
The eastern one of these	Fishes ⅔°	N7½°	4
The western one of the 2 in the belly	Water Bearer 26°	N4½°	4
The eastern one of these	Water Bearer 29⅔°	N3½°	4
The star in the tail of the same Fish	Fishes 6°	N6⅓°	4
The first star from the tail, in the cord	Fishes 11°	N5¾°	6
The eastern one of these	Fishes 13°	N3¾°	6
The western star of the 3 bright ones following	Fishes 17⅙°	N2¼°	4
The middle one of these	Fishes 20½°	N1⅙°	4
The eastern one of the 3	Fishes 23°	S⅙°	4
The northern one of the 2 little ones in the curve under these	Fishes 22⅓°	S2°	6
The southern one of these	Fishes 23°	S5°	6
The western one of the 3 after the curve	Fishes 26½°	S2⅓°	4
The middle one of these	Fishes 28⅔°	S4⅔°	4
The eastern one of the 3	Ram ⅔°	S7¾°	4
The star in the knot of the 2 cords	Ram 2½°	S8½°	3
The star west of the knot, in the northern cord	Ram ½°	S1⅔°	4
The southern one of the next 3 after it	Ram ⅙°	N1⅚°	5
The middle one of these	Ram ⅔°	N5⅓°	3
The northern one of the 3, and at the tail's end	Ram ½°	N9°	4
The northern one of the 2 in the mouth of the eastern Fish	Ram 2°	N21¾°	5
The southern one of these	Ram 1⅔°	N21⅔°	5
The eastern of the 3 little ones in the head	Fishes 28⅔°	N20°	6

Configurations	*Longitude*	*Latitude*	*Magn.*
The middle one of these	Fishes 27⅔°	N19⅚°	6
The western one of the 3	Fishes 27°	N20⅓°	6
The western star of the 3 in the southern fin after the star in Andromeda's elbow	Fishes 25⅔°	N14⅓°	4
The middle one of these	Fishes 26⅓°	N13¼°	4
The eastern one of the 3	Fishes 27⅔°	N12°	4
The northern one of the 2 in the belly	Ram 2⅙°	N17°	4
The southern one of these	Fishes 29⅚°	N15⅓°	4
The star in the eastern fin near the tail	Ram 0°	N11¾°	4
In all, 34 stars, of which 2 are of the 3rd magnitude, 22 of the 4th, 3 of the 5th, 7 of the 6th.			
The Unfigured Stars about the Fishes			
The western star of the 2 northern ones of the square under the western Fish	Fishes 1⅙°	S2⅔°	4
The eastern one of these	Fishes 2¼°	S2½°	4
The western star of the southern side	Fishes ⅔°	S5½°	4
The eastern one of the southern side	Fishes 2⅓°	S5½°	4
In all 4 stars of 4th magnitude.			
—Thus, in all, 346 stars of the zodiac, of which 5 are of 1st magnitude, 9 of 2nd, 64 of 3rd, 133 of 4th, 105 of 5th, 27 of 6th, 3 nebulae, and the Hair.			
Constellation of the Sea-Monster			
The star at the nostril tip	Ram 17⅔°	S7¾°	4
The eastern star of the 3 in the muzzle, at the tip of the jaw	Ram 17⅔°	S12⅓°	3
The middle one of these in the middle of the mouth	Ram 12⅔°	S11½°	3
The western one of the 3, in the cheek	Ram 10½°	S14°	3
The star in the brow and eye	Ram 10⅙°	S8⅙°	4
The star north of this in the hair of the head	Ram 12⅔°	S6⅓°	4
The star west of these in the flowing hair	Ram 7⅔°	S4⅙°	4
The northern star of the western side of the square in the chest	Ram 3°	S24½°	4
The southern star of the western side	Ram 3⅓°	S28°	4
The northern star of the eastern side	Ram 6⅔°	S25⅙°	4
The southern star of the eastern side	Ram 7°	S27½°	3
The middle one of the 3 in the body	Fishes 22°	S25⅓°	3
The southern one of these	Fishes 23°	S30⅚°	4
The northern one of the 3	Fishes 25°	S20°	3
The eastern one of the 2 near the tail	Fishes 19⅔°	S15⅔°	3
The western one of these	Fishes 15°	S15⅔°	3
The northern star of the eastern side of the square near the tail	Fishes 11°	S13⅔°	5
The southern one of the eastern side	Fishes 10⅔°	S14⅔°	5
The northern star of the western side	Fishes 9⅓°	S13°	5+
The southern one of the western side	Fishes 9°	S14°	5+
The northern star of the 2 at the tail's tip	Fishes 4⅓°	S9⅔°	3−
The star at the tail's southern end	Fishes 5⅔°	S20⅓°	3
In all, 22 stars, of which 10 are of 3rd magnitude, 8 of 4th, 4 of 5th.			
Constellation of Orion			
The nebula in Orion's head	Bull 27°	S13⅓°	neb.
The bright red star in the right shoulder	Twins 2°	S17°	1−
The star in the left shoulder	Bull 24°	S17½°	2
The eastern star below this	Bull 25°	S18°	4−

Configurations	*Longitude*	*Latitude*	*Magn.*
The star in the right elbow	Twins 4⅓°	S14½°	4
The star in the right forearm	Twins 6⅓°	S11⅚°	6
The eastern double star of the southern side of the square in the right hand	Twins 6½°	S10°	4
The western star of the southern side	Twins 6°	S9¾°	4
The eastern star of the northern side	Twins 7⅓°	S8¼°	6
The western star of the northern side	Twins 6⅔°	S8¼°	6
The western star of the 2 in the club	Twins 1⅔°	S3¾°	5
The eastern one of these	Twins 4⅔°	S4¼°	5
The eastern one of the 4 stars in a straight line in the back	Bull 27⅚°	S19⅔°	4
The star west of this one	Bull 26⅓°	S20°	6
The star still west of this last	Bull 25⅓°	S20⅙°	6
The last and western one of the 4	Bull 24⅙°	S20⅔°	5
The northern one of those in the skin held by the left hand	Bull 20½°	S8°	4
The 2nd one from the northernmost	Bull 19⅓°	S8⅙°	4
The 3rd from the northernmost	Bull 18°	S10¼°	4
The 4th from the northernmost	Bull 16⅓°	S12⅚°	4
The 5th from the northernmost	Bull 15⅙°	S14¼°	4
The 6th from the northernmost	Bull 14⅚°	S15⅚°	3
The 7th from the northernmost	Bull 14⅚°	S17⅙°	3
The 8th from the northernmost	Bull 15⅓°	S20⅓°	3
The last and southernmost of those in the skin	Bull 16⅓°	S21½°	3
The western star of the 3 in the belt	Bull 25⅓°	S24⅙°	2
The middle one of these	Bull 27⅓°	S24⅚°	2
The eastern one of the 3	Bull 28⅙°	S25⅔°	2
The star in the handle of the dagger	Bull 23⅚°	S25⅚°	3
The northern one of the 3 bunched at the tip of the dagger	Bull 26½°	S28⅓°	4
The middle one of these	Bull 26⅔°	S29⅙°	3−
The southern one of the 3	Bull 27°	S29⅚°	3
The eastern star of the 2 under the tip of the handle	Bull 27⅔°	S30⅔°	4
The western one of these	Bull 26½°	S30⅚°	4
The bright star in the left foot common with the Water	Bull 19⅚°	S31½°	1
The northern one of those above the ball of the anklejoint in the shin	Bull 21°	S30¼°	4+
The star outside under the left heel	Bull 23⅓°	S31⅙°	4
The star under the right and eastern knee	Twins ⅙°	S33½°	3+
In all, 38 stars, of which 2 are of the 1st magnitude, 4 of the 2nd, 8 of the 3rd, 15 of the 4th, 3 of the 5th, 5 of the 6th, a nebula.			
Constellation of the River			
The star after the star in the foot of Orion at the beginning of the River	Bull 18⅓°	S31⅚°	4+
The star north of this one in the bend near Orion's shin	Bull 18½°	S28¼°	4
The eastern one of the 2 after this one	Bull 18°	S29⅚°	4
The western one of these	Bull 14⅔°	S28¼°	4
Again the eastern one of the next 2	Bull 13⅙°	S25⅚°	4
The western one of these	Bull 10⅙°	S25⅓°	4
The eastern one of the 3 after this last	Bull 6⅓°	S26°	5
The middle one of these	Bull 5½°	S27°	4
The western one of the 3	Bull 2⅚°	S27⅚°	4
The eastern star of the 4 in the next interval	Ram 27°	S32⅚°	3
The star west of this one	Ram 24⅓°	S31°	4
The star west again of this last	Ram 24⅙°	S28⅚°	3

Configurations	*Longitude*	*Latitude*	*Magn.*
The western one of the 4	Ram 22°	S28°	3
Likewise the eastern one of the 4 in the next interval	Ram 17⅙°	S25½°	3
The star west of this last	Ram 14½°	S23⅚°	4
The star again west of this	Ram 12⅙°	S23½°	3
The western one of the 4	Ram 10½°	S23¼°	4
The first star in the bend of the River and touching the chest of the Sea-Monster	Ram 5⅙°	S32⅙°	4
The star east of this	Ram 5⅚°	S34⅚°	4
The western one of the next 3	Ram 8⅚°	S38½°	4
The middle one of these	Ram 13⅚°	S38⅙°	4
The eastern one of the 3	Ram 17½°	S39°	4
The northern one of the western side of the 4 in a trapezium	Ram 21⅓°	S41⅓°	4
The southern one of the western side	Ram 21½°	S42½°	5
The western one of the eastern side	Ram 22⅙°	S43¼°	4
The eastern one of this side and last of the 4	Ram 24⅔°	S43⅓°	4
The northern one of the 2 distant stars to the east and close together	Bull 4⅙°	S50⅓°	4
The southern one of these	Bull 5°	S51¾°	4
The eastern one of the next 2 after the turn	Ram 28⅙°	S53⅚°	4
The western one of these	Ram 25⅚°	S53⅙°	4
The eastern one of the 3 in the next interval	Ram 17⅚°	S53°	4
The middle one of these	Ram 14⅚°	S53½°	4
The western one of the 3	Ram 11⅚°	S52½°	4
The last and bright star of the River	Ram ⅙°	S53½°	1
In all, 34 stars, of which 1 is of 1st magnitude, 5 of 3rd, 26 of 4th, 2 of 5th.			
Constellation of the Hare			
The northern star of the western side of the square down over the ears	Bull 19⅔°	S35°	5
The southern one of the western side	Bull 19⅚°	S36½°	5
The northern one of the eastern side	Bull 21⅓°	S35⅔°	5
The southern one of the eastern side	Bull 21⅓°	S36⅔°	5
The star in the chin	Bull 19⅙°	S39¼°	4+
The star in the left forefoot	Bull 16⅙°	S45¼°	4+
The star in the middle of the body	Bull 25⅚°	S41½°	3
The star under the belly	Bull 24⅚°	S44⅓°	3
The northern one of the 2 in the hind-feet	Twins 1°	S44⅙°	4+
The southern one of these	Bull 29°	S45⅚°	4+
The star in the loin	Twins 0°	S38⅓°	4+
The star at the tail's tip	Twins 2⅔°	S38⅙°	4+
In all, 12 stars, of which 2 are of 3rd magnitude, 6 of 4th, 4 of 5th.			
Constellation of the Dog			
The brightest and red star in the face called the Dog	Twins 17⅔°	S39⅙°	1
The star in the ears	Twins 19⅔°	S35°	4
The star in the head	Twins 21⅓°	S36½°	5
The northern one of the 2 in the neck	Twins 23⅓°	S37¾°	4
The southern one of these	Twins 25⅓°	S40°	4
The star in the chest	Twins 20½°	S42⅔°	5
The northern one of the 2 in the right knee	Twins 16⅙°	S41¼°	5
The southern one of these	Twins 16°	S42½°	5
The star at the end of the forefoot	Twins 11°	S41⅓°	3
The western star of the 2 in the left knee	Twins 14⅔°	S46½°	5
The eastern one of these	Twins 16⅙°	S45⅚°	5
The eastern star of the 2 in the left shoulder	Twins 24⅔°	S46⅙°	4
The western one of these	Twins 21⅔°	S47°	5

Configurations	*Longitude*	*Latitude*	*Magn.*
The star at the beginning of the left thigh	Twins 26⅔°	S48¾°	3−
The star under the belly betwen the thighs	Twins 23⅔°	S51½°	3
The star in the joint of the right foot	Twins 23°	S55⅙°	4
The star at the tip of the right foot	Twins 9⅔°	S53¾°	3
The star in the tail	Crab 2⅙°	S50⅔°	3−
In all, 18 stars, of which 1 is of 1st magnitude, 5 of 3rd, 5 of 4th, 7 of 5th.			
The Unfigured Stars about the Dog			
The star north of the Dog's head	Twins 19½°	S25¼°	4
The southernmost one of the 4 in a straight line under the hind-feet	Twins 10°	S61½°	4
The star north of this	Twins 11⅓°	S58¾°	4
The star still north of this	Twins 13°	S57°	4
The last and northern star of the 4	Twins 14⅙°	S56°	4
The western star of the 3 in a straight line east of these 4	Bull 28°	S55½°	4
The middle one of these	Twins ⅓°	S57⅔°	4
The eastern one of the 3	Twins 2⅓°	S59⅚°	4
The eastern star of the 2 bright ones under these	Bull 29°	S59⅔°	2
The western star of these	Bull 26°	S57⅔°	2
The last and southern star of the foregoing	Bull 22⅙°	S59½°	4
In all, 11 stars, of which 2 are of 2nd magnitude, 9 of 4th.			
Constellation of Procyon (Little Dog)			
The star in the neck	Twins 25°	S14°	4
The bright star in the hinder parts called Procyon	Twins 29⅙°	S16⅙°	1
In all, 2 stars, of which 1 is of 1st magnitude, 1 of 4th.			
Constellation of the Argus			
The western star of the 2 in the stern-post ornament	Crab 10⅓°	S42½°	5
The eastern one of these	Crab 14⅓°	S43⅓°	3
The northern one of the 2 close together above the Shield in the stern	Crab 8⅚°	S45°	4
The southern one of these	Crab 8⅔°	S46⅙°	4
The star west of these	Crab 5⅓°	S45½°	4
The bright star in the middle of the Shield	Crab 6⅓°	S47¼°	3
The western star of the 3 below the Shield	Crab 5⅓°	S49¾°	4
The eastern one of these	Crab 9⅓°	S49½°	4
The middle one of the 3	Crab 8½°	S49¼°	4
The star in the goose neck of the stern	Crab 14°	S49⅚°	4
The northern one of the 2 in the stern-keel	Crab 4°	S53°	4
The southern one of these	Crab 4°	S58⅔°	3
The northern of those in the deck of the poop	Crab 10⅙°	S55½°	5
The western one of the next 3	Crab 12⅙°	S58⅔°	5
The middle one of these	Crab 13⅔°	S57¼°	4
The eastern one of the 3	Crab 16½°	S57⅚°	4
The bright one east of these, on the deck	Crab 21⅙°	S58⅔°	2
The western one of the 2 dim stars under the bright one	Crab 18⅙°	S60°	5
The eastern one of these	Crab 21°	S59⅓°	5
The eastern star of the 2 above the bright one	Crab 23⅓°	S56⅔°	5
The western one of these	Crab 24⅓°	S57⅔°	5
The northern star of the 3 in the shields near the mastholder	Lion 5⅔°	S51½°	4+
The middle one of these	Lion 6⅙°	S55⅔°	4+
The southern one of the 3	Lion 4°	S57⅙°	4+

Configurations	*Longitude*		*Latitude*	*Magn.*
The northern one of the 2 close together under these	Lion	9 1/6°	S60°	4+
The southern one of these	Lion	9°	S61 1/4°	4
The southern one of the 2 in the middle of the mast	Lion	1/6°	S51 5/6°	3
The northern one of these	Crab	29 1/3°	S49°	3
The western star of the 2 near the tip of the mast	Crab	28°	S43 1/3°	4
The eastern one of these	Crab	29°	S43 1/2°	4
The star below the 3rd and eastern shield	Lion	14 1/6°	S54 1/2°	2
The star on the section of the deck	Lion	17 1/2°	S51 1/4°	2−
The star between the oars in the keel	Crab	11 1/6°	S63°	4
The dim star east of this	Crab	19°	S64 1/2°	6
The bright star east of this below the deck	Lion	0°	S63 5/6°	2
The bright star south of this in the lower keel	Lion	8 1/2°	S69 2/3°	2
The western star of the 3 east of this	Lion	15 1/6°	S65 2/3°	3
The middle one of these	Lion	21 1/3°	S65 5/6°	3
The eastern one of the 3	Lion	26°	S67 1/3°	2
Of the 2 east of these, the western one near the section	Virgin	1°	S62 5/6°	3
The eastern one of these	Virgin	8°	S62 1/4°	3
The western one of the 2 in the northern and western oar	Twins	4°	S65 5/6°	4+
The eastern one of these	Twins	20 1/6°	S65 2/3°	3+
Of the 2 in the remaining oar the western star called Canopus	Twins	17 1/6°	S75°	1
The last and eastern one of these	Twins	29°	S71 3/4°	3+
In all, 45 stars, of which 1 is of 1st magnitude, 6 of 2nd, 11 of 3rd, 19 of 4th, 7 of 5th, 1 of 6th.				
Constellation of the Water-Snake				
Of the 2 western stars in the head, the southern one in the nostrils	Crab	14°	S15°	4
The northern one of these, above the eye	Crab	13 1/3°	S13 1/6°	4
Of the 2 stars east of these, the northern one near the top of the head	Crab	15 1/3°	S11 1/2°	4
The southern one of these, in the open mouth	Crab	15 1/2°	S14 1/4°	4
The star east of them all in the jaw	Crab	17 1/2°	S12 1/4°	4
The western one of the 2 at the beginning of the neck	Crab	20 1/3°	S11 5/6°	5
The eastern one of these	Crab	23 1/3°	S13 2/3°	4
The middle one of the next 3 in the curve of the neck	Crab	28 5/6°	S15 1/3°	4
The eastern one of the 3	Lion	2/3°	S14 5/6°	4
The southernmost of these	Crab	28 1/2°	S17 1/6°	4
The dim northern star of the 2 close together in the south	Crab	29 1/6°	S19 3/4°	6
The bright one of these 2	Lion	0°	S20 1/2°	2
The western star of the 3 east of the curve	Lion	6°	S26 1/2°	4
The middle one of these	Lion	8 2/3°	S26°	4
The eastern one of the 3	Lion	11 1/6°	S23 1/4°	4
The western one of the next 3 almost in a straight line	Lion	18°	S24 2/3°	3
The middle one of these	Lion	20°	S23 1/4°	4
The eastern one of the 3	Lion	23°	S22 1/6°	3
The northern one of the 2 after the base of the Bowl	Virgin	1 1/2°	S25 3/4°	4+
The southern one of these	Virgin	2 1/3°	S30 1/6°	4
The western one of the 3 in the triangle after these	Virgin	12 1/6°	S31 1/3°	4
The middle and southern one of these	Virgin	14 1/2°	S33 1/6°	4
The eastern one of the 3	Virgin	16 1/6°	S31 1/3°	3
The star behind the Raven near the tail	Balance	0°	S13 2/3°	4+
The star at the tip of the tail	Balance	13 1/2°	S17 2/3°	4+
In all, 25 stars, of which 1 is of 2nd magnitude, 3 of 3rd, 19 of 4th, 1 of 5th, 1 of 6th.				

Configurations	*Longitude*		*Latitude*	*Magn.*
The Unfigured Stars about the Water-Snake				
The star in the southern part of the head	Crab	12½°	S23¼°	3
The star far east of those in the neck	Lion	11°	S16⅓°	3
In all, 2 stars of 3rd magnitude.				
Constellation of the Bowl				
The star in the base of the Bowl common to the Water-Snake	Lion	26⅓°	S23°	4
The southern one of the 2 in the middle of the Bowl	Virgin	2½°	S19½°	4
The northern one of these	Virgin	0°	S18°	4
The star on the southern edge of the face	Virgin	7°	S18½°	4+
The star on the northern edge	Lion	29⅓°	S13⅔°	4
The star on the southern handle	Virgin	9⅙°	S16⅙°	4−
The star on the northern handle	Virgin	1⅔°	S11½°	4
In all, 7 stars of 4th magnitude.				
Constellation of the Raven				
The star in the beak and common with the Water-snake	Virgin	15⅓°	S21⅔°	3
The star in the neck near the head	Virgin	14⅓°	S19⅔°	3
The star in the breast	Virgin	16⅔°	S18⅙°	5
The star in the western and right wing	Virgin	13½°	S14⅚°	3
The western star of the 2 in the eastern wing	Virgin	16⅔°	S12½°	3
The eastern one of these	Virgin	17°	S11¾°	4
The star at the end of the foot, common with the Water-Snake	Virgin	20½°	S18⅙°	3
In all, 7 stars, of which 5 are of 3rd magnitude, 1 of 4th, 1 of 5th.				
Constellation of the Centaur				
The southernmost of the 4 in the head	Balance	10½°	S21⅔°	5+
The northernmost of these	Balance	10°	S18⅚°	5+
The western star of the remaining 2 middles ones	Balance	9⅙°	S20½°	4+
The eastern and last one of these 4	Balance	10°	S20°	5+
The star on the left and western shoulder	Balance	6⅙°	S25⅔°	3
The star on the right shoulder	Balance	15⅔°	S22½°	3
The star on the left shoulder blade	Balance	9⅙°	S27½°	4
The northern star of the 2 western ones of the 4 in the wand	Balance	18⅙°	S22⅓°	4
The southern one of these	Balance	19⅙°	S23¾°	4
Of the remaining 2, the star at the tip of the wand	Balance	22°	S18¼°	4
The remaining one south of this	Balance	22½°	S20⅚°	4
The western one of the 3 in the right side	Balance	13⅓°	S28⅓°	4+
The middle one of these	Balance	14°	S29⅓°	4+
The eastern one of the 3	Balance	15⅙°	S28°	4+
The star in the right arm	Balance	16⅓°	S26½°	4+
The star in the right forearm	Balance	22⅚°	S25¼°	3
The star at the tip of the right hand	Balance	27½°	S24¼°	4
The bright star at the beginning of the human body	Balance	18°	S33½°	3+
The eastern star of the 2 dim stars north of this one	Balance	17⅔°	S31°	5
The western one of these	Balance	16⅚°	S33°	5
The star at the beginning of the back	Balance	12⅙°	S34⅚°	5
The star west of this last on the horse's back	Balance	9°	S37⅔°	5
The eastern star of the 3 in the loins	Balance	5⅚°	S40°	3
The middle one of these	Balance	5°	S43°	4
The western one of the 3	Balance	2⅔°	S41°	5
The western star of the 2 close together in the right thigh	Balance	2⅔°	S46⅙°	3
The eastern one of these	Balance	3½°	S46¾°	4

Configurations	*Longitude*	*Latitude*	*Magn.*
The star in the chest under the horse's armpit	Balance 18⅓°	S40¾°	4
The western star of the 2 under the belly	Balance 16⅓°	S43°	2
The eastern one of these	Balance 17⅔°	S43¾°	3
The star in the bend of the right foot	Balance 10°	S51⅙°	2
The star in the ankle of the same foot	Balance 15⅓°	S51⅔°	2
The star under the ankle of the left foot	Balance 6⅓°	S55⅙°	4
The star in the frog of the same foot	Balance 11⅙°	S55⅓°	2
The star at the tip of the right forefoot	Scorpion 8⅓°	S41⅙°	1
The star in the knee of the left foot	Balance 24⅙°	S45⅓°	2
The star outside under the right hind-foot	Balance 14⅔°	S49⅙°	4
In all, 37 stars, of which 1 is of 1st magnitude, 5 of 2nd, 7 of 3rd, 16 of 4th, 8 of 5th.			
Constellation of the Wild Beast (Lupus)			
The star at the end of the hind-foot near the Centaur's hand	Balance 28°	S24⅚°	3
The star in the hand of the same foot	Balance 25⅚°	S29⅙°	3
The western star of the 2 on the shoulder-blade	Scorpion 1°	S21¼°	4
The eastern one of these	Scorpion 4⅙°	S21°	4
The star in the middle of Wild Beast's body	Scorpion 3°	S25⅙°	4
The star in the belly under the flank	Scorpion ⅙°	S27°	5
The star in the thigh	Scorpion ½°	S29°	5
The northern one of the 2 near the beginning of the thigh	Scorpion 4⅔°	S28½°	5
The southern one of these	Scorpion 3⅔°	S30⅙°	5
The star at the end of the loins	Scorpion 5⅔°	S33⅙°	5
The southern star of the 3 at the tip of the tail	Balance 22°	S31⅓°	5
The middle one of the 3	Balance 21⅚°	S30½°	4
The northern one of these	Balance 23°	S29⅓°	4+
The southern star of the 2 in the neck	Scorpion 8⅚°	S17°	4
The northern one of these	Scorpion 9⅓°	S15⅓°	4+
The western one of the 2 in the muzzle	Scorpion 5⅔°	S13⅓°	4
The eastern one of these	Scorpion 6⅔°	S11⅚°	4
The southern one of the 2 in fore-foot	Balance 27⅙°	S11⅚°	4+
The northern one of these	Balance 26½°	S10°	4+
In all, 19 stars, of which 2 are of 3rd magnitude, 11 of 4th, 6 of 5th.			
Constellation of the Censer (Ara)			
The northern star of the 2 in the base	Scorpion 27⅔°	S22⅔°	5
The southern one of these	Archer 3°	S25¾°	4
The star in the middle of the altar-like vessel	Scorpion 26⅓°	S26½°	4+
The northern one of the 3 in the brazier	Scorpion 20⅔°	S1⅓°	5
The southern one of the remaining 2 contiguous ones	Scorpion 25⅙°	S34⅙°	4+
The northern one of these	Scorpion 25°	S33⅓°	4
The star at the end of the burning	Scorpion 20⅚°	S34¼°	4
In all, 7 stars, of which 5 are of 4th magnitude, 2 of 5th.			
Constellation of the Southern Crown			
The outside western star of the southern edge	Archer 9⅙°	S21½°	4
The eastern star of those on the Crown	Archer 11⅔°	S21°	5
The star east of this one	Archer 13⅙°	S23°	5
The star again east of this one	Archer 14⅚°	S20°	4
The star after this one in front of the Archer's groin	Archer 16⅙°	S18½°	5
The star after this one and north of the bright one in the knee	Archer 17°	S17⅙°	4
The star north of this one	Archer 16⅚°	S16°	4
The star still north of this one	Archer 16½°	S15⅙°	4

Configurations	*Longitude*	*Latitude*	*Magn.*
The eastern one of the 2 western ones following this last on the northern edge	Archer 15⅙°	S15⅓°	6
The western one of these 2 dim ones	Archer 14⅔°	S14⅚°	6
The star rather west of this one	Archer 11⅚°	S14⅔°	5
The star still west of this	Archer 9⅔°	S15⅚°	5
The last one south of this last	Archer 9⅙°	S18½°	5
In all, 13 stars, of which 5 are of 4th magnitude, 6 of 5th, 2 of 6th.			
Constellation of the Southern Fish			
The star in the mouth, the same as that at the beginning of the Water	Water Bearer 7°	S20⅓°	1
The western star of the 3 at the southern edge of the head	Water Bearer ⅔°	S20⅓°	4
The middle one of these	Water Bearer 4⅙°	S22¼°	4
The eastern one of the 3	Water Bearer 5⅓°	S22½°	4
The star near the gills	Water Bearer 4⅓°	S16¼°	4+
The star in the southern spinal fin	Goat 25⅙°	S19½°	5
The eastern one of the 2 in the belly	Water Bearer 1⅙°	S15⅙°	5
The western one of these	Goat 28⅚°	S14⅔°	4
The eastern one of the 3 in the northern fin	Goat 25⅙°	S15°	4
The middle one of these	Goat 21⅚°	S16½°	4
The western one of the 3	Goat 21°	S18⅙°	4
The star at the tip of the tail	Goat 20⅙°	S22¼°	4
In all, 12 stars, of which 1 is of the 1st magnitude, 9 of 4th, 2 of 5th.			
The Unfigured Stars about the Southern Fish			
The western one of the 3 bright stars west of the Fish	Goat 8°	S22⅓°	3−
The middle one of these	Goat 11⅙°	S22⅓°	3−
The eastern one of the 3	Goat 11°	S21⅓°	3−
The dim star west of this	Goat 12°	S20⅚°	5
The southern star of the remaining 2 to the north	Goat 13⅚°	S17°	4
The northern one of these	Goat 13⅚°	S14⅚°	4
In all, 6 stars, of which 3 are of 3rd magnitude, 2 of 4th, 1 of 5th.			

Thus there are 316 stars in the southern part, of which 7 are of 1st magnitude, 18 of 2nd, 63 of 3rd, 164 of 4th, 54 of 5th, 9 of 6th, and 1 nebula.

Thus altogether there are 1022 stars, of which 15 are of 1st magnitude, 45 of 2nd, 208 of 3rd, 474 of 4th, 217 of 5th, 49 of 6th, 9 dim ones, 5 nebulae, and the Hair.

2. On the Position of the Milky Way

Now this is the exposition we give of the fixed stars, and next we add on the things having to do with the position of the Milky Way as far as we can and to the extent we have observed each of its parts, trying to represent its particular appearances.

It is easily seen that the Milky Way is not simply a circle but a zone having quite the color of milk, whence its name; and that it is not regular and ordered, but different in width, color, density, and position; and that in one part it is double. These particulars we find needing careful observation.

Now, the double part of this zone has one of its junctions near the Censer [Ara] and the other at the Bird [Cygnus]. The western zone nowhere touches the other, for it divides at the junction at the Censer and at the junction at the Bird, the eastern zone joining the other part of the Milky Way and making one

zone which the great circle drawn in its middle would traverse. We shall first discuss this zone beginning with its southernmost parts.

These parts go through the Centaur's feet, and are thinner and fainter. The star in the bend of the right hind-foot is a little south of the northern line of the Milky Way; likewise the star in the left fore-knee and the star under the right hind-ankle. The star in the left hind shank lies in the middle of the Milky Way, the star in the same ankle and the star in the right fore-ankle being very nearly 2° north of the southern arc. And the parts about the hind-feet are rather denser.

Next, the northern arc of the Milky Way is very nearly 1½° from the star in the loin of the Beast [Lupus], and the southern arc intercepts the star at the Censer's ember, touching the more northern of the two consecutive stars in the brazier, and the more southern of the two in the base. The star in the northern part of the brazier and the star in the middle of the brazier lie in the Milky Way itself. And these parts are thinner.

Next, the northern part of the Milky Way intercepts the three joints preceding the Scorpion's centre and the nebular mass following the centre. The southern arc touches the star in the Archer's right fore-ankle and intercepts the star in his left hand. The star in the southern part of the Archer is outside the Milky Way, and the star at the arrow's point is in the middle of it. In the northern part of the Archer each of the stars lies a little more than 1° from either arc: the southern one from the southern arc, and the northern one from the opposite arc. And the parts about the three joints are slightly denser, the parts about the arrow's point appearing quite dense and smoky.

The following parts are slightly thinner, and extend along the Eagle, keeping the same width. The star in the tip of the tail of the Serpent which the Serpentarius [Ophiouchus] holds, lies in the open sky a little more than 1° from the western arc of the Milky Way. The two western stars of the bright ones lying under it lie in the Milky Way itself, the more southern one 1° from the eastern arc and the more northern 2°. Of those in the Eagle's right shoulder the eastern star touches the same arc; and the western one is cut off within it, and likewise the bright western star of those in the left wing. And the bright star in the back and the two in a straight line with it just fail to touch the same arc.

After these parts, the whole Arrow [Sagitta] is enclosed in the Milky Way, and the star at the point is 1° from the eastern arc, and the star at the notched end is 2° from the western arc. The parts near the Eagle are slightly denser, and the rest slightly thinner.

The Milky Way next passes through the Bird. The northerwestern arc is bounded in its curve by the star in the Bird's southern shoulder, by the star under it in the wing, and by the two stars in the southern foot. The southeastern arc is bounded by the star in the tip of the southern flat of the wing and it encloses within itself the two figured stars under the same wing which are nearly 2° from it. The parts about the wings are slightly thicker.

The parts following are included in this zone and are much denser and as if coming from another beginning. For they lean to the extreme parts of the other zone, and, making a gap with it on the southern side, they now join with the zone we have just spoken of which is very thin at the junction. After the gap in density with respect to the other zone, they begin from the bright star in the Bird's rump and from the nebular mass in the northern knee. Then, turning

slightly as far as the star in the southern knee, they extend thinned out, as far as the tiara of Cepheus, and are bound side by the southern star of the three in the tiara and by th three at which they make two eminences, one pointing to other to the southeast.

After these parts, the Milky Way encloses the whole of Ca star at the tip of the foot. The southern arc is bounded by peia's head, and the northern arc by the star in the foot of star in Cassiopeia's calf. The other stars about her all lie i Near the arcs the parts are of a thinner mass, while thos Cassiopeia show an extended density.

And next, the right side of Perseus is enveloped by the M star outside Perseus' right knee bounds the northern side wh and the bright star in the right rib and the two eastern stars of it bound the southern side which is the densest. The n handle, the star in the head, the star in the right shoulder, right elbow are all enveloped in it. The quadrilateral in the ri the star in the calf of the same leg lie in the middle of the M in the right heel being itself a little within the southern sid

And after these parts the zone, showing a slightly thinner the Charioteer. The star in the right shoulder called the Goa two in the right forearm just fail to touch the northeaster Way. The little star above the left foot in the part around t southwestern side, and the star ½° above the right foot is wi In the left forearm the two consecutive stars, called the Kid of the zone.

And, next, the Milky Way goes through the feet of the rather extensive density below the stars in the tip of the fee of the three in a straight line under the right foot of the Cha star of the two in Orion's club, and the northern stars of the bound the western arc of the Milky Way. The bright sta oteer's right hand and the star in the eastern foot of the east nearly 1° within the eastern side. And the stars in the other feet lie in the middle of the Milky Way.

From there the zone passes by the Little Dog and the Dog of the Little Dog quite far outside the Milky Way to the almost wholly outside to the west. For a cloud here envel back, and next it almost touches the three east of it in the D lone star above the Dog's head, outside and farther off, is ver

side, and the two bright stars in the same piece of the keel ar
2° within the western arc.

From these the Milky Way now joins with the zone through t
feet. And the mass through the Argus is rather light, but the pa
little shield are thicker than it, and likewise those about the s
about the segment of the keel.

The zone we have just spoken of, dividing off as we said from
scribed at the Censer and beginning from these, envelops the three
Scorpion's body and leaves the eastern star of the three in the bo
the western arc. And the star in the fourth joint lies in the clear s
the two zones, nearly equidistant from both and a little more than

And after this, the western zone turns eastward 1° likewise, an
western side of the Milky Way with the star in the right knee of O
the eastern side with the star in the same shin. The western star in
same foot touches the same side. And next again the star under th
of the Serpentarius bounds the western arc, and the principal star
the tip of the same hand bounds the eastern arc. At the point the
large gap of clear space in which lie the two stars in the Serpent's
one in the tip. And the whole part of the zone just given is a mass o
air-like except for the part enveloping the three joints. For that pa
dense.

And after this gap the Milky Way again begins from the four sta
right shoulder of the Serpentarius, and the bright line star touchin
tail bounds the eastern arc of this zone, the farthest away to the
four stars spoken of above bounding the opposite arc.

From here the zone, after this, narrows into the western parts
beak so that it gives the appearance of a gap. But the rest of it fro
the Bird's beak to the star in the Bird's breast is rather wider and
star in the Bird's throat lies in the middle of the concentration. Bu
runs northward from the stars in the breast to the star in the sh
right wing and the two consecutive stars in the tip of the right foot
as we said before, there is a clear gap relative to the other zone fr
mentioned stars of the Bird to the bright star in the Rump.

3. On the Construction of the Solid Sphere

Now, the appearances of the Milky Way have this position. And
we may construct, by means of a solid sphere, a model in accord
potheses demonstrated with respect to the sphere of the fixed star
to which it appeared to be revolved, along with the planets, by the

many sections as appear feasible. Then, making out of a strong tensile material two circles with surfaces at right angles and everywhere accurately turned (the lesser everywhere touching the curved surface of the sphere, and the other a bit larger) we shall draw lines in the middle of the curved surface of each, dividing their widths exactly in two. By means of these lines, cutting one or the other of the sides so defined into semicircles, we shall divide the semicircles resulting from these cuts into 180 sections. This done, we suppose the lesser circle always to be through both sets of poles—that is, of the equator and of the ecliptic, and also through the tropic points on the surface of this cut. Boring it in the middle diametrically opposite at the limits of the cut, we shall fit it on pins at the poles of the ecliptic taken on the sphere so that it can revolve over the whole spherical surface.

Now, to get some constant beginning for the constellations of the fixed stars, since it is not plausible to mark the tropic and equinoctial points on the sphere's ecliptic (for the distance of the constellations to them is not preserved), we shall mark the brightest of them, I mean the star in the Dog's mouth [Sirius], on the circle drawn at right angles to the ecliptic at the section beginning the division, at the given latitude from the ecliptic in the direction of its south pole. For each of the other fixed stars, according to the sequence of the Catalogue, we shall make marks by the turning the circle which is divided by the cut, about the ecliptic's poles. For we always carry the surface of its cut side to the point on the ecliptic as many degrees away from the beginning of the numbers at the section of the Dog as the desired star is in longitude from the Dog according to the Catalogue. And we go to the point of the side which has been divided and swung about, according as the star in the Catalogue is either towards the north or south pole of the ecliptic. And there we mark the point of the star, affixing a yellow or some other distinct color in proportion and in accordance with the size of the magnitudes of each.

Now, we shall make the figures of each of the constellations as simple as possible, enclosing the stars under the same contour, only with lines not differing much from the color of the whole sphere so that the advantage in distinguishing them may not be lost and the juxtaposition of many colors may not blot out the likeness to the true picture; and so that we may easily retain the comparison made in our observations when we have become habituated to the original image of the stars in the spherical representation.

We also arrange the position of the Milky Way in accordance with the places and figures and further in accordance with the densities and gaps, already explained. And we fit the greater circle, which is always to be the meridian, on the smaller one which is enveloping the sphere, about the same poles as those of the equator. And the pole-points are inserted, diametrically opposite, in the greater circle or meridian, at the extremities of the side which has been cut and divided and which is to lie above. But the pole-points are inserted in the smaller circle, which passes through both sets of poles, at the extremities diametrically opposite of arcs extending the 23°51′ of inclination either side of the ecliptic's poles. And little solids are put at the cuts of the circle where the holes of the poles are[1].

[1]The smaller circle is attached to the solid sphere, and these poles are the ecliptic's and do not change with respect to the fixed stars. But the larger circle is attached to the smaller at points 23°51′ away from these. These are the poles of the equator which change with respect to the fixed stars.

Each time we place the cut side of the smaller circle, which is always the same of course as the meridian through the tropic points, at that point of division of the ecliptic which is as far away from the beginning of the Dog as the Dog at the given time is from the summer tropic. At the beginning of the reign of Antonine that is 12⅓° west. We fit the meridian circle at right angles to the horizon, which is the base plane, so that it cuts its apparent surface in two and can revolve in its own plane in such a way that each time we can raise the north pole above the horizon by the amount proper to the given latitudes. And this is done with the help of the division of the meridian circle.

It makes no difference if we cannot put the equator and tropic points on the sphere itself. For the point on the divided side of the meridian circle, between the poles of the equator and lying the 90° of a quadrant's distance from either one, will serve the same purpose as the points of the equator. And the points lying 23°51′ either side of this point will serve the same purpose as either of the tropics, the northern one the same as the summer tropic, and the southern one the same as the winter tropic. And so, when the desired stars are carried by the prime east-west revolution to the divided side of the meridian circle, then, by means of that division, their distances with respect to the equator and tropics can be gotten as if on the circle through the poles of the equator.

4. On the Configurations Proper to the Fixed Stars

Now that the character of the constellations of the fixed stars has been demonstrated, there remains the discussion of their configurations. Of the configurations concerning the fixed stars besides those dealing only with their relations to each other (as when they are in a straight line or in a triangular figure or some such thing), there are those which are considered with respect only to the planets and sun and moon or parts of the ecliptic, and those which are considered with respect only to the earth. And then there are those which are considered with respect to the earth, and at the same time with respect to the planets and sun and moon or parts of the ecliptic.

Those configurations of the fixed stars which are with respect only to the planets and parts of the ecliptic, are gotten in general whenever the fixed stars and the planets are on one and the same great circle drawn through the ecliptic's poles, or on a different one but forming triangular or tetragonal or hexagonal distances—that is, forming an angle which is either right or greater or less than the third of a right angle. And they are gotten in particular in the case of those fixed stars over which some one of the planets can pass (and these are those arranged in the band of the zodiac containing the latitudinal passages of the planets): with respect to the five planets, at their apparent appulses or occultations, and with respect to the sun and moon, at the heliacal or lunar settings, conjunctions, and heliacal or lunar risings. We call heliacal or lunar setting any star begins to disappear in the rays of the luminaries; conjunction whenever the star is in a straight line with the centre; and heliacal or lunar rising whenever it begins to appear from the rays.

There are four configurations of the fixed stars with respect to the earth alone: by some they are called in general "centres," and in particular, they are rising, upper culmination, setting, and lower culmination. Where the equator is at the zenith, all the fixed stars both rise and set, and once at each revolution have an upper and lower culmination; for then the equator's poles touch the

horizon, making none of the parallel circles either one of perpetual apparition or one of perpetual occultation. Where the poles are at the zenith, no one of the fixed stars either rises or sets; for then the equator has the horizon's position and one of the hemispheres formed by it revolves always above the earth and the other always below the earth, so that each of the fixed stars culminates twice in one revolution, those above the earth and those below. In the other latitudes between these, where there are some circles always visible and some always invisible, the stars enclosed by these relative to the poles neither rise nor set, and two culminations are effected at each revolution—those within the circle of perpetual apparition above the earth, and those within the circle of perpetual occultation below the earth. And the rest on the greater parallels both rise and set, culminating once above and once below in each revolution.

And their time from one of these centres back again to the same is always the same. For it embraces sensibly one revolution. And the time from one of the centres to the diametrically opposite one relative to the meridian is always the same, for it embraces half a revolution. With respect to the horizon, when the equator is at the zenith, the same is again true. For the time of each embraces half a revolution, since each of the parallels is bisected not only by the meridian but also by the horizon. But in the case of the other inclinations neither the time above nor the time below is itself equal for all, nor in each case is the time above equal to the time below, except for those on the equator itself which alone in the oblique sphere is bisected by the horizon; for all the other parallels are cut in dissimilar and unequal arcs. Consequently, the time from the rising or setting to one of the culminations is in each case equal to the time from the same culmination to the rising or setting, because the meridian divides the sections of the parallels above the earth and those below into equal parts. But the times from the rising or setting to either culmination are unequal in the inclined sphere, and equal in the right sphere, since there only are the lower sections equal to the upper sections. Therefore, in the right sphere, the stars which culminate together always rise and set together in so far as their shift about the poles of the ecliptic is imperceptible. But in the oblique sphere those stars which culminate together neither rise nor set together, the more southern always rising later than the more northern and set earlier.

And the configurations of the fixed stars, considered with respect to the earth or planets or parts of the ecliptic together, are gotten in general from the simultaneous risings or culminations or settings either with one of the planets or with some part of the ecliptic; and in particular they are considered with respect to the sun according to nine kinds.

The first kind of configuration, called early morning east wind, is when the star at rising is with the sun. And of this there is a kind called invisible morning after-rising when the star, beginning its heliacal setting, rises immediately after the sun. And there is a kind called true morning co-rising when the star rises together with and at the same time as the sun on the horizon. And there is a kind called visible morning fore-rising when the star, beginning its heliacal rising, rises before the sun.

The second configuration, called early morning culmination, is when the star, with the sun rising at the horizon, is culminating either above or below the earth. And again of this there is a kind called invisible morning after-culmination when the star culminates immediately after sunrise. And there is a kind

called true morning co-culmination when the star culminates as the sun rises. And there is a kind called morning fore-culmination when the sun rises immediately after the star's culmination.

The third kind of configuration, called early morning west wind, is when the star is on the western horizon while the sun is on the eastern. Again of this there is a kind called invisible morning after-setting when the star sets immediately after the sun rises. And there is a kind called true morning co-setting when the star sets at the same time the sun rises. And there is a kind called visible morning fore-setting when the sun rises immediately after the star sets.

The fourth configuration, called noontide east wind, is when the star is on the eastern horizon while the sun is on the meridian. And again of this there is a kind called invisible daytime when the star rises as the sun culminates above the earth; and a kind called visible night time when the star rises as the sun culminates below the earth.

The fifth configuration called noontide culmination is when the sun and star are on the meridian together. And of this there are two invisible daytime kinds when, with the sun culminating above the earth, the star either culminates with it above the earth or culminates diametrically opposite below the earth. And there are two night-time kinds with the sun culminating below the earth. Of these there is the invisible when the star culminates with the sun below the earth, and the visible when the star culminates diametrically opposite above the earth.

The sixth configuration, called noontide west wind, is when the star is on the western horizon while the sun is on the meridian. And again of this there is the invisible daytime kind when the star sets as the sun culminates above the earth, and there is the visible night-time kind when the star sets as the sun culminates below the earth.

The seventh configuration, called late east wind, is when the star is on the eastern horizon while the sun is on the western. And again of this there is the kind called visible evening after-rising when the star rises immediately after sunset. And there is the kind called true evening co-rising when the star rises exactly as the sun sets. And there is the kind called invisible evening fore-rising when the sun sets immediately after the star rises.

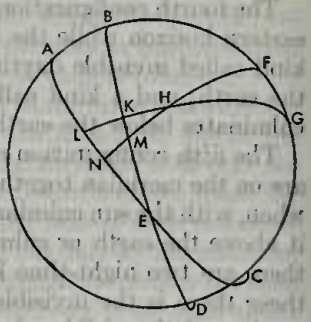
The eighth configuration, called late culmination, is when, with the sun on the western horizon, the star is on the meridian either above or below the earth. And of this again there is the kind called visible evening after-culmination when the star culminates immediately after sunset. And there is the kind called true evening co-culmination when the star culminates exactly as the sun sets. And there is the kind called invisible evening fore-culmination when the sun sets immediately after the star culminates.

The ninth configuration, called late west wind, is when the star is on the western horizon together with the sun. And of this kind again there is the kind called visible evening fore-setting when the star, beginning its heliacal setting, sets immediately after the sun. And there is the kind called true evening co-setting when the star sets exactly with the sun. And there is the kind called invisible evening fore-setting when the star, beginning its heliacal rising, sets before the sun.

5. On the Co-risings, Co-culminations, and Co-settings of the Fixed Stars

Now with matters this way, the times of the true co-risings, co-culminations, and co-settings considered with respect to the sun's centre can be gotten geometrically by us from their position according to the star catalogue. Because the points on the ecliptic with which each of the fixed stars culminates, rises, and sets, can be shown geometrically by the following theorems.

For first, with a view to the co-culminations, let there be the circle *ABCD* through both sets of poles, the equator's and the ecliptic's; and let there be the semicircle *AEC* of the equator about the pole *F*, and the semicircle *BED* of the ecliptic about the pole *G*. Let the segment *GHKL* of a great circle be drawn through the poles of the ecliptic. And let the fixed star in question be thought at the point *H* on it. For their positions with respect to the circles so drawn are available from our observation and record. And let the segment *FHMN* of the great circle through the poles of the equator and through the star at *H* be drawn.



Now, it is evident that the star at *H* culminates with the points *M* and *N* of the equator and ecliptic; and that these points and the arc *HN* are given will be clear from these considerations. For, by things demonstrated in the first part of the Composition, since the arcs *GL* and *NF* of great circles are drawn through to the arcs *AG* and *AN* of two great circles,

chord 2 arc *GA* : chord 2 arc *AF* comp. chord 2 arc *GL* : chord 2 arc *LH*, chord 2 arc *NH* : chord 2 arc *FN*.

But each of the arcs *AF*, *FN*, and *GK* is supposed a quadrant, and from the star's record are given the arc *KH* in latitude, the arc *KB* in longitude, and arcs *FG* and *KL* from the inclination of the ecliptic already demonstrated. Therefore it is clear that the desired arcs *GA*, *AF*, *GL*, *LH*, and *NF* are given, and the remaining arc *NH* is hence given.

Again, since

chord 2 arc *FG* : chord 2 arc *GA* comp. chord 2 arc *FH* : chord 2 arc *HN*, chord 2 arc *NL* : chord 2 arc *LA*

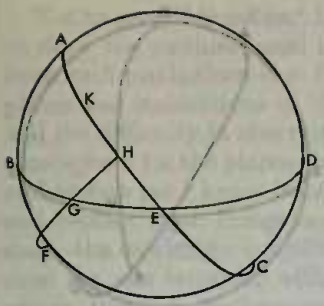
and the desired arcs *FG*, *GA*, *FH*, and *HN* are given by the preceding steps, and the arc *LA* from the arc *KB* by means of the co-risings of the equator and ecliptic in the right sphere, therefore the remaining arc *NL* is given.

The points of the equator and ecliptic co-rising and co-setting with the fixed stars are easily gotten from the co-culminations in this way.

For let there be the meridian circle *ABCD*, the semicircle of the equator *AEC* about pole *F*, and the semicircle of the horizon *BED*. And let the star rise at the point *G* of the horizon. Through the points *F* and *G* let the quadrant *FGH* of a great circle be drawn.

Now, since again the arcs *FH* and *EB* have been drawn through to the arcs of two great circles *AF* and *AE*, therefore

chord 2 arc *FB* : chord 2 arc *BA*
comp. chord 2 arc *FG* : chord 2



arc GH, chord 2 arc HE : chord 2 arc AE. But each of the desired arcs FA, FH, and EA embraces a quadrant, and the arc FB is given from the elevation of the poles, and the equator's point H and the arc HG by means of the co-culminations. And, therefore, the remaining arc HE is given.

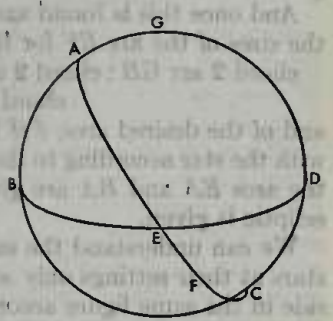
It is easily understood that in the case of the co-settings—if we take an arc equal to arc HE west of H, such as HK—the star will set with the point K of the equator because the setting at that time takes place on the arc equal to BG, and an angle west of the meridian is intercepted equal to that contained, according to this figure, to the east by the arcs AF and FH.

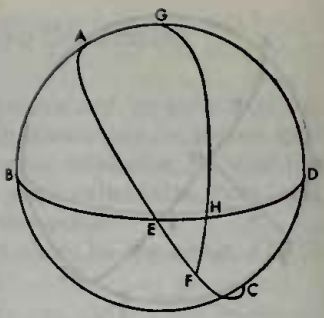
From the things demonstrated for the co-risings and co-settings of the equator and ecliptic for each latitude, the part of the ecliptic rising with the point E of the equator and with the star will be given; and the part setting with point K and the star also. It is clear that, at the times the sun is truly at those points of the ecliptic, the risings, culminations, and settings of the fixed stars considered with respect to its centre, and called the true co-centerings, will be accomplished.

6. On the Heliacal Risings and Settings of the Fixed Stars

Now, in the case of the heliacal risings and settings we no longer find the geometrical method worked out from their positions alone sufficient (since, for example, with whatever point of the ecliptic the star is shown to rise by these means, it is still impossible similarly to find out at what distance of the sun from the horizon beneath the earth it will first appear or be hidden). For this distance of arc cannot be equal for all the stars nor for the same everywhere, but differs according to the magnitudes of the stars, their latitudinal elongations from the sun, and the change in the inclinations of the ecliptic.

If we suppose $ABCD$ to be the meridian circle, $AEFC$ the semicircle of the ecliptic, and BED the semicircle of the horizon about pole G, then it is clear that (when the stars are rising at point E of the ecliptic) if the greater begins to appear when the sun, for instance, is below the earth a distance of arc EF, then the smaller (if it has an equal latitudinal elongation from the sun) will first appear when the sun is distant by a greater arc EF and makes thinner rays. Again, in the case of stars of equal magnitude, if the star nearer the point E in latitude appears first at a distance EF, then the one farther away will appear at a less distance, because, with the sun the same distance below the earth, there are more rays near the ecliptic and the sun than remote from them. And, in the case of stars of equal magnitude and rising at the same latitudinal elongation, the more the ecliptic is inclined to the horizon and makes a less angle DEF, at so much greater a distance EF will the star first appear.





For if we add on, as in the following figure, the semicircle *HFK* through the poles of the horizon and through the sun at *F* (which is, of course, at right angles to the horizon) then the sun's distance under the earth for the same stars is always equal to arc *FH*. For the rays above the earth are similar for equal distances considered in this way. And, with arc *HF* remaining the same as we said, arc *EF* will be less when the ecliptic is perpendicular to the horizon, and greater when it is inclined.

It is therefore necessary to take observations for each of the stars to get the sun's distance beneath the earth along the ecliptic. If, now, the distance along the circle at right angles to the horizon (as arc *FH* in the above figure) does not remain the same in all places for the same stars because similar rays do not shine the same in the denser air of the northern climate, then we shall need observations for not only one climate, but also for each of the rest. But if the arc corresponding to *FH* remains everywhere the same for the same stars, as is likely (for the stars' rays would necessarily suffer in the same way from the difference in the air) then the distances observed for one climate will suffice for examining the rest geometrically, whether the inclination of the ecliptic changes because of difference in place or because of the demonstrated shift of the sphere of the fixed stars towards its eastern parts.

For let the distance *EF* be given from observation in some latitude. Since then again the arcs *BH* and *FA* have been drawn through to the arcs *GB* and *GF* of two great circles, therefore,

chord 2 arc *AB* : chord 2 arc *BG* comp. chord 2 arc *AE* : chord 2 arc *EF*,
chord 2 arc *FH* : chord 2 arc *HG*.

But of the desired arcs, *BG* and *HG* are each a quadrant. And since point *E* is supposed to be that with which the star rises, the point *A* of culmination is also given from the Table of Ascensions, so that arc *AE* is therefore given and *EF* from observation. The arc *AG* is given by calculation of the distance of the point *A* from the equator which, in turn, is given in the Table of Obliquity, and from the distance of the equator from the zenith along the same meridian, which is equal to the pole's elevation. And, therefore, the remaining arc *FH* will be given.

And once this is found and is kept the same, we shall get, by means of this, the sizes of the arc *EF* for the other inclinations. For again

chord 2 arc *GB* : chord 2 arc *AB* comp. chord 2 arc *GH* : chord 2 arc *FH*,
chord 2 arc *FE* : chord 2 arc *EA*;

and of the desired arcs, *FH* is now assumed, and the point *E* is given as rising with the star according to the latitude in question as shown before; and likewise the arcs *EA* and *BA* are given. And, therefore, the remaining arc *EF* of the ecliptic is given.

We can understand the same kind of method for the disappearances of the stars at their settings only with the position of the ecliptic drawn on the other side in the same figure according to the inclination so that *BD* is taken as the western horizon.

Not to pass this subject by entirely, we think we have treated it sufficiently to show the methods used in this theory. And because the generality induced from such predictions is so fluctuating not only from the great difference of the places and inclinations of the ecliptic but also from the multitude of the stars and the difficulty in observing their heliacal risings, since the observers and the atmosphere for the places observed can make the time of the first glimpse unlike and unsure (as I know from trial and from the differences in the observations) and besides these things because, through the shift of the sphere of the fixed stars, the co-risings, co-culminations, and co-settings cannot remain constant, even for one latitude, with those calculated with the present numbers and proofs, therefore we have avoided such a waste of time, being content for the present with the approximations which can be gotten each time from the catalogue or from the arrangement of the sphere. For we see that the signs of the condition of the atmosphere gotten from the heliacal risings and settings almost never keep any order or unchangeableness, if one attributes the cause of the differences to the weather and not to the places of the ecliptic; so that this cause acts only in a very general way and is not established by the beginnings of the heliacal risings and settings as it is by the configurations of the moon with respect to the sun in its general distances, and by the moon's inclinations in the particular distances.

BOOK NINE

1. Concerning the Order of the Spheres of the Sun and Moon and Five Planets

Now, certainly whatever one could say in general about the fixed stars, to the extent that the appearances up until now fall under our apprehension, would be pretty much like this. But since this Composition still lacks a treatment of the five planets, we shall give an exposition of them, going as far as possible with what they have in common to avoid repetition, and then adding on the plan of each one in particular.

First, then, concerning the order of their spheres, all of which have their positions about the poles of the ecliptic, we see the foremost mathematicians agree that all these spheres are nearer the earth than the sphere of the fixed stars, and farther from the earth than that of the moon; that the three—of which Saturn's is the largest, Jupiter's next earthward, and Mars' below that—are all farther from the earth than the others and that of the sun. On the other hand, the spheres of Venus and Mercury are placed by the earlier mathematicians below the sun's, but by some of the later ones above the sun's because of their never having seen the sun eclipsed by them. But this judgment seems to us unsure since these planets could be below the sun and never yet have been in any of the planes through the sun and our eye but in another, and therefore not have appeared in a line with it; just as in the case of the moon's conjunctive passages there are for the most part no eclipses.

Since there is no other way of getting at this because of the absence of any sensible parallax in these stars, from which appearance alone linear distances are gotten, the order of the earlier mathematicians seems the more trustworthy, using the sun as a natural dividing line between those planets which can be any angular distance from the sun and those which cannot but which always move near it. Besides, it does not place them far enough at their perigees to produce a sensible parallax.

2. On the Aim of the Planetary Hypotheses

So much, then, for the orders of the spheres. Now, since our problem is to demonstrate, in the case of the five planets as in the case of the sun and moon, all their apparent irregularities as produced by means of regular and circular motions (for these are proper to the nature of divine things which are strangers to disparities and disorders) the successful accomplishment of this aim as truly belonging to mathematical theory in philosophy is to be considered a great thing, very difficult and as yet unattained in a reasonable way by anyone. For, since, in the case of the researches about the periodic movements of each planet, whatever slight error the eye makes in systematic observations produces a sensible difference more quickly when the examination has been over a shorter interval than when over a greater one, the time for which we have observations of

the planets recorded (being short for grasping such a considerable lay-out, furnishes an unsure prediction over long periods. In the case of research about the anomalies, the fact that there are two anomalies appearing for each of the planets, and that they are unequal in magnitude and in the times of their returns, works a good deal of confusion. For one of the anomalies is seen to have relation to the sun, and the other to the parts of the zodiac, but both are mixed together so it is very hard to determine what belongs to each[1]; and most of the old observations were thrown together carelessly and grossly. The more continuous of them contain stations and apparitions, and the apprehension of these properties is not certain. The stations cannot indicate the exact time, since the planet's local motion remains imperceptible for many days before and after its station; and the apparitions not only make the places immediately disappear along with the stars as they are seen for the first or last time, but also can be utterly misleading as to the times because of the differences in the atmosphere and in the eye of the observer. In general, the observations made with reference to some fixed star at a rather great angular distance, unless because of these things one attends to them wisely and clear-sightedly, furnish a magnitude from their measurements hard to calculate and subject to guesswork. And this is so, not only because the lines between the observed stars make different angles with the ecliptic and by no means right angles—whence in the variety of the zodiac's inclinations a great uncertainty is apt to follow in the determination of the longitudinal and latitudinal positions—but also because the same angular distances appear to the eye greater near the horizon and smaller near the culminations, and so they can be measured as sometimes greater and sometimes smaller than the real angular distance.

[1]It is important to notice that all the five planets, just like the moon, have an anomaly with respect to the sun as well as an anomaly with respect to the zodiac. Since Ptolemy merely expounds the theory as a deduction, it might be well to explain the appearances from which these things could be gotten. It must be remembered that Ptolemy had long astronomical tables of data which had been kept for nearly a thousand years, and which served as appearances over a long period of time.

The appearances of Venus and Mercury are fairly simple. These two stars move on the ecliptic, more or less, and swing back and forth from one side of the sun to the other, Mercury never getting more than some 25° from the sun, and Venus never more than some 45°. This makes them alternately morning and evening stars. This swinging in the heliacal anomaly and, from a greatest elongation from the mean sun (say eastern) back to the next greatest eastern elongation, is called one cycle of heliacal anomaly. Furthermore the time from a western elongation to an eastern is longer than from the eastern to the next western. It is also observed that, for different positions of the mean sun on the ecliptic, the greatest elongations are smaller or larger, but always the same for the same position.

It is further observed that, if one considers the sum of the greatest eastern and greatest western elongation for each position of the mean sun, this sum is greatest at one position, and on either side gets smaller and smaller until it is least at the position exactly opposite on the ecliptic. This variation of the apparent heliacal anomaly is called the zodiacal anomaly.

The appearances of Mars, Jupiter, and Saturn are more complicated. These planets move generally from west to east along the ecliptic more or less at different speeds, Mars making a complete circuit through the fixed stars in about 2 years, Jupiter in about 11, and Saturn in about 30. They can be at any angular distance from the sun, unlike Venus and Mercury which are constrained to remain within certain bounds. But they are tied to the sun in a different way. Whenever the mean sun is nearly opposite any one of these three planets, the planet stops its eastward motion, and this is called a station. And then, as the mean sun gets more directly opposite, the planet moves westward, and this is called a regression or retrogradation. As the mean sun moves on, the planet appears to stop again, and then move eastward again. As the mean sun moves toward the planet, the planet moves faster and faster

And so I consider Hipparchus to have been most zealous after the truth, both because of all these things and especially because of his having left us more examples of accurate observations than he ever got from his predecessors. He sought out the hypotheses of the sun and moon, and demonstrated as far as possible and by every available means that they were accomplished through uniform circular movements, but he did not attempt to give the principle of the hypotheses of the five planets, as far as we can tell from those memoirs of his which have come down to us, but only arranged the observations in a more useful way and showed the appearances to be inconsistent with the hypotheses of the mathematicians of that time. For not only did he think it necessary as it seemed to declare that, because of the double anomaly of each planet, the regressions of each are unequal and of such and such a magnitude, while the other mathematicians gave their geometrical demonstrations on one and the same anomaly and regression, but he also thought that these movements could not be effected either by eccentric circles, or by circles concentric with the ecliptic but bearing epicycles, or even by both together, although the zodiacal anomaly was of one magnitude and the anomaly with respect to the sun of another. For these are the means used by nearly all those who have wished to demonstrate uniform circular movement by the so-called perpetual table, but in a false and inconsequential way, some getting nowhere at all, some following the problem to a limited extent. But Hipparchus reasoned that no one who has progressed through the whole of mathematics to such a point of accuracy and zeal for truth would be content to stop at this like the rest; but that anyone who was to persuade himself and those in touch with him would have to demonstrate the magnitude and periods of each of the anomalies by clear and consistent appearances; and, putting both together, he would have to find out the position and order of the circles by which these anomalies are produced and the mode of their movement and finally show about all the appearances to be consistent with the peculiar property of this hypothesis of the circles. I think this is difficult, and it seemed so to him. We have said all this not through ostentation, but in order that, if we are forced by the problem itself either (1) to use something contrary to the general argument, as when, for example, for ease we make our demonstrations of the circles described by the movement in the planetary spheres as if they were simple and in the same plane with the ecliptic; or if we are forced (2) to presuppose something without immediate foundation in the appearances, an apprehension gotten from continuous trial and adjustment; or (3) to suppose not everywhere the same mode of movement or inclination of the circles—in order that, I say, we may then reasonably agree that (1) using something of the sort that no appreciable difference is to result from it will not falsify the subject in hand; (2) that things supposed without proof, once they are conceived in such a way as to be consistent with appearances, cannot be

eastward until the mean sun has overtaken it; then it moves more and more slowly eastward until it again appears to stop. The time from a station to a corresponding station is one cycle of heliacal anomaly. Furthermore, the time from a station through the eastward motion of the planet to the next station is much longer than that from a station through the westward or regressive motion of the planet to the next station.

Again these speeds vary for different positions of the mean sun on the ecliptic. And this variation is called the zodiacal anomaly.

All the planets wander slightly from one side of the ecliptic to the other. The theory of this latitudinal anomaly is explained in Book XIII.

found without some plan and knowledge even if the way of getting hold of them is hard to explain (after all, generally speaking, the cause of first principles is either nothing or hard to interpret in its nature); and (3) that, since the appearances relative to the stars are also found to be dissimilar, one should not reasonably think it strange or absurd to vary the mode of the hypotheses of the circles, especially when, along with saving the regular circular movement absolutely everywhere, each of the appearances is demonstrated in its more lawful and general character.

And so we have used for the demonstrations of each planet only those observations which cannot be disputed, that is those taken at contact or great proximity with the stars or even with the moon, and above all those taken with the astrolabe where the eye is lined up with the diametrically opposite sights in the circles, sees on every side equal angular distances by means of similar arcs, and can accurately apprehend the passages relative to the middle of each star in longitude and latitude by moving to and fro to the observed stars the astrolabe's ecliptic circle and the diametrically opposite sights in the circles through its poles.

3. On the Periodic Returns of the Five Planets

Now that these things have been explained in advance, we shall first set out the least periodic joint returns (that is very nearly joint) of each of the five planets as calculated by Hipparchus, with a correction of our own gotten from the comparison of the positions made after the demonstrations of the anomalies, as we shall show in that place. We have put them first in order to have set out at hand, for the calculations of the anomalies, the particular mean movements of each one, both in longitude and in anomaly, since there would be no appreciable difference even if one should use the mean passages in their more approximate form. In general, one is to understand by movement in longitude the movement of the epicycle's centre on the eccentric, and by anomaly the movement of the star on the epicycle.

Now, we find 57 cycles of anomaly of Saturn exactly completed in 59 solar years of our kind (that is from one tropic or equinoctial point back to the same) plus $1+\frac{1}{2}+\frac{1}{4}$ day very nearly, and in 2 revolutions of the star plus $(1+\frac{2}{3}+\frac{1}{20})^\circ$ of a revolution; for, in the case of the three stars continually being overtaken by the sun, the number of revolutions run by the sun in the time of return of each is equal to the number of longitudinal revolutions of the star together with the number of its returns in anomaly.[1] And we find 65 cycles of anomaly of Jupiter exactly completed in 71 solar years of the same kind less $4+\frac{1}{2}+\frac{1}{3}+\frac{1}{15}$ days very nearly, and in 6 revolutions of the star less $(4+\frac{1}{2}+\frac{1}{3})^\circ$ of a revolution, counting from a tropic point back to the same. Likewise we find 37 cycles of anomaly of Mars in 79 of our solar years plus $3+\frac{1}{6}+\frac{1}{20}$ days very nearly, and in 42 revolutions of the star plus $3\frac{1}{6}^\circ$ of a revolution, counting from a tropic

[1]This fact that, for the planets Mars, Jupiter, and Saturn, the number of revolutions of the mean sun is always equal to the number of longitudinal revolutions of the star plus the number of its returns of anomaly, is of great importance later on in Book X, Chapter 6. It also becomes important in the heliocentric theories. For while it remains a useful accident for Ptolemy, it is a necessary consequence of the primary assumptions of Copernicus.

The fact, again, that even these three planets, which are not tied to the sun by an immediate appearance as Venus and Mercury, yet have an anomaly which is referred to the sun, is a sign pointing towards a heliocentric hypothesis.

point back to the same; 5 cycles of anomaly of Venus in 8 of our solar years less $2+\frac{1}{4}+\frac{1}{20}$ days, and in 8 revolutions of the star less $2\frac{1}{4}°$ of a revolution, equal in number to the revolutions of the sun; and 145 cycles of anomaly of Mercury in 46 years of the same kind plus $1\frac{1}{30}$ day, and in 46 revolutions of the star plus 1° of a revolution, again equal in number to the revolutions of the sun.

But if, for each one, we reduce the time of return into days in accordance with the year's time as already given by us, and the number of cycles of anomaly into the 360° of a circle, then we shall have, in the case of Saturn, $21{,}551^{18}/_{60}$ days and 20,520° of anomaly; in the case of Jupiter, $25{,}927^{37}/_{60}$ days and 27,400° of anomaly; in the case of Mars, $28{,}857^{53}/_{60}$ days and 13,320° of anomaly; in the case of Venus, $2{,}919^{49}/_{60}$ days and 1,800° of anomaly; and in the case of Mercury, $16{,}802^{24}/_{60}$ days and 52,200° of anomaly.

Dividing, then, in each case, the number of degrees of anomaly by the number of days, we shall have the daily mean movement of anomaly: very nearly $57^{i}7^{ii}43^{iii}41^{iv}43^{v}40^{vi}$ for Saturn, $54^{i}9^{ii}2^{iii}46^{iv}26^{v}$ for Jupiter, $27^{i}41^{ii}40^{iii}19^{iv}20^{v}58^{vi}$ for Mars, $36^{i}59^{ii}25^{iii}53^{iv}11^{v}28^{vi}$ for Venus, and $3^{o}6^{i}24^{ii}6^{iii}59^{iv}35^{v}50^{vi}$ for Mercury.

Taking a twenty-fourth of each of these, we shall have the hourly mean movement of anomaly: $2^{i}22^{ii}49^{iii}19^{iv}14^{v}19^{vi}10^{vii}$ for Saturn, $2^{i}15^{ii}22^{iii}36^{iv}56^{v}5^{vi}$ for Jupiter, $1^{i}9^{ii}14^{iii}10^{iv}48^{v}22^{vi}25^{vii}$ for Mars, $1^{i}32^{ii}28^{iii}34^{iv}42^{v}58^{vi}40^{vii}$ for Venus, and $7^{i}46^{ii}0^{iii}17^{iv}28^{v}59^{vi}35^{vii}$ for Mercury.

Again, multiplying the daily mean movements of each by thirty, we shall have the monthly mean movement of anomaly: $28°33^{i}51^{ii}50^{iii}51^{iv}50^{v}$ for Saturn, $27°4^{i}31^{ii}23^{iii}13^{iv}$ for Jupiter, $13°50^{i}50^{ii}9^{iii}40^{iv}29^{v}$ for Mars, $18°29^{i}42^{ii}56^{iii}35^{iv}44^{v}$ for Venus, and $93°12^{i}3^{ii}29^{iii}47^{iv}55^{v}$ for Mercury.

And likewise multiplying the daily mean movements by the 365 days of the Egyptian year, we shall have the yearly mean movement of anomaly: $347°32^{i}0^{ii}48^{iii}50^{iv}38^{v}20^{vi}$ for Saturn, $329°25^{i}1^{ii}52^{iii}28^{iv}10^{v}$ for Jupiter, $168°28^{i}30^{ii}17^{iii}42^{iv}32^{v}50^{vi}$ for Mars, $225°1^{i}32^{ii}28^{iii}34^{iv}39^{v}15^{vi}$ for Venus, and a surplus of $53°56^{i}42^{ii}32^{iii}32^{iv}59^{v}10^{vi}$ for Mercury.

In the same way, multiplying also each of the yearly movements by 18, as in the case of the tables of the sun and moon, we shall have the mean surplus of anomaly of the Egyptian 18-year period: $135°36^{i}14^{ii}39^{iii}11^{iv}30^{v}$ for Saturn, $169°30^{i}33^{ii}44^{iii}27^{iv}$ for Jupiter, $152°33^{i}5^{ii}18^{iii}45^{iv}51^{v}$ for Mars, $90°27^{i}44^{ii}34^{iii}23^{iv}46^{v}30^{vi}$ for Venus, and $251°0^{i}45^{ii}45^{iii}53^{iv}45^{v}$ for Mercury.

From these numbers we shall get the mean movements in longitude so that, without reducing the number of revolutions to degrees, we may, in each case, divide them into the time proposed. It is clear that for Venus and Mercury the mean movements in longitude will be the same as those already set out for the sun, and for the other three stars they will be, for each number, the difference between the mean movement of anomaly and the sun's mean movement in longitude. And in this way we shall find Saturn's daily mean movement in longitude to be $2^{o}0^{i}33^{ii}31^{iv}28^{v}51^{vi}$, Jupiter's $4^{i}59^{ii}14^{iii}26^{iv}46^{v}31^{vi}$, and Mars' $31^{i}26^{ii}36^{iii}53^{iv}51^{v}33^{vi}$; Saturn's hourly mean movement in longitude $5^{ii}1^{iii}23^{iv}48^{v}42^{vi}7^{vii}30^{viii}$, Jupiter's $12^{ii}28^{iii}6^{iv}6^{v}56^{vi}17^{vii}30^{viii}$, and Mars' $1^{i}18^{ii}36^{iii}32^{iv}14^{v}39^{vi}$; Saturn's monthly mean movement in longitude $1°0^{i}16^{ii}45^{iii}44^{iv}25^{v}30^{vi}$, Jupiter's $2°29^{i}37^{ii}13^{iii}23^{iv}15^{v}30^{vi}$, and Mars' $15°43^{i}18^{ii}26^{iii}55^{iv}46^{v}30^{vi}$; Saturn's yearly mean movement in longitude $12°13^{i}23^{ii}56^{iii}30^{iv}30^{v}15^{vi}$, Jupiter's $30°20^{i}22^{ii}52^{iii}52^{iv}38^{v}35^{vi}$, and Mars' $191°16^{i}54^{ii}27^{iii}38^{iv}35^{v}45^{vi}$; and Saturn's mean

movement in longitude over 18 years to be $220°1^{i}10^{ii}57^{iii}9^{iv}4^{v}30^{vi}$, Jupiter's a surplus of $186°6^{i}51^{ii}51^{iii}53^{iv}34^{v}30^{vi}$, and Mars' a surplus of $203°4^{i}20^{ii}17^{iii}34^{iv}43^{v}30^{vi}$.

Now we shall again arrange, in the order of the stars for the ready use of each, the tables of these mean movements, in 45 rows and in lots of three like the others. The first lots will contain the combinations of 18-year periods, the second the yearly and hourly combinations, and the third the monthly and daily ones. And here follow the tables:

4. Tables of the Mean Movements in Longitude and in Anomaly of the Five Stars

SATURN, Anomaly at epoch, 34°2′

18-years	*Longitude at epoch, Goat 26°43′* Degrees in longitude							*Apogee at epoch, Scorpion 14°10′* Degrees of anomaly						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
18	220	1	10	57	9	4	30	135	36	14	39	11	30	0
36	80	2	21	54	18	9	0	271	12	29	18	23	0	0
54	300	3	32	51	27	13	30	46	48	43	57	34	30	0
72	160	4	43	48	36	18	0	182	24	58	36	46	0	0
90	20	5	54	45	45	22	30	318	1	13	15	57	30	0
108	240	7	5	42	54	27	0	93	37	27	55	9	0	0
126	100	8	16	40	3	31	30	229	13	42	34	20	30	0
144	320	9	27	37	12	36	0	4	49	57	13	32	0	0
162	180	10	38	34	21	40	30	140	26	11	52	43	30	0
180	40	11	49	31	30	45	0	276	2	26	31	55	0	0
198	260	13	0	28	39	49	30	51	38	41	11	6	30	0
216	120	14	11	25	48	54	0	187	14	55	50	18	0	0
234	340	15	22	22	57	58	30	322	51	10	29	29	30	0
252	200	16	33	20	7	3	0	98	27	25	8	41	0	0
270	60	17	44	17	16	7	30	234	3	39	47	52	30	0
288	280	18	55	14	25	12	0	9	39	54	27	4	0	0
306	140	20	6	11	34	16	30	145	16	9	6	15	30	0
324	0	21	17	8	43	21	0	280	52	23	45	27	0	0
342	220	22	28	5	52	25	30	56	28	38	24	38	30	0
360	80	23	39	3	1	30	0	192	4	53	3	50	0	0
378	300	24	50	0	10	34	30	327	41	7	43	1	30	0
396	160	26	0	57	19	39	0	103	17	22	22	13	0	0
414	20	27	11	54	28	43	30	238	53	37	1	24	30	0
432	240	28	22	51	37	48	0	14	29	51	40	36	0	0
450	100	29	33	48	46	52	30	150	6	6	19	47	30	0
468	320	30	44	45	55	57	0	285	42	20	58	59	0	0
486	180	31	55	43	5	1	30	61	18	35	38	10	30	0
504	40	33	6	40	14	6	0	196	54	50	17	22	0	0
522	260	34	17	37	23	10	30	332	31	4	56	33	30	0
540	120	35	28	34	32	15	0	108	7	19	35	45	0	0
558	340	36	39	31	41	19	30	243	43	34	14	56	30	0
576	200	37	50	28	50	24	0	19	19	48	54	8	0	0
594	60	39	1	25	59	28	30	154	56	3	33	19	30	0
612	280	40	12	23	8	33	0	290	32	18	12	31	0	0
630	140	41	23	20	17	37	30	66	8	32	51	42	30	0
648	0	42	34	17	26	42	0	201	44	47	30	54	0	0
666	220	43	45	14	35	46	30	337	21	2	10	5	30	0
684	80	44	56	11	44	51	0	112	57	16	49	17	0	0
702	300	46	7	8	53	55	30	248	33	31	28	28	30	0
720	160	47	18	6	3	0	0	24	9	46	7	40	0	0
738	20	48	29	3	12	4	30	159	46	0	46	51	30	0
756	240	49	40	0	21	9	0	295	22	15	26	3	0	0
774	100	50	50	57	30	13	30	0	58	30	5	14	30	0
792	320	52	1	54	39	18	0	206	34	44	44	26	0	0
810	180	53	12	51	48	22	30	342	10	59	23	37	30	0

SATURN, *Continued*

Single Years	*Saturn's Degrees in longitude*							*Saturn's Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	12	13	23	56	30	30	15	347	32	0	48	50	38	20
2	24	26	47	53	1	0	30	335	4	1	37	41	16	40
3	36	40	11	49	31	30	45	322	36	2	26	31	55	0
4	48	53	35	46	2	1	0	310	8	3	15	22	33	20
5	61	6	59	42	32	31	15	297	40	4	4	13	11	40
6	73	20	23	39	3	1	30	285	12	4	53	3	50	0
7	85	33	47	35	33	31	45	272	44	5	41	54	28	20
8	97	47	11	32	4	2	0	260	16	6	30	45	6	40
9	110	0	35	28	34	32	15	247	48	7	19	35	45	0
10	122	13	59	25	5	2	30	235	20	8	8	26	23	20
11	134	27	23	21	35	32	45	222	52	8	57	17	1	40
12	146	40	47	18	6	3	0	210	24	9	46	7	40	0
13	158	54	11	14	36	33	15	197	56	10	34	58	18	20
14	171	7	35	11	7	3	30	185	28	11	23	48	56	40
15	183	20	59	7	37	33	45	173	0	12	12	39	35	0
16	195	34	23	4	8	4	0	160	32	13	1	30	13	20
17	207	47	47	0	38	34	15	148	4	13	50	20	51	40
18	220	1	10	57	9	4	30	135	36	14	39	11	30	0

Hours	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	0	5	1	23	48	42	0	2	22	49	19	14	19
2	0	0	10	2	47	37	24	0	4	45	38	38	28	38
3	0	0	15	4	11	26	6	0	7	8	27	57	42	57
4	0	0	20	5	35	14	48	0	9	31	17	16	57	17
5	0	0	25	6	59	3	31	0	11	54	6	36	11	36
6	0	0	30	8	22	52	13	0	14	16	55	55	25	55
7	0	0	35	9	46	40	55	0	16	39	45	14	40	14
8	0	0	40	11	10	29	37	0	19	2	34	33	54	33
9	0	0	45	12	34	18	19	0	21	25	23	53	8	52
10	0	0	50	13	58	7	1	0	23	48	13	12	23	12
11	0	0	55	15	21	55	43	0	26	11	2	31	37	31
12	0	1	0	16	45	44	25	0	28	33	51	50	51	50
13	0	1	5	18	9	33	8	0	30	56	41	10	6	9
14	0	1	10	19	33	21	50	0	33	19	30	29	20	28
15	0	1	15	20	57	10	32	0	35	42	19	48	34	47
16	0	1	20	22	20	59	14	0	38	5	9	7	49	7
17	0	1	25	23	44	47	56	0	40	27	58	27	30	26
18	0	1	30	25	8	36	38	0	42	50	47	46	17	45
19	0	1	35	26	32	25	20	0	45	13	37	5	32	4
20	0	1	40	27	56	14	2	0	47	36	26	24	46	23
21	0	1	45	29	20	2	45	0	49	59	15	44	0	42
22	0	1	50	30	43	51	27	0	52	22	5	3	15	2
23	0	1	55	32	7	40	9	0	54	44	54	22	29	21
24	0	2	0	33	31	28	51	0	57	7	43	41	43	40

SATURN, *Continued*

Months	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
30	1	0	16	45	44	25	30	28	33	51	50	51	50	0
60	2	0	33	31	28	51	0	57	7	43	41	43	40	0
90	3	0	50	17	13	16	30	85	41	35	32	35	30	0
120	4	1	7	2	57	42	0	114	15	27	23	27	20	0
150	5	1	23	48	42	7	30	142	49	19	14	19	10	0
180	6	1	40	34	26	33	0	171	23	11	5	11	0	0
210	7	1	57	20	10	58	30	199	57	2	56	2	50	0
240	8	2	14	5	55	24	0	228	30	54	46	54	40	0
270	9	2	30	51	39	49	30	257	4	46	37	46	30	0
300	10	2	47	37	24	15	0	255	38	38	28	38	20	0
330	11	3	4	23	8	40	30	314	12	30	19	30	10	0
360	12	3	21	8	53	6	0	342	46	22	10	22	0	0

Days	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	2	0	33	31	28	51	0	57	7	43	41	43	40
2	0	4	1	7	2	57	42	1	54	15	27	23	27	20
3	0	6	1	40	34	26	33	2	51	23	11	5	11	0
4	0	8	2	14	5	55	24	3	48	30	54	46	54	40
5	0	10	2	47	37	24	15	4	45	38	38	28	38	20
6	0	12	3	21	8	53	6	5	42	46	22	10	22	0
7	0	14	3	54	40	21	57	6	39	54	5	52	5	40
8	0	16	4	28	11	50	48	7	37	1	49	33	49	20
9	0	18	5	1	43	19	39	8	34	9	33	15	33	0
10	0	20	5	35	14	48	30	9	31	17	16	57	16	40
11	0	22	6	8	46	17	21	10	28	25	0	39	0	20
12	0	24	6	42	17	46	12	11	25	32	44	20	44	0
13	0	26	7	15	49	15	3	12	22	40	28	2	27	40
14	0	28	7	49	20	43	54	13	19	48	11	44	11	20
15	0	30	8	22	52	12	45	14	16	55	55	25	55	0
16	0	32	8	56	23	41	36	15	14	3	39	7	38	40
17	0	34	9	29	55	10	27	16	11	11	22	49	22	20
18	0	36	10	3	26	39	18	17	8	19	6	31	6	0
19	0	38	10	36	58	8	9	18	5	26	50	12	49	40
20	0	40	11	10	29	37	0	19	2	34	33	54	33	20
21	0	42	11	44	1	5	51	19	59	42	17	36	17	0
22	0	44	12	17	32	34	42	20	56	50	1	18	0	40
23	0	46	12	51	4	3	33	21	53	57	44	59	44	20
24	0	48	13	24	35	32	24	22	51	5	28	41	28	0
25	0	50	13	58	7	1	15	23	48	13	12	23	11	40
26	0	52	14	31	38	30	6	24	45	20	56	4	55	20
27	0	54	15	5	9	58	57	25	42	28	39	46	39	0
28	0	56	15	38	41	27	48	26	39	36	23	28	22	40
29	0	58	16	12	12	56	39	27	36	44	7	10	6	20
30	1	0	16	45	44	25	30	28	33	51	50	51	50	0

JUPITER, Anomaly at epoch, 146°4′

18-years	Longitude at epoch, Balance 4°41′ — Degrees in longitude							Apogee at epoch, Virgin 2°9′ — Degrees of anomaly						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
18	186	6	51	51	53	34	30	169	30	33	44	27	0	0
36	12	13	43	43	47	9	0	339	1	7	28	54	0	0
54	198	20	35	35	40	43	30	148	31	41	13	21	0	0
72	24	27	27	27	34	18	0	318	2	14	57	48	0	0
90	210	34	19	19	27	52	30	127	32	48	42	15	0	0
108	36	41	11	11	21	27	0	297	3	22	26	42	0	0
126	222	48	3	3	15	1	30	106	33	56	11	9	0	0
144	48	54	54	55	8	36	0	276	4	29	55	36	0	0
162	235	1	46	47	2	10	30	85	35	3	40	3	0	0
180	61	8	38	38	55	45	0	255	5	37	24	30	0	0
198	247	15	30	30	49	19	30	64	36	11	8	57	0	0
216	73	22	22	22	42	54	0	234	6	44	53	24	0	0
234	259	29	14	14	36	28	30	43	37	18	37	51	0	0
252	85	36	6	6	30	3	0	213	7	52	22	18	0	0
270	271	42	57	58	23	37	30	22	38	26	6	45	0	0
288	97	49	49	50	17	12	0	192	8	59	51	12	0	0
306	283	56	41	42	10	46	30	1	39	33	35	39	0	0
324	110	3	33	34	4	21	0	171	10	7	20	6	0	0
342	296	10	25	25	57	55	30	340	40	41	4	33	0	0
360	122	17	17	17	51	30	0	150	11	14	49	0	0	0
378	308	24	9	9	45	4	30	319	41	48	33	27	0	0
396	134	31	1	1	38	39	0	129	12	22	17	54	0	0
414	320	37	52	53	32	13	30	298	42	56	2	21	0	0
432	146	44	44	45	25	48	0	108	13	29	46	48	0	0
450	332	51	36	37	19	22	30	277	44	3	31	15	0	0
468	158	58	28	29	12	57	0	87	14	37	15	42	0	0
486	345	5	20	21	6	31	30	256	45	11	0	9	0	0
504	171	12	12	13	0	6	0	66	15	44	44	36	0	0
522	357	19	4	4	53	40	30	235	46	18	29	3	0	0
540	183	25	55	56	47	15	0	45	16	52	13	30	0	0
558	9	32	47	48	40	49	30	214	47	25	57	57	0	0
576	195	39	39	40	34	24	0	24	17	59	42	24	0	0
594	21	46	31	32	27	58	30	193	48	33	26	51	0	0
612	207	53	23	24	21	33	0	3	19	7	11	18	0	0
630	34	0	15	16	15	7	30	172	49	40	55	45	0	0
648	220	7	7	8	8	42	0	342	20	14	40	12	0	0
666	46	13	59	0	2	16	30	151	50	48	24	39	0	0
684	232	20	50	51	55	51	0	321	21	22	9	6	0	0
702	58	27	42	43	49	25	30	130	51	55	53	33	0	0
720	244	34	34	35	43	0	0	300	22	29	38	0	0	0
738	0	41	26	27	36	34	30	109	53	3	22	27	0	0
756	256	48	18	19	30	9	0	279	23	37	6	54	0	0
774	82	55	10	11	23	43	30	88	54	10	51	21	0	0
792	269	2	2	3	17	18	0	258	24	44	35	48	0	0
810	95	8	53	55	10	52	30	67	55	18	20	15	0	0

JUPITER, *Continued*

Single years	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	IV	°	I	II	III	IV	V	VI
1	30	20	22	52	52	58	35	329	25	1	52	28	10	0
2	60	40	45	45	45	57	10	298	50	3	44	56	20	0
3	91	1	8	38	38	55	45	268	15	5	37	24	30	0
4	121	21	31	31	31	54	20	237	40	7	29	52	40	0
5	151	41	54	24	24	52	55	207	5	9	22	20	50	0
6	182	2	17	17	17	51	30	176	30	11	14	49	0	0
7	212	22	40	10	10	50	5	145	55	13	7	17	10	0
8	242	43	3	3	3	48	40	115	20	14	59	45	20	0
9	273	3	25	55	56	47	15	84	45	16	52	13	30	0
10	303	23	48	48	49	45	50	54	10	18	44	41	40	0
11	333	44	11	41	42	44	25	23	35	20	37	9	50	0
12	4	4	34	34	35	43	0	353	0	22	29	38	0	0
13	34	24	57	27	28	41	35	322	25	24	22	6	10	0
14	64	45	20	20	21	40	10	291	50	26	14	34	20	0
15	95	5	43	13	14	38	45	261	15	28	7	2	30	0
16	125	26	6	6	7	37	20	230	40	29	59	30	40	0
17	155	46	28	59	0	35	55	200	5	31	51	58	50	0
18	186	6	51	51	53	34	30	169	30	33	44	27	0	0

Hours	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	IV
1	0	0	12	28	6	6	56	0	2	15	22	36	56	5
2	0	0	24	56	12	13	52	0	4	30	45	13	52	10
3	0	0	36	24	18	20	48	0	6	46	7	50	48	15
4	0	0	49	52	24	27	45	0	9	1	30	27	44	20
5	0	1	2	20	30	34	41	0	11	16	53	4	40	25
6	0	1	14	48	36	41	37	0	13	32	15	41	36	30
7	0	1	27	16	42	48	34	0	15	47	38	18	32	35
8	0	1	39	44	48	55	30	0	18	3	0	55	28	40
9	0	1	52	12	55	2	26	0	20	18	23	32	24	45
10	0	2	4	41	1	9	22	0	22	33	46	9	20	50
11	0	2	17	9	7	16	19	0	24	49	8	46	16	55
12	0	2	29	37	13	23	15	0	27	4	31	23	13	0
13	0	2	42	5	19	30	11	0	29	19	54	0	9	5
14	0	2	54	33	25	37	8	0	31	35	16	37	5	10
15	0	3	7	1	31	44	4	0	33	50	39	14	1	15
16	0	3	19	29	37	51	0	0	36	6	1	50	57	20
17	0	3	31	57	43	57	56	0	38	21	24	27	53	25
18	0	3	44	25	50	4	53	0	40	36	47	4	49	30
19	0	3	56	53	56	11	49	0	42	52	9	41	45	35
20	0	4	9	22	2	18	45	0	45	7	32	18	41	40
21	0	4	21	50	8	25	42	0	47	22	54	55	37	45
22	0	4	34	18	14	32	38	0	49	38	17	32	33	50
23	0	4	46	46	20	39	34	0	51	53	40	9	29	55
24	0	4	59	14	26	46	31	0	54	9	2	46	26	0

JUPITER, *Continued*

Months	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
30	2	29	37	13	23	15	30	27	4	31	23	13	0	0
60	4	59	14	26	46	31	0	54	9	2	46	26	0	0
90	7	28	51	40	9	46	30	81	13	34	9	39	0	0
120	9	58	28	53	33	2	0	108	18	5	32	52	0	0
150	12	28	6	6	56	17	30	135	22	36	56	5	0	0
180	14	57	43	20	19	33	0	162	27	8	19	18	0	0
210	17	27	20	33	42	48	30	189	31	39	42	31	0	0
240	19	56	57	47	6	4	0	216	36	11	5	44	0	0
270	22	26	35	0	29	19	30	243	40	42	28	57	0	0
300	24	56	12	13	52	35	0	270	45	13	52	10	0	0
330	27	25	49	27	15	50	30	297	49	45	15	23	0	0
360	29	55	26	40	39	6	0	324	54	16	38	36	0	0

Days	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	4	59	14	26	46	31	0	54	9	2	46	26	0
2	0	9	58	28	53	33	2	1	48	18	5	32	52	0
3	0	14	57	43	20	19	33	2	42	27	8	19	18	0
4	0	19	56	57	47	6	4	3	36	36	11	5	44	0
5	0	24	56	12	13	52	35	4	30	45	13	52	10	0
6	0	29	55	26	40	39	6	5	24	54	16	38	36	0
7	0	34	54	41	7	25	37	6	19	3	19	25	2	0
8	0	39	53	55	34	12	8	7	13	12	22	11	28	0
9	0	44	53	10	0	58	39	8	7	21	24	57	54	0
10	0	49	52	24	27	45	10	9	1	30	27	44	20	0
11	0	54	51	38	54	31	41	9	55	39	30	30	46	0
12	0	59	50	53	21	18	12	10	49	48	33	17	12	0
13	1	4	50	7	48	4	43	11	43	57	36	3	38	0
14	1	9	49	22	14	51	14	12	38	6	38	50	4	0
15	1	14	48	36	41	37	45	13	32	15	41	36	30	0
16	1	19	47	51	8	24	16	14	26	24	44	22	56	0
17	1	24	47	5	35	10	47	15	20	33	47	9	22	0
18	1	29	46	20	1	57	18	16	14	42	49	55	48	0
19	1	34	45	34	28	43	49	17	8	51	52	42	14	0
20	1	39	44	48	55	30	20	18	3	0	55	28	40	0
21	1	44	44	3	22	16	51	18	57	9	58	15	6	0
22	1	49	43	17	49	3	22	19	51	19	1	1	32	0
23	1	54	42	32	15	49	53	20	45	28	3	47	58	0
24	1	59	41	46	42	36	24	21	39	37	6	34	24	0
25	2	4	41	1	9	22	55	22	33	46	9	20	50	0
26	2	9	40	15	36	9	26	23	27	55	12	7	16	0
27	2	14	39	30	2	55	57	24	22	4	14	53	42	0
28	2	19	38	44	29	42	28	25	16	13	17	40	8	0
29	2	24	37	58	56	28	59	26	10	22	20	26	34	0
30	2	29	37	13	23	15	30	27	4	31	23	13	0	0

MARS, Anomaly at epoch, 327°13′

18-years	*Longitude at epoch, Ram 3°32′* — *Degrees in longitude*							*Apogee at epoch, Crab 16°40′* — *Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
18	203	4	20	17	34	43	30	152	33	5	18	45	51	0
36	46	8	40	35	9	27	0	305	6	10	37	31	42	0
54	249	13	0	52	44	10	30	97	39	15	56	17	33	0
72	92	17	21	10	18	54	0	250	12	21	15	3	24	0
90	295	21	41	27	53	37	30	42	45	26	33	49	15	0
108	138	26	1	45	28	21	0	195	18	31	52	35	6	0
126	341	30	22	3	3	4	30	347	51	37	11	20	57	0
144	184	34	42	20	37	48	0	140	24	42	30	6	48	0
162	27	39	2	38	12	31	30	292	57	47	48	52	39	0
180	230	43	22	55	47	15	0	85	30	53	7	38	30	0
198	73	47	43	13	21	58	30	238	3	58	26	24	21	0
216	276	52	3	30	56	42	0	30	37	3	45	10	12	0
234	119	56	23	48	31	25	30	183	10	9	3	56	3	0
252	323	0	44	6	6	9	0	335	43	14	22	41	54	0
270	166	5	4	23	40	52	30	128	16	19	41	27	45	0
288	9	9	24	41	15	36	0	280	49	25	0	13	36	0
306	212	13	44	58	50	19	30	73	22	30	18	59	27	0
324	55	18	5	16	25	3	0	225	55	35	37	45	18	0
342	258	22	25	33	59	46	30	18	28	40	56	31	9	0
360	101	26	45	51	34	30	0	171	1	46	15	17	0	0
378	304	31	6	9	9	13	30	323	34	51	34	2	51	0
396	147	35	26	26	43	57	0	116	7	56	52	48	42	0
414	350	39	46	44	18	40	30	268	41	2	11	34	33	0
432	193	44	7	1	53	24	0	61	14	7	30	20	24	0
450	36	48	27	19	28	7	30	213	47	12	49	6	15	0
468	239	52	47	37	2	51	0	6	20	18	7	52	6	0
486	82	57	7	54	37	34	30	158	53	23	26	37	57	0
504	286	1	28	12	12	18	0	311	26	28	45	23	48	0
522	129	5	48	29	47	1	30	103	59	34	4	9	39	0
540	332	10	8	47	21	45	0	256	32	39	22	55	30	0
558	175	14	29	4	56	28	30	49	5	44	41	41	21	0
576	18	18	49	22	31	12	0	201	38	50	0	27	12	0
594	221	23	9	40	5	55	30	354	11	55	19	13	3	0
612	64	27	29	57	40	39	0	146	45	0	37	58	54	0
630	267	31	50	15	15	22	30	299	18	5	56	44	45	0
648	110	36	10	32	50	6	0	91	51	11	15	30	36	0
666	313	40	30	50	24	49	30	244	24	16	34	16	27	0
684	156	44	51	7	59	33	0	36	57	21	53	2	18	0
702	359	49	11	25	34	16	30	189	30	27	11	48	9	0
720	202	53	31	43	9	0	0	342	3	32	30	34	0	0
738	45	57	52	0	43	43	30	134	36	37	49	19	51	0
756	249	2	12	18	18	27	0	287	9	43	8	5	42	0
774	92	6	32	35	53	10	30	79	42	48	26	51	33	0
792	295	10	52	53	27	54	0	232	15	53	45	37	24	0
810	138	15	13	11	2	37	30	24	48	59	4	23	15	0

MARS, *Continued*

Single years	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	191	16	54	27	38	35	45	168	28	30	17	42	32	50
2	22	33	48	55	17	11	30	336	57	0	35	25	5	40
3	213	50	43	22	55	47	15	145	25	30	53	7	38	30
4	45	7	37	50	34	23	0	313	54	1	10	50	11	20
5	236	24	32	18	12	58	45	122	22	31	28	32	44	10
6	67	41	26	45	51	34	30	290	51	1	46	15	17	0
7	258	58	21	13	30	10	15	99	19	32	3	57	49	50
8	90	15	15	41	8	46	0	267	48	2	21	40	22	40
9	281	32	10	8	47	21	45	76	16	32	39	22	55	30
10	112	49	4	36	25	57	30	244	45	2	57	5	28	20
11	304	5	59	4	4	33	15	53	13	33	14	48	1	10
12	135	22	53	31	49	9	0	221	42	3	32	30	34	0
13	326	39	47	59	21	44	45	30	10	33	50	13	6	50
14	157	56	42	27	0	20	30	198	39	4	7	55	39	40
15	349	13	36	54	38	56	15	7	7	34	25	38	12	30
16	180	30	31	22	17	32	0	175	36	4	43	20	45	20
17	11	47	25	49	56	7	45	344	4	35	1	3	18	10
18	203	4	20	17	34	43	30	152	33	5	18	45	51	0

Hours	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	1	18	36	32	14	39	0	1	9	14	10	48	22
2	0	2	37	13	4	29	18	0	2	18	28	21	36	44
3	0	3	55	49	36	43	6	0	3	27	42	32	25	7
4	0	5	14	26	8	58	35	0	4	36	56	43	13	29
5	0	6	33	2	41	13	14	0	5	46	10	54	1	52
6	0	7	51	39	13	27	53	0	6	55	25	4	50	14
7	0	9	10	15	45	42	32	0	8	4	39	15	38	36
8	0	10	28	52	17	57	11	0	9	13	53	26	26	59
9	0	11	47	28	50	11	49	0	10	23	7	37	15	21
10	0	13	6	5	22	26	28	0	11	32	21	48	3	44
11	0	14	24	41	54	41	7	0	12	41	35	58	52	6
12	0	15	43	18	26	55	46	0	13	50	50	9	40	29
13	0	17	1	54	59	10	25	0	15	0	4	20	28	51
14	0	18	20	31	31	25	4	0	16	9	18	31	17	13
15	0	19	39	8	3	39	43	0	17	18	32	42	5	36
16	0	20	57	44	35	54	22	0	18	27	46	52	53	58
17	0	22	16	21	8	9	0	0	19	37	1	3	42	21
18	0	23	34	57	40	23	39	0	20	46	15	14	30	43
19	0	24	53	34	12	38	18	0	21	55	29	25	19	5
20	0	26	12	10	44	52	57	0	23	4	43	36	7	28
21	0	27	30	47	17	7	36	0	24	13	57	46	55	50
22	0	28	49	23	49	22	15	0	25	23	11	57	44	13
23	0	30	8	0	21	36	54	0	26	32	26	8	32	35
24	0	31	26	36	53	51	33	0	27	41	40	19	20	58

MARS, *Continued*

Months	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
30	15	43	18	26	55	46	30	13	50	50	9	40	29	0
60	31	26	36	53	51	33	0	27	41	40	19	20	58	0
90	47	9	55	20	47	19	30	41	32	30	29	1	27	0
120	62	53	13	47	43	6	0	55	23	20	38	41	56	0
150	78	36	32	14	38	52	30	69	14	10	48	22	25	0
180	94	19	50	41	34	39	0	83	5	0	58	2	54	0
210	110	3	9	8	30	25	30	96	55	51	7	43	23	0
240	125	46	27	35	26	12	0	110	46	41	17	23	52	0
270	141	29	46	2	21	58	30	124	37	31	27	4	21	0
300	157	13	4	29	17	45	0	138	28	21	36	44	50	0
330	172	56	22	56	13	31	30	152	19	11	46	25	19	0
360	188	39	41	23	9	18	0	166	10	1	56	5	48	0

Days	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	31	26	36	53	51	33	0	27	41	40	19	20	58
2	1	2	53	13	47	43	6	0	55	23	20	38	41	56
3	1	34	19	50	41	34	39	1	23	5	0	58	2	54
4	2	5	46	27	35	26	12	1	50	46	41	17	23	52
5	2	37	13	4	29	17	45	2	18	28	21	36	44	50
6	3	8	39	41	23	9	18	2	46	10	1	56	5	48
7	3	40	6	18	17	0	51	3	13	51	42	15	26	46
8	4	11	32	55	10	52	24	3	41	33	22	34	47	44
9	4	42	59	32	4	43	57	4	9	15	2	54	8	42
10	5	14	26	8	58	35	30	4	36	56	43	13	29	40
11	5	45	52	45	52	27	3	5	4	38	23	32	50	38
12	6	17	19	22	46	18	36	5	32	20	3	52	11	36
13	6	48	45	59	40	10	9	6	0	1	44	11	32	34
14	7	20	12	36	34	1	42	6	27	43	24	30	53	32
15	7	51	39	13	27	53	15	6	55	25	4	50	14	30
16	8	23	5	50	21	44	48	7	23	6	45	9	35	28
17	8	54	32	27	15	36	21	7	50	48	25	28	56	26
18	9	25	59	4	9	27	54	8	18	30	5	48	17	24
19	9	57	25	41	3	19	27	8	46	11	46	7	38	22
20	10	28	52	17	57	11	0	9	13	53	26	26	59	20
21	11	0	18	54	51	2	33	9	41	35	6	46	20	18
22	11	31	45	31	44	54	6	10	9	16	47	5	41	16
23	12	3	12	8	38	45	39	10	36	58	27	25	2	14
24	12	34	38	45	32	37	12	11	4	40	7	44	23	12
25	13	6	5	22	26	28	45	11	32	21	48	3	44	10
26	13	37	31	59	20	20	18	12	0	3	28	23	5	8
27	14	8	58	36	14	11	51	12	27	45	8	42	26	6
28	14	40	25	13	8	3	24	12	55	26	49	1	47	4
29	15	11	51	50	1	54	57	13	23	8	29	21	8	2
30	15	43	18	26	55	46	30	13	50	50	9	40	29	0

VENUS, Anomaly at epoch, 71°7′

18-years	*Longitude at epoch, Fishes 45°* — *Degrees in longitude*							*Apogee at epoch, Bull 16°10′* — *Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
18	355	37	25	36	20	34	30	90	27	44	34	23	46	30
36	351	14	51	12	41	9	0	180	55	29	8	47	33	0
54	346	52	16	49	1	43	30	271	23	13	43	11	19	30
72	342	29	42	25	22	18	0	1	50	58	17	35	6	0
90	338	7	8	1	42	52	30	92	18	42	51	58	52	30
108	333	44	33	38	3	27	0	182	46	27	26	22	39	0
126	329	21	59	14	24	1	30	273	14	12	0	46	25	30
144	324	59	24	50	44	36	0	3	41	56	35	10	12	0
162	320	36	50	27	5	10	30	94	9	41	9	33	58	30
180	316	14	16	3	25	45	0	184	37	25	43	57	45	0
198	311	51	41	39	46	19	30	275	5	10	18	21	31	30
216	307	29	7	16	6	54	0	5	32	54	52	45	18	0
234	303	6	32	52	27	28	30	96	0	39	27	9	4	30
252	298	43	58	28	48	3	0	186	28	24	1	32	51	0
270	294	21	24	5	8	37	30	276	56	8	35	56	37	30
288	289	58	49	41	29	12	0	7	23	53	10	20	24	0
306	285	36	15	17	49	46	30	97	51	37	44	44	10	30
324	281	13	40	54	10	21	0	188	19	22	19	7	57	0
342	276	51	6	30	30	55	30	278	47	6	53	31	43	30
360	272	28	32	6	51	30	0	9	14	51	27	55	30	0
378	268	5	57	43	12	4	30	99	42	36	2	19	16	30
396	263	43	23	19	32	39	0	190	10	20	36	43	3	0
414	259	20	48	55	53	13	30	280	38	5	11	6	49	30
432	254	58	14	32	13	48	0	11	5	49	45	30	36	0
450	250	35	40	8	34	22	30	101	33	34	19	54	22	30
468	246	13	5	44	54	57	0	192	1	18	54	18	9	0
486	241	50	31	21	15	31	30	282	29	3	28	41	55	30
504	237	27	56	57	36	6	0	12	56	48	3	5	42	0
522	233	5	22	33	56	40	30	103	24	32	37	29	28	30
540	228	42	48	10	17	15	0	193	52	17	11	53	15	0
558	224	20	13	46	37	49	30	284	20	1	46	17	1	30
576	219	57	39	22	58	24	0	14	47	46	20	40	48	0
594	215	35	4	59	18	58	30	105	15	30	55	4	34	30
612	211	12	30	35	39	33	0	195	43	15	29	28	21	0
630	206	49	56	12	0	7	30	286	11	0	3	52	7	30
648	202	27	21	48	20	42	0	16	38	44	38	15	54	0
666	198	4	47	24	41	16	30	107	6	29	12	39	40	30
684	193	42	13	1	1	51	0	197	34	13	47	3	27	0
702	189	19	38	37	22	25	30	288	1	58	21	27	13	30
720	184	57	4	13	43	0	0	18	29	42	55	51	0	0
738	180	34	29	50	3	34	30	108	57	27	30	14	46	30
756	176	11	55	26	24	9	0	199	25	12	4	38	33	0
774	171	49	21	2	44	43	30	289	52	56	39	2	19	30
792	167	26	46	39	5	18	0	20	20	41	13	26	6	0
810	163	4	12	15	25	52	30	110	48	25	47	49	52	30

VENUS, *Continued*

Single years	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	359	45	24	45	21	8	35	225	1	32	28	34	39	15
2	359	30	49	30	42	17	10	90	3	4	57	9	18	30
3	359	16	14	16	3	25	45	315	4	37	25	43	57	45
4	359	1	39	1	24	34	20	180	6	9	54	18	37	0
5	358	47	3	46	45	42	55	45	7	42	22	53	16	15
6	358	32	28	32	6	51	30	270	9	14	51	27	55	30
7	358	17	53	17	28	0	5	135	10	47	20	2	34	45
8	358	3	18	2	49	8	40	0	12	19	48	37	14	0
9	357	48	42	48	10	17	15	225	13	52	17	11	53	15
10	357	34	7	33	31	25	50	90	15	24	45	46	32	30
11	357	19	32	18	52	34	25	315	16	57	14	21	11	45
12	357	4	57	4	13	43	0	180	18	29	42	55	51	0
13	356	50	21	49	34	51	35	45	20	2	11	30	30	15
14	356	35	46	34	56	0	10	270	21	34	40	5	9	30
15	356	21	11	20	17	8	45	135	23	7	8	39	48	45
16	356	6	36	5	38	17	20	0	24	39	37	14	28	0
17	355	52	0	50	59	25	55	225	26	12	5	49	7	15
18	355	37	25	36	20	34	30	90	27	44	34	23	46	30

Hours	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	2	27	50	43	3	1	0	1	32	28	34	42	58
2	0	4	55	41	26	6	2	0	3	4	57	9	25	57
3	0	7	23	32	9	9	3	0	4	37	25	44	8	56
4	0	9	51	22	52	12	5	0	6	9	54	18	51	54
5	0	12	19	13	35	15	6	0	7	42	22	53	34	53
6	0	14	47	4	18	18	7	0	9	14	51	28	17	52
7	0	17	14	55	1	21	9	0	10	47	20	3	0	50
8	0	19	42	45	44	24	10	0	12	19	48	37	43	49
9	0	22	10	36	26	26	11	0	13	52	17	12	26	48
10	0	24	38	27	10	30	12	0	15	24	45	47	9	46
11	0	27	6	17	53	33	14	0	16	57	14	21	52	45
12	0	29	34	8	36	36	15	0	18	29	42	56	35	44
13	0	32	1	59	19	39	16	0	20	2	11	31	18	42
14	0	34	29	50	2	42	18	0	21	34	40	6	1	41
15	0	36	57	40	45	45	19	0	23	7	8	40	44	40
16	0	39	25	31	28	48	20	0	24	39	37	15	27	38
17	0	41	53	22	11	51	21	0	26	12	5	50	10	37
18	0	44	21	12	54	54	23	0	27	44	34	24	53	36
19	0	46	49	3	37	57	24	0	29	17	2	59	36	34
20	0	49	16	54	21	0	25	0	30	49	31	34	19	33
21	0	51	44	45	4	3	27	0	32	22	0	9	2	32
22	0	54	12	35	47	6	28	0	33	54	28	43	45	30
23	0	56	40	26	30	9	29	0	35	26	57	18	28	29
24	0	59	8	17	13	12	31	0	36	59	25	53	11	28

VENUS, *Continued*

Months	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
30	29	34	8	36	36	15	30	18	29	42	56	35	44	0
60	59	8	17	13	12	31	0	36	59	25	53	11	28	0
90	88	42	25	49	48	46	30	55	29	8	49	47	12	0
120	118	16	34	26	25	2	0	73	58	51	46	22	56	0
150	147	50	43	3	1	17	30	92	28	34	42	58	40	0
180	177	24	51	39	37	33	0	110	58	17	39	34	24	0
210	206	59	0	16	13	48	30	129	28	0	36	10	8	0
240	236	33	8	52	50	4	0	147	57	43	32	45	52	0
270	266	7	17	29	26	19	30	166	27	26	29	21	36	0
300	295	41	26	6	2	35	0	184	57	9	25	57	20	0
330	325	15	34	42	38	50	30	203	26	52	22	33	4	0
360	354	49	43	19	15	6	0	221	56	35	19	8	48	0

Days	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	59	8	17	13	12	31	0	36	59	25	53	11	28
2	1	58	16	34	26	25	2	1	13	58	51	46	22	56
3	2	57	24	51	39	37	33	1	50	58	17	39	34	24
4	3	56	33	8	52	50	4	2	27	57	43	32	45	52
5	4	55	41	26	6	2	35	3	4	57	9	25	57	20
6	5	54	49	43	19	15	6	3	41	56	35	19	8	48
7	6	53	58	0	32	27	37	4	18	56	1	12	20	16
8	7	53	6	17	45	40	8	4	55	55	27	5	31	44
9	8	52	14	34	58	52	39	5	32	54	52	58	43	12
10	9	51	22	52	12	5	10	6	9	54	18	51	54	40
11	10	50	31	9	25	17	41	6	46	53	44	45	6	8
12	11	49	39	26	38	30	12	7	23	53	10	38	17	36
13	12	48	47	43	51	42	43	8	0	52	36	31	29	4
14	13	47	56	1	4	55	14	8	37	52	2	24	40	32
15	14	47	4	18	18	7	45	9	14	51	28	17	52	0
16	15	46	12	35	31	20	16	9	51	50	54	11	3	28
17	16	45	20	52	44	32	47	10	28	50	20	4	14	56
18	17	44	29	9	57	45	18	11	5	49	45	57	26	24
19	18	43	37	27	10	57	49	11	42	49	11	50	37	52
20	19	42	45	44	24	10	20	12	19	48	37	43	49	20
21	20	41	54	1	37	22	51	12	56	48	3	37	0	48
22	21	41	2	18	50	35	22	13	33	47	29	30	12	16
23	22	40	1	36	3	47	53	14	10	46	55	23	23	44
24	23	39	18	53	17	0	24	14	47	46	21	16	35	12
25	24	38	27	10	30	12	55	15	24	45	47	9	46	40
26	25	37	35	27	43	25	26	16	1	45	13	2	58	8
27	26	36	43	44	56	37	57	16	38	44	38	56	9	36
28	27	35	52	2	9	50	28	17	15	44	4	49	21	4
29	28	35	0	19	23	2	59	17	52	43	30	42	32	32
30	29	34	8	36	36	15	30	18	29	42	56	35	44	0

MERCURY, Anomaly at epoch, 21°55′

18-years	*Longitude at epoch, Fishes 0°45* — *Degrees in longitude*							*Apogee at epoch, Balance 1°10* — *Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
18	355	37	25	36	20	34	30	251	0	45	45	53	45	0
36	351	14	51	12	41	9	0	142	1	31	31	47	30	0
54	346	52	16	49	1	43	30	33	2	17	17	41	15	0
72	342	29	42	25	22	18	0	284	3	3	3	35	0	0
90	338	7	8	1	42	52	30	175	3	48	49	28	45	0
108	333	44	33	38	3	27	0	66	4	34	35	22	30	0
126	329	21	59	14	24	1	30	317	5	20	21	16	15	0
144	324	59	24	50	44	36	0	208	6	6	7	10	0	0
162	320	36	50	27	5	10	30	99	6	51	53	3	45	0
180	316	14	16	3	25	45	0	350	7	37	38	57	30	0
198	311	51	41	39	46	19	30	241	8	23	24	51	15	0
216	307	29	7	16	6	54	0	132	9	9	10	45	0	0
234	303	6	32	52	27	28	30	23	9	54	56	38	45	0
252	298	43	58	28	48	3	0	274	10	40	42	32	30	0
270	294	21	24	5	8	37	30	165	11	26	28	26	15	0
288	289	58	49	41	29	12	0	56	12	12	14	20	0	0
306	285	36	15	17	49	46	30	307	12	58	0	13	45	0
324	281	13	40	54	10	21	0	198	13	43	46	7	30	0
342	276	51	6	30	30	55	30	89	14	29	32	1	15	0
360	272	28	32	6	51	30	0	340	15	15	17	55	0	0
378	268	5	57	43	12	4	30	231	16	1	3	48	45	0
396	263	43	23	19	32	39	0	122	16	46	49	42	30	0
414	259	20	48	55	53	13	30	13	17	32	35	36	15	0
432	254	58	14	32	13	48	0	264	18	18	21	30	0	0
450	250	35	40	8	34	22	30	155	19	4	7	23	45	0
468	246	13	5	44	54	57	0	46	19	49	53	17	30	0
486	241	50	31	21	15	31	30	297	20	35	39	11	15	0
504	237	27	56	57	36	6	0	188	21	21	25	5	0	0
522	233	5	22	33	56	40	30	79	22	7	10	58	45	0
540	228	42	48	10	17	15	0	330	22	52	56	52	30	0
558	224	20	13	46	37	49	30	221	23	38	42	46	15	0
576	219	57	39	22	58	24	0	112	24	24	28	40	0	0
594	215	35	4	59	18	58	30	3	25	10	14	33	45	0
612	211	12	30	35	39	33	0	254	25	56	0	27	30	0
630	206	49	56	12	0	7	30	145	26	41	46	21	15	0
648	202	27	21	48	20	42	0	36	27	27	32	15	0	0
666	198	4	47	24	41	16	30	287	28	13	18	8	45	0
684	193	42	13	1	1	51	0	178	28	59	4	2	30	0
702	189	19	38	37	22	25	30	69	29	44	49	56	15	0
720	184	57	4	13	43	0	0	320	30	30	35	50	0	0
738	180	34	29	50	3	34	30	211	31	16	21	43	45	0
756	176	11	55	26	24	9	0	102	32	2	7	37	30	0
774	171	49	21	2	44	43	30	353	32	47	53	31	15	0
792	167	26	46	39	5	18	0	244	33	33	39	25	0	0
810	163	4	12	15	25	52	30	135	34	19	25	18	45	0

MERCURY, *Continued*

Single years	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	359	45	24	45	21	8	35	53	56	42	32	32	59	10
2	359	30	49	30	42	17	10	107	53	25	5	5	58	20
3	359	16	14	16	3	25	45	161	50	7	37	38	57	30
4	359	1	39	1	24	34	20	215	46	50	10	11	56	40
5	358	47	3	46	45	42	55	269	43	32	42	44	55	50
6	358	32	28	32	6	51	30	323	40	15	15	17	55	0
7	358	17	53	17	28	0	5	17	36	57	47	50	54	10
8	358	3	18	2	49	8	40	71	33	40	20	23	53	20
9	357	48	42	48	10	17	15	125	30	22	52	56	52	30
10	357	34	7	33	31	25	50	179	27	5	25	29	51	40
11	357	19	32	18	52	34	25	233	23	47	58	2	50	50
12	357	4	57	4	13	43	0	287	20	30	30	35	50	0
13	356	50	21	49	34	51	35	341	17	13	3	8	49	10
14	356	35	46	34	56	0	10	35	13	55	35	41	48	20
15	356	21	11	20	17	8	45	89	10	38	8	14	47	30
16	356	6	36	5	38	17	20	143	7	20	40	47	46	40
17	355	52	0	50	59	25	55	197	4	3	13	20	45	50
18	355	37	25	36	20	34	30	251	0	45	45	53	45	0

Hours	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	2	27	50	43	3	1	0	7	46	0	17	28	59
2	0	4	55	41	26	6	2	0	15	32	0	34	57	59
3	0	6	23	32	9	9	3	0	23	18	0	52	26	58
4	0	9	51	22	52	12	5	0	31	4	1	9	55	58
5	0	12	19	13	35	15	6	0	38	50	1	27	24	57
6	0	14	47	4	18	18	7	0	46	36	1	44	53	57
7	0	17	14	55	1	21	9	0	54	22	2	2	22	57
8	0	19	42	45	44	24	10	1	2	8	2	19	51	56
9	0	22	10	36	27	27	11	1	9	54	2	37	20	56
10	0	24	38	27	10	30	12	1	17	40	2	54	49	55
11	0	27	6	17	53	33	14	1	25	26	3	12	18	55
12	0	29	34	8	36	36	15	1	33	12	3	29	47	55
13	0	32	1	59	19	39	16	1	40	58	3	47	16	54
14	0	34	29	50	2	42	18	1	48	44	4	4	45	54
15	0	36	57	40	45	45	19	1	56	30	4	22	14	53
16	0	39	25	31	28	48	20	2	4	16	4	39	43	53
17	0	41	53	22	11	51	21	2	12	2	4	57	12	52
18	0	44	21	12	54	54	23	2	19	48	5	14	41	52
19	0	46	49	3	37	57	24	2	27	34	5	32	10	52
20	0	49	16	54	21	0	25	2	35	20	5	49	39	51
21	0	51	44	45	4	3	27	2	43	6	6	7	8	51
22	0	54	12	35	47	6	28	2	50	52	6	24	37	50
23	0	56	40	26	30	9	29	2	58	38	6	42	6	50
24	0	59	8	17	13	12	31	3	6	24	6	59	35	50

MERCURY, *Continued*

Months	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
30	29	34	8	36	36	15	30	93	12	3	29	47	55	0
60	59	8	17	13	12	31	0	186	24	6	59	35	50	0
90	88	42	25	49	48	46	30	279	36	10	29	23	45	0
120	118	16	34	26	25	2	0	12	48	13	59	11	40	0
150	147	50	43	3	1	17	30	106	0	17	28	59	35	0
180	177	24	51	39	37	33	0	199	12	20	58	47	30	0
210	206	59	0	16	13	48	30	292	24	24	28	35	25	0
240	236	33	8	52	50	4	0	25	36	27	58	23	20	0
270	266	7	17	29	26	19	30	118	48	31	28	11	15	0
300	295	41	26	6	2	35	0	212	0	34	57	59	10	0
330	325	15	34	42	38	50	30	305	12	38	27	47	5	0
360	354	49	43	19	15	6	0	38	24	41	57	35	0	0

Days	*Degrees in longitude*							*Degrees of anomaly*						
	°	I	II	III	IV	V	VI	°	I	II	III	IV	V	VI
1	0	59	8	17	13	12	31	3	6	24	6	59	35	50
2	1	58	16	34	26	25	2	6	12	48	13	59	11	40
3	2	57	24	51	39	37	33	9	19	12	20	58	47	30
4	3	56	33	8	52	50	4	12	25	36	27	58	23	20
5	4	55	41	26	6	2	35	15	32	0	34	57	59	10
6	5	54	49	43	19	15	6	18	38	24	41	57	35	0
7	6	53	58	0	32	27	37	21	44	48	48	57	10	50
8	7	53	6	17	45	40	8	24	51	12	55	56	46	40
9	8	52	14	34	58	52	39	27	57	37	2	56	22	30
10	9	51	22	52	12	5	10	31	4	1	9	55	58	20
11	10	50	31	9	25	17	41	34	10	25	16	55	34	10
12	11	49	39	26	38	30	12	37	16	49	23	55	10	0
13	12	48	47	43	51	42	43	40	23	13	30	54	45	50
14	13	47	56	1	4	55	14	43	29	37	37	54	21	40
15	14	47	4	18	18	7	45	46	36	1	44	53	57	30
16	15	46	12	35	31	20	16	49	42	25	51	53	33	20
17	16	45	20	52	44	32	47	52	48	49	58	53	9	10
18	17	44	29	9	57	45	18	55	55	14	5	52	45	0
19	18	43	37	27	10	57	49	59	1	38	12	52	20	50
20	19	42	45	44	24	10	20	62	8	2	19	51	56	40
21	20	41	54	1	37	22	51	65	14	26	26	51	32	30
22	21	41	2	18	50	35	22	68	20	50	33	51	8	20
23	22	40	10	36	3	47	53	71	27	14	40	50	44	10
24	23	39	18	53	17	0	24	74	33	38	47	50	20	0
25	24	38	27	10	30	12	55	77	40	2	54	49	55	50
26	25	37	35	27	43	25	26	80	46	27	1	49	31	40
27	26	36	43	44	56	37	57	83	52	51	8	49	7	30
28	27	35	52	2	9	50	28	86	59	15	15	48	43	20
29	28	35	0	19	23	2	59	90	5	39	22	48	19	10
30	29	34	8	36	36	15	30	93	12	3	29	47	55	0

5. Preliminaries of the Hypotheses of the Five Planets

Now, the relation of the anomalies to the longitudinal passage of the five planets follows the exposition of these mean movements, and we have attempted a general outline of it in the following way.

For, as we said, the very simple movements together sufficient for the problem in hand are two: one effected by circles eccentric to the ecliptic, and the other by circles concentric with the ecliptic but bearing epicycles. And likewise also the apparent anomalies for each star considered singly are two: one observed with respect to the parts of the zodiac and the other with respect to the configurations of the sun. In the case of this latter anomaly, we find, from different configurations observed in contiguity and in the same parts of the zodiac, that, for the five planets, the time from the greatest movement to the mean movement is always longer than from the mean to the least. And such a property (*σύμπτωμα*) cannot follow from the hypothesis of eccentricity, but its contrary follows, because the greatest passage is always effected at the perigee, and in both hypotheses the arc from the perigee to the point of mean passage is less than that from this point to the apogee. But it can occur in the hypothesis of epicycles when the greatest passage is not effected at the perigee as in the case of the moon, but at the apogee—that is, when the star, starting from the apogee, moves not westward as the moon, but eastward in the opposite direction. And so we suppose this anomaly to be produced by epicycles.

But in the case of the anomaly observed with respect to the parts of the zodiac, we find, by means of the arcs of the zodiac taken at the same phases or configurations, that, on the contrary, the time from the least movement to the mean is always greater than the time from the mean movement to the greatest. But this property can follow from either hypothesis, and at the beginning of the composition of the sun [III, 3] we showed how they were alike in this. But since it is more proper to the eccentric hypothesis, we suppose this anomaly is effected according to it, and also because the other anomaly is peculiar to the epicyclic hypothesis.

But immediately on applying the particular positions observed to the courses constructed from the combination of both hypotheses and continually examining them together, we find things cannot proceed so simply: (1) The planes in which we describe the eccentric circles are not immobile, so that the straight line through both their centres and the ecliptic's centre, along which the apogees and perigees are sighted, always remains at the same angular distances from the tropic and equinoctial points. (2) The epicycles do not have their centres borne on the eccentric circles whose centres are those with respect to which the epicycles' centres revolve in a regular eastward motion and cut off equal angles in equal times. But (1) the eccentrics' apogees make a slight regular shift eastward from the tropic points around the ecliptic's centre and nearly as much for each planet as the sphere of the fixed stars is found to make—that is, one degree in a hundred years, as far as one can detect from present data. And (2) the epicycles' centres are borne on circles equal to the eccentrics effecting the anomaly, but described about other centres. And these other centres, in the case of all except Mercury, bisect the straight lines between the centres of the eccentrics effecting the anomaly and the centre of the ecliptic.[1] But in the case of Mercury

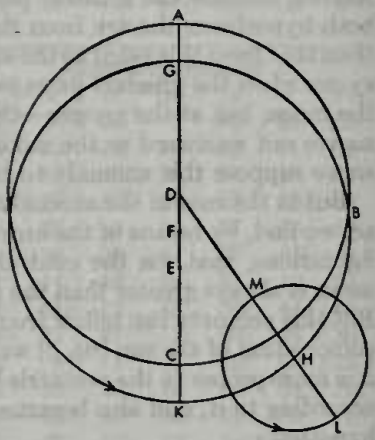
[1]These three points—the centre of the equant, the centre of the deferent, and the centre of the ecliptic—with the deferent's centre midway between the other two, by means of the

alone, this other centre is the same distance from the centre revolving it [centre of equant] as this centre revolving it is in turn from the centre effecting the anomaly on the side of the apogee, and as this last centre effecting the anomaly is in turn from the centre placed at the eye. For, in the case of that star alone as also with the moon, we find the eccentric circle revolved by the aforesaid centre, contrariwise to the epicycle, back westward one revolution in a year's time, since it appears to be twice perigee in one revolution, just as the moon is also twice so in one month's time.

6. On the Mode and Difference of these Hypotheses

The mode of the hypotheses just derived would be more easy to understand in this way:

In the case of the hypothesis of all the planets except Mercury, first let there be conceived the eccentric circle *ABC* about the centre *D*, and the diameter *ADC* through *D* and the centre of the ecliptic. And on this diameter let *E* be made the centre of the ecliptic, the point *A* the apogee, and *C* the perigee. And let *DE* be bisected at *F;* and with *F* as centre and *DA* as radius, let circle *GHK* be drawn, equal of course to circle *ABC*. And with *H* as centre let the epicycle *LM* be drawn, and let the straight line *LHMD* be joined.



Then first we suppose the plane of the eccentric circles to be inclined to that of the ecliptic, and again the plane of the epicycle to that of the eccentric, because of the latitudinal passage of the stars to be demonstrated by us hereafter. But to make things easy as far as the longitudinal passages are concerned, we suppose that they are all conceived in the one plane of the ecliptic, since there will be no appreciable difference in longitude resulting from such inclinations as will be found for each of the stars.

Then we say that the whole plane revolves eastward in the direction of the signs about centre *E*, moving the apogees and perigees one degree in a hundred years; that the epicycle's diameter *LHM* in turn is revolved regularly by centre *D* eastward in the direction of the signs at the rate of the star's longitudinal return; and that at the same time it revolves the points of the epicycle *L* and *M*, its centre *H* always borne on the eccentric *GHK*, and the star itself. And the star in turn moves on the epicycle *LM*, regularly with respect to the diameter always pointing to centre *D*, and makes its returns at the rate of the mean cycle

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Copernican transformation, come to represent the two foci and the centre of the Keplerian ellipse with exactly the same ratios preserved.

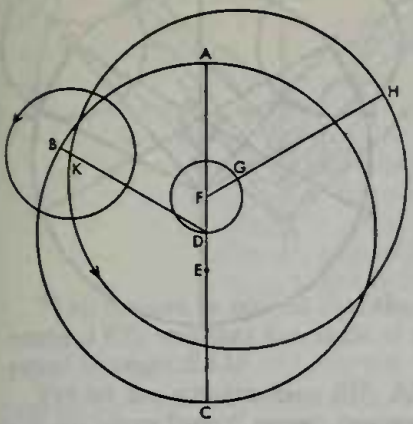
It is to be remarked that this broadening of the principle of celestial mechanics to allow an epicycle's centre to move regularly about a point other than the centre of its deferent was for Copernicus the major scandal of the Ptolemaic system and one which his own allowed him to remove only at the expense of the appearances, as Kepler well saw.

All this is explained in more detail in the first note of Book XII and in Appendix B to this work.

of the anomaly with respect to the sun, moving eastward in the order of the signs at the apogee *L*.

And we could visualize the characteristic property of the hypothesis of Mercury in this way.

For let there be the eccentric circle of anomaly *ABC* about centre *D*, and the diameter *ADEC* through *D*, the ecliptic's centre *E*, and the apogee *A*. And let *DF* be taken on *AC* in the direction of the apogee *A* and equal to *DE*.



Then, other things remaining the same (that is, the whole plane shifting the apogee eastward about *E* as a centre the same amount as for the other stars, and the epicycle being revolved regularly eastward about *D* as centre by the straight line *DB*, and again the star moving on the epicycle in the same way as the others), here the centre of the other eccentric, equal to the first and on which the epicycle's centre will always lie, will be revolved by the straight line *FGH* about point *F* contrariwise to the epicycle—that is, westward in the direction opposite to the signs—regularly and at the same speed as the epicycle, so that each of the straight lines *DB* and *FGH*, in one year's time, is restored once with respect to the points of the ecliptic, and twice with respect to each other. And this centre of the other eccentric will always be a distance from the point *F* equal to either of the straight lines *ED* or *DF* (for instance, *FG*) so that the little circle described by its movement westward, with *F* as centre and *FG* as radius, is always bounded by the centre of the first eccentric which remains fixed; and the moving eccentric is described in every case with *G* as centre and with radius *GH*, equal to *DA*, as for instance here *HK*, and the epicycle always has its centre on it, as here at point *K*.

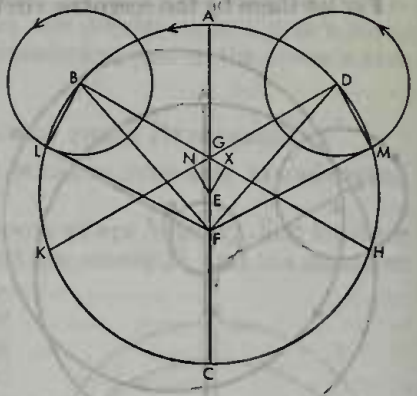
We can follow the suppositions even more precisely from the demonstrations we are to give of the magnitudes for each planet. In these demonstrations the motives for formulating the hypotheses will somehow show up in clearer outline.

Yet one must premise that, since the longitudinal cycles do not make their returns with the points of the zodiac and with the apogees or perigees of the eccentric circles, because of their shift, therefore the longitudinal movements set out by us in the foregoing manner do not contain returns considered with respect to the apogees of the eccentrics but returns that are relative to the tropic and equinoctial points in accordance with our year.

Then it must be shown first that, also according to these hypotheses, whenever the mean longitudinal position of the star is equidistant from the apogees and perigees in either direction, the difference of zodiacal anomaly is equal for either distance, and also the greatest elongation on the epicycle on the corresponding side of the mean position.

For about centre *E* and diameter *AEC*, let there be the eccentric circle *ABC* on which the epicycle's centre is borne. On this diameter let *F* be taken as the ecliptic's centre, and *G* as the centre of the eccentric making the anomaly—that

is, the centre around which we say the epicycle's mean passage is regularly effected. And let the straight lines *BGH* and *DGK* be drawn through, both the same angular distance from the apogee *A*, so that angles *AGB* and *AGD* are equal. And let epicycles be described about points *B* and *D*, and let straight lines *BF* and *BD* be joined, and from *F* the eye-point let the straight lines *FL* and *FM* be drawn tangent to the epicycles on the corresponding sides.



I say that angle *FBG* of the difference of zodiacal anomaly is equal to angle *GDF;* and that angle *BFL*, the greatest elongation on the epicycle, is likewise equal to angle *DFM*. For in this way the magnitudes of the elongations resulting from the combination of the eccentric and of the greatest elongations from the mean position will also be equal.

Then let the straight lines *BL* and *DM* be drawn from *B* and *D* perpendicular to *FL* and *FM*, and straight lines *EN* and *EX* from *E* perpendicular to *BH* and *DK*. Since

angle *XGE* = angle *NGE*,

and the angles at *N* and *X* are right, and *EG* is common to the equiangular triangles, therefore

NG = *XG*,

EN = *EX*.

Therefore *BH* and *DK* are chords equidistant from the centre *E;* therefore they are equal and their halves also. And so, by subtraction,

BG = *DG*.

But *GF* is common, and

angle *BGF* = angle *DGF*,

and are within equal sides; therefore also

base *BF* = base *DF*,

and

angle *GBF* = angle *DGF*.

And also, as radii of the epicycles,

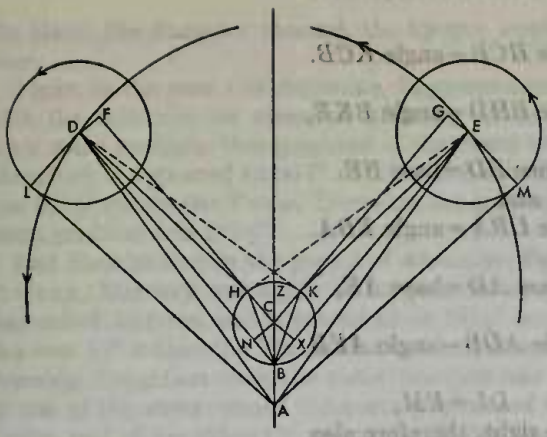
BL = *DM*,

and the angles at *L* and *M* are right; therefore also

angle *BFL* = angle *DFM*.

Which things it was required to prove.

Then again, for the hypothesis of Mercury, let there be the diameter *ABC* through the circles' centres and apogee; and let *A* be supposed the centre of the ecliptic; *B* the centre of the eccentric effecting the anomaly, and *C* the point about which moves the centre of the eccentric bearing the epicycle. And again let the straight lines *BD* and *BE* of the epicycle's regular eastward motion be drawn through on either side, and the straight lines *CF* and *CG* of the eccentric's westward revolution at the same rate, so that of course the angles at *C* and *B* are



equal, and BD parallel to CF, and BE to CG. And let the centres of the eccentrics be taken on CF and CG, and let them be H and K. And let the eccentrics described about these centres, eccentrics bearing the epicycles, pass through the points D and E. Again, with equal epicycles described about points D and E, let the straight lines AD and AE be joined, and let the straight lines AL and AM be drawn tangent to the corresponding sides of the epicycles.

Then it must be proved that the angle of zodiacal anomaly ADB is equal to angle AEB; and that the angle of greatest elongation for the epicycle DAL is equal to angle EAM.

For let the straight lines BH, BK, HD, and KE be joined; and let CN and CX be drawn from C perpendicular to BD and BE; and DF and EG from D and E perpendicular to CF and CG; and DL and EM perpendicular to AL and AM. Since then

$$\text{angle } CBN = \text{angle } CBX,$$

and the angles at N and X are right, and the straight line CB is common, therefore

$$CN = CX,$$

that is

$$DF = EG.$$

But also

$$HD = \text{KE},^1$$

and the angles at F and G are right. And so

$$\text{angle } DHF = \text{angle } EKG;$$

and

$$\text{angle } CHB = \text{angle } CKB,$$

because it is supposed

$$HC = CK,$$

[1]For, as below,

$$HB = KB,$$

and as radii of the same moving eccentric

$$HE = KD.$$

And by hypothesis

$$\text{angle } DBC = \text{angle } EBC;$$

and, since by hypothesis

$$\text{arc } HZ = \text{arc } ZK,$$

therefore

$$\text{angle } HBC = \text{angle } KBC,$$

and by addition

$$\text{angle } DBZ = \text{angle } EBZ.$$

Hence, by congruent triangles DKB and EHB,

$$HD = KE.$$

CB is common, and

angle HCB = angle KCB.

And so also, by subtraction,

angle BHD = angle BKE,

and

base BD = base BE.

But again BA is also common, and

angle DBA = angle EBA.

And so

base AD = base AE,

and

angle ADB = angle AEB

And in the same way, since

$DL = EM$,

and the angles at L and M are right, therefore also

angle DAL = angle EAM.

Which things it was required to prove.[1]

7. Demonstration of Mercury's Apogee and of its Shift

With this theory established, we first took this way of finding the whereabouts on the ecliptic of Mercury's apogee. For we examined the observations of the greatest elongations in which the morning positions of the star were equal to the evening ones in their angular distance from the mean position of the sun. When these had been found, it followed from what we have shown that the point midway between the two positions included the eccentric's apogee.

Then we took some observations for this purpose, few because such combinations are seldom met with, but capable of showing up the subject in hand. Here are the more recent of them.

For in the year 16 of Hadrian, Egyptianwise Phamenoth 16-17 in the evening, we observed Mercury, by the manipulation of the astrolabe, at its greatest elongation from the sun's mean position. And then, being sighted with the bright star of the Hyades [Aldebaran], it appeared to be 1° within the Fishes. But at that time the mean position of the sun $(9+\frac{1}{2}+\frac{1}{4})°$ within the Water Bearer. Therefore the greatest evening elongation from the mean position was $21\frac{1}{4}°$.

And in the year 18 of Hadrian, Egyptianwise Epiphi 18-19 at dawn, Mercury was at its greatest elongation, appeared very slight and dim, was sighted with the bright star of the Hyades [Aldebaran], and appeared to be $(18+\frac{1}{2}+\frac{1}{4})°$ within the Bull. But at that time the mean sun was 10° within the Twins. And therefore at that time the greatest morning elongation from the mean position was equally $21\frac{1}{4}°$. And so, since in one observation the star's mean position was $(9+\frac{1}{2}+\frac{1}{4})°$ within the Water Bearer, and in the other 10° within the Twins, and since the point on the ecliptic midway between them is $(10-\frac{1}{8})°$ within

[1] In other words, Ptolemy has proved that if the epicycle's centre is equidistant on either side from the apogee, then the angles are equal. It cannot be proved in the case of the hypotheses of Mercury that the converse is true, since as we shall see later Mercury has two perigees and, therefore, there are other points of symmetry. Ptolemy, however, will use the converse as true for his observations. This is possible because he knows about where the apogee of the eccentric lies and, within a limited range on either side of that approximate point, the converse is true.

the Ram, the diameter through the apogee would be in that position at that time.

Again, in the year 1 of Antonine, Egyptianwise Epiphi 20-21 in the evening, with the astrolabe we observed Mercury at its greatest elongation from the sun's mean position. Being sighted at that time with the star of the Lion's heart [Regulus], it appeared to be 7° within the Crab. But at that time the mean sun was 10½° within the Twins. Therefore the greatest evening elongation from the mean position was 26½°.

And likewise also in the year 4 of Antonine, Egyptianwise Phamenoth 18-19 at dawn, Mercury was again at its greatest elongation, was sighted with the star called Antares, and appeared to be 13½° within the Goat, while the mean sun was 10° within the Water Bearer. And therefore at that time the greatest morning elongation from the mean position was likewise 26½°. And so, since at one of the observations the mean position of the star was 10½° within the Twins, and at the other 10° within the Water Bearer, therefore the point of the ecliptic midway between them is 10¼° within the Balance, and at that time the diameter through the apogee would be in that position.

From these observations, then, we find the apogee falling very nearly 10° within the Ram or the Balance; and from the old observations of the greatest elongations, about 6° within the same signs, just as one would calculate it to be.

For in the year 23 according to Dionysius, Hydron 29 in the morning, Mercury was three moon's to the north of the brightest star in the Goat's tail. But this fixed star was then, according to our starting points (that is, the tropic and equinoctial points), 22⅓° within the Goat, the same evidently as Mercury. And it is evident the mean sun was 18⅙° within the Water Bearer; for that time was the year 486 of Nabonassar, Egyptianwise Choïak 17-18 at dawn. Therefore, the greatest morning elongation from the mean position was (25+½+⅓)°.

We did not find a greatest evening elongation exactly equal to this one in any of the observations that have come down to us, but from two of them we calculated one very nearly equal in the following manner.

For in the same year 23 according to Dionysius, Tauron 4 in the evening, Mercury was three moons beyond the straight line through the horns of the Bull, and seemed to be passing more than three moons south of the common star.[1] And so again, according to our starting points, it was 23⅔° within the Bull. And that was again the year 486 of Nabonassar, Egyptianwise Phamenoth 30-Pharmouthi 1 in the evening, when the mean sun was 29½° within the Ram. Therefore, the greatest evening elongation from the mean position was 24⅙°.

In the year 28 according to Dionysius, Didymon 7 in the evening, Mercury was in a straight line with the heads of the Twins, and was to the south of the southern one by a third part of a moon less than double the distance between the heads. And so again at that time Mercury was, according to our starting points, 29⅓° within the Twins. And that was the year 491 of Nabonassar, Egyptianwise Pharmouthi 5-6 in the evening, at which time the mean sun was (2+½+⅓)° within the Twins. And therefore the elongation itself was 26½°.

Since, when the mean position was 29½° within the Ram, the greatest elongation was 24⅙°, and, when the mean position was (2+½+⅓)° within the Twins, the elongation was 26½°, and since the greatest morning elongation whose cor-

[1]The star described in the Catalogue of the Stars as "the star at the tip of the northern horn, the same as the star in the Charioteer's right foot."

respondent we sought was $(25+\frac{1}{2}+\frac{1}{3})°$, we extracted from the difference of the two observations just reported, where the mean position would have to be in order for the greatest evening elongation to be $(25+\frac{1}{2}+\frac{1}{3})°$. For the difference between the two mean positions comes to $33\frac{1}{3}°$, and between the greatest elongations to $2\frac{1}{3}°$, so that very nearly 24° corresponds to the $1\frac{2}{3}°$ difference between $(25+\frac{1}{2}+\frac{1}{3})°$ and $24\frac{1}{6}°$. If we add these 24° to the $29\frac{1}{2}°$ within the Ram, we shall have the mean position where the greatest evening elongation will be equal to the greatest morning elongation of $(25+\frac{1}{2}+\frac{1}{3})°$, and that will be $23\frac{1}{2}°$ within the Bull. And the point midway between $18\frac{1}{6}°$ within the Water Bearer and $23\frac{1}{2}°$ within the Bull is about $(5+\frac{1}{2}+\frac{1}{3})°$ within the Ram.

Again, in the year 24 according to Dionysius, Leonton 28 in the evening, Mercury was west of Spica, according to Hipparchus' calculations, by little more than 3°. And so it was then $19\frac{1}{2}°$ within the Virgin according to our starting points. And that was the year 486 of Nabonassar, Egyptianwise Paÿni 30 in the evening, at which time the mean sun was $(27+\frac{1}{2}+\frac{1}{3})°$ within the Lion. Therefore, the greatest evening elongation from the mean position was $21\frac{2}{3}°$, whose exact morning correspondent we again calculate by means of the following observations.

For in the year 75 according to the Chaldeans, Dius 14 in the morning, Mercury was half a cubit on the upper side of the southern Balance. And so it was then, according to our starting points, $14\frac{1}{6}°$ within the Balance. And that was the year 512 of Nabonassar, Egyptianwise Thoth 9-10, at dawn, at which time the mean sun was $5\frac{1}{2}°$ within the Scorpion. Therefore, the greatest morning elongation was 21°.

In the year 67 according to the Chaldeans, Apellaius 5 in the morning, Mercury was half a cubit on the upper side of the northern forehead of the Scorpion. And so at that time, in our terms, it was $2\frac{1}{3}°$ within the Scorpion. And that was the year 504 of Nabonassar, Egyptianwise Thoth 27-28 at dawn, at which time the mean sun was $(24+\frac{1}{2}+\frac{1}{3})°$ within the Scorpion. Therefore the elongation itself was $22\frac{1}{2}°$. Then again—since in these two observations the difference between the mean positions comes to $19\frac{2}{3}°$, and between the greatest elongations to $1\frac{1}{2}°$, and since for this reason 9° corresponds to the $\frac{2}{3}°$ by which the $21\frac{2}{3}°$ of the elongation required exceed the 21° of the smaller elongation—if we add these 9° to the $5\frac{1}{2}°$ within the Scorpion, we shall have the mean position at which the greatest morning elongation is equal to the $21\frac{2}{3}°$ of the evening elongation. And this mean position is $14\frac{1}{6}°$ within the Scorpion. And again, the point midway between $(27+\frac{1}{2}+\frac{1}{3})°$ within the Lion and $14\frac{1}{6}°$ within the Scorpion is very nearly 6° within the Balance.

From these appearances, then, and from the agreement of the appearances particular to the other stars, we find it consistent for the diameters through the perigees and apogees of the five planets to make a shift about the ecliptic's centre eastward in the order of the signs, and for that shift to be at the same rate as that of the sphere of the fixed stars. For, as we showed [VII, 2], it shifts very nearly 1° in a hundred years, and so the time from the old observations, where Mercury's apogee was about 6°, to our own observations, where it has come to be about 10°, is found to embrace nearly 400 years.

8. That Mercury also Becomes Twice Perigee in One Revolution

Following on this, we examined the magnitudes of the greatest elongations produced when the sun's mean position was at the apogee itself, and again when it was diametrically opposite. And here one can very well understand the usefulness of this kind of sighting [*i.e.*, with the astrolabe], since, even if some observed stars do not appear near each other because of their predetermined positions (and this happens for the most part in the case of Mercury because most of the fixed stars can rarely appear at a distance from the sun equal to Mercury's), yet by sighting with the far distant stars it is possible to get accurately their longitudinal and latitudinal positions.

Now in the year 19 of Hadrian, Egyptianwise Athyr 14-15, the morning Mercury was at its greatest elongation, and, sighted with the star of the Lion's heart, it appeared to be $20\frac{1}{5}°$ within the Virgin, while the mean sun was about $9\frac{1}{4}°$ within the Balance. And so the greatest elongation came to $19\frac{1}{20}°$.

And in the same year, Pachon 19 in the evening, it was at a greatest elongation, and, sighted with the bright star of the Hyades [Aldebaran], appeared to be $4\frac{1}{3}°$ within the Bull, while the mean sun was $11\frac{1}{2}°$ within the Ram. And so the greatest elongation in this case came to $23\frac{1}{4}°$, and from that it was clear the eccentric's apogee was in the Balance and not in the Ram.

Now, these things being given, let the straight line ABC be the diameter through the apogee; and let the point B be supposed the ecliptic's centre where the eye is; A the point 10° within the Balance; and C the point 10° within the Ram. Let equal epicycles be described about A and C, one with point D on it, the other with point E; and let the straight lines BD and BE be produced from B tangent to them; and let the perpendiculars AD and CE be drawn from the centres to the points of contact.

Since, then, the greatest morning elongation in the Balance was observed to be $19\frac{1}{20}°$ from the mean position,

$$\text{angle } ABD = 19°3'$$
$$= 38°6' \text{ to 2 rt.}$$

And so, on the circle about right triangle ABD,

$$\text{arc } AD = 38°6',$$

and

$$\text{chord } AD \fallingdotseq 39^{p}9'$$

where

$$\text{hypt. } AB = 120^{p}.$$

Again, since the greatest evening elongation in the Ram was observed to be $23\frac{1}{4}°$ from the mean position,

$$\text{angle } CBE = 23°15'$$
$$= 46°30' \text{ to 2 rt.}$$

And so also, on the circle about right triangle CBE,

$$\text{arc } CE = 46°30'$$

and

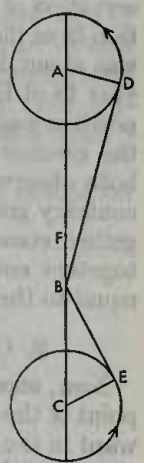
$$\text{chord } CE = 47^{p}22'$$

where

$$\text{hypt. } CB = 120^{p}.$$

And because as radii of the epicycles

$$AD = CE,$$



therefore

$$BC = 99^{p}9'$$

and, by addition,

$$ABC = 219^{p}9'$$

where

$$CE = 39^{p}9'$$

and

$$AB = 120^{p}.$$

And so, if the straight line ABC be bisected at F, then as half of ABC

$$AF = 109^{p}34',$$

and

$$BF = 10^{p}25'.$$

Then it is clear that either the point F is the centre of the eccentric or the centre around which the centre of that eccentric circle is revolved. For in this way only could the epicycle's centre be equidistant from F for either of the diametrically opposite positions, as was proved. But since, if F were itself the centre of that eccentric on which the epicycle's centre always lies, that eccentric would be fixed, and of all the positions that in the Ram would alone be perigee because BC is the least of all the straight lines from B to the circle described about F; and since the position in the Ram is not to be the nearest the earth, but positions in the Twins and the Water Bearer are even nearer the earth than it and very nearly equal to each other, therefore it is clear that the centre of this eccentric is carried about point F in the direction contrary to that of the epicycle's revolution — that is, westward contrary to the order of the signs — once in one cycle. For in this way the epicycle's centre will be twice perigee in that cycle.

It is immediately understandable from the observations just set out [IX, 7], that the epicycle is nearer the earth in the Twins and the Water Bearer than in the Ram. For in the observation of the year 16 of Hadrian, Phamenoth 16, the greatest evening elongation from the mean position was $21\frac{1}{4}°$; and in the observation of the year 4 of Antonine, Phamenoth 18, the greatest morning elongation from the mean position was $26\frac{1}{2}°$, while the mean sun for both observations was about 10° within the Water Bearer. And again, in the observation of the year 18 of Hadrian, Epiphi 19, the greatest morning elongation from the mean position was $21\frac{1}{4}°$; and in the observation of the year 1 of Antonine, Epiphi 20, the greatest evening elongation from the mean position was $26\frac{1}{2}°$, while for both observations the mean sun was about 10° within the Twins. And so the contrary greatest elongations in the Water Bearer and in the Twins, added together, come to $(47+\frac{1}{2}+\frac{1}{4})°$, while both the greatest elongations in the Ram together embrace $46\frac{1}{2}°$, because the evening elongation was observed to be equal to the morning one of $23\frac{1}{4}°$.

9. On the Ratio and Magnitude of Mercury's Anomalies

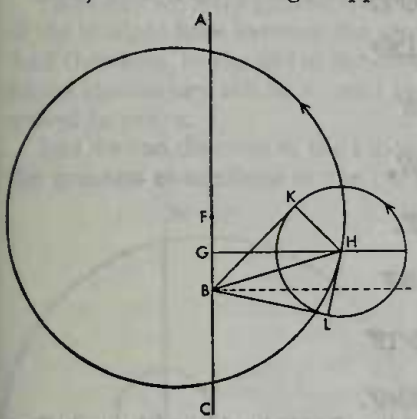
Now, once these things have been dealt with, it remains to show around what point of the straight line AB the epicycle's yearly return in regular motion eastward in the order of the signs takes place; and how far from F is the centre of the eccentric making its return at an equal speed westward. We have also used for this research two observations of greatest elongations both morning and evening, with the mean position in either case situated a quadrant's distance

from the apogee on the same side, for in this position very nearly the greatest difference of zodiacal anomaly is produced.

For in the year 14 of Hadrian, Egyptianwise Mesore 18 in the evening, as we find in the observations taken by Theon, "Mercury," he says, "was at its greatest elongation from the sun, being $(3+\frac{1}{2}+\frac{1}{3})°$ east of the star in the Lion's heart [Regulus]." And so, according to our starting points, it was very nearly $6\frac{1}{3}°$ within the Lion, while the mean sun at that time was about $10\frac{1}{12}°$ within the Crab. And so the greatest elongation was $26\frac{1}{4}°$.

And in the year 2 of Antonine, Egyptianwise Mesore 24 at dawn, observing its greatest elongation with the astrolabe and sighting it with the bright star of the Hyades [Aldebaran], we found Mercury to be $20\frac{1}{12}°$ within the Twins, while the mean sun was again about $10\frac{1}{3}°$ within the Crab. And so the greatest morning elongation was $20\frac{1}{4}°$.

Now, with these things supposed, again let there be the diameter $AFBC$ through the points 10° within the Balance and the Ram. And as in the last figure, let A be supposed the point where the epicycle's centre lies when it is 10° within the Balance; C the point where it lies when it is 10° within the Ram; B the centre of the ecliptic; and F the point about which the eccentric's centre makes its westward shift.



First let it be required to find how far from the point B is the centre about which we say the epicycle's regular eastward movement takes place.

Then let it be the point G, and let a straight line be drawn through G at right angles to AC, so that it will be a quadrant's distance from the apogee A; and let H be taken on it as the centre of the epicycle, according to the observations just set forth, because in these the mean position was a quadrant's distance from the apogee, the mean sun being about 10° within the Crab. And with the epicycle KL described about H, let the straight lines BK and BL be drawn from B tangent to it. And let HK, HL, and BH be joined.[1]

Now, since at the mean position just taken, the greatest morning elongation is supposed $20\frac{1}{4}°$, and the evening $26\frac{1}{4}°$, therefore

$$\text{angle } KBL = 46°30';$$

and therefore, being half of angle KBL,

$$\text{angle } KBH = 46°30' \text{ to 2 rt.}$$

And so also, on the circle about right triangle BHK,

$$\text{arc } HK = 46°30',$$

and

$$\text{chord } HK = 47^{p}22'$$

where

$$\text{hypt. } BH = 120^{p}.$$

[1]We have added to the figure a broken line through B parallel to GH to represent the line to the mean sun which moves regularly, of course, about the earth or the centre of the ecliptic.

And therefore where

epic. rad. $HK = 39^p9'$

and it was shown [p. 300]

$BF = 10^p25'$,

there

$BH = 99^p9'$.

Again, since the difference of 6° between these greatest elongations embraces twice the difference of zodiacal anomaly, and since this difference is contained by angle BHG (for we have proved that before [p. 295]), therefore

angle $BHG = 3°$

$= 6°$ to 2 rt.

And so, on the circle about right triangle BGH,

arc $BG = 6°$,

and

chord $BG = 6^p17'$

where

hypt. $BH = 120^p$.

And where

$BH = 99^p9'$

and likewise

$BF = 10^p25'$,

there

$BG = 5^p12'$.

Therefore

$BG =$ half BF

and

$BG = BF = 5^p12'$

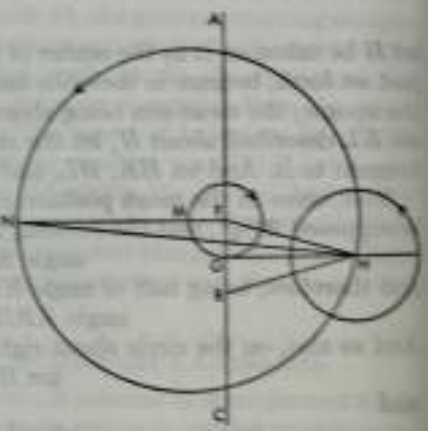
where

epic. rad. $= 39^p9'$.

Again, in the same figure, let the straight line FMN be drawn through F at right angles to AC on the side opposite GH. And it is clear the centre of that eccentric on which the epicycle's centre H lies will be on this straight line FMN, because of the equal period of joint return in opposite directions made by GH and FN. And let

$FN = FA$,

so that FN like FA is composed of the eccentric's radius and of the straight line between the eccentric's centres and the point F. And let the eccentric's centre be taken on it and let it be M. And let FH be joined.



Then, since angle MFG is right, and since angle HFG hardly differs at all from a right angle so that NFH does not differ appreciably from a straight line, and since it has been shown [p. 300] that, if

epic. rad. $= 39^p30'$,

then

$$NF = AF = 120°34'$$

and

$$FH = BH = 60°9',$$

therefore, by addition,

$$NFH = 208°43'$$

and the eccentric's radius will be the half of NFH, that is

$$NM = 104°22'$$

and, by subtraction,

$$\text{i. btw. c. } FM = 3°12'.$$

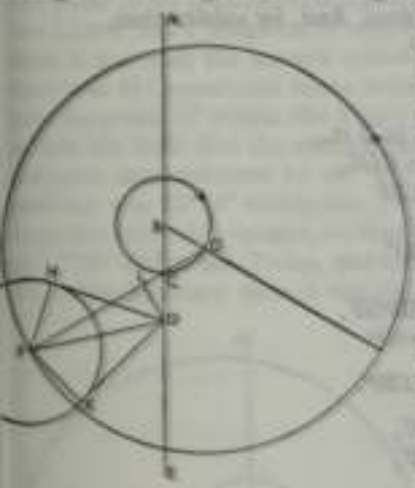
But it was shown [p. 302]

$$BG = 3°12',$$
$$GF = 3°12'$$

where the parts are the same.

Therefore we have proved that, to the 104°22′ of the eccentric's radius, each of the straight lines between the centres is 3°12′, and the epicycle's radius 30°9′. And therefore, to the 60° of the eccentric's radius, each of the straight lines between the centers will be 3°, and the epicycle's radius 22°30′. Which it was required to prove.

And we can discover in the following way that, these things being supposed, the greatest elongations at the perigees are consistent with the observations,[1] that is, that, when the mean position is 10° within the Water Bearer or 10° within the Twins and is distant from the apogee the side of the inscribed equilateral triangle, then the angle at the eye subtending the epicycle is very nearly $(47+\frac{1}{2}\frac{1}{4}+\frac{1}{8}\frac{1}{4})°$.



For let there be the diameter through the apogee $ABCDE$, and let point A on it be supposed at the apogee; B the point about which the eccentric's centre moves westward; C the point about which the epicycle's centre moves eastward; and D the centre of the ecliptic. And let both movements be taken about their proper centres, regularly and equally, in opposite directions from the apogee A, through an angular distance equal to the side of the inscribed equilateral triangle. And let CF be the straight line revolving the epicycle, and BG the straight line revolving the

[1] That these points should be the perigees of the moving eccentric must have been known to Ptolemy through empirical mathematical methods—that is, by trying points very small distances away from the given point on either side and calculating the distances. For instance, if we calculate the square of the distance BH (i.e. the distance of the epicycle's centre to the earth) first when the epicycle's centre is 118° from the apogee, secondly when it is 120° from the apogee, and thirdly when it is 122° from the apogee, we get the following values respectively: 3,101.49, 3,101.04, and 3,101.89. According to Mr. George Comstock who has worked on this problem, it is evident from the methods of the differential calculus that the perigee's being at or very near the point 120° from the apogee is dependent on the particular ratio of the line between the centres to the eccentric's radius. Therefore, there is no general geometric proof.

eccentric's centre. And let G be the eccentric's centre, and F the epicycle's. And with the epicycle described about F, let DH and DK be drawn tangent to it; and let CG, DF, FH, and FK be joined. And let DL be drawn from D perpendicular to CF.

It is required to prove that

$$\text{angle } HDK = (47+\tfrac{1}{2}+\tfrac{1}{4})^\circ.$$

Since, then, each of the angles ABG and ACL subtends a side of the inscribed equilateral triangle and equals 120°, so that

$$\text{angle } CBG = \text{angle } DCL = 60^\circ,$$

and since

$$\text{angle } BGC = \text{angle } BCG$$

because it is supposed

$$BC = BG,$$

and since the other two angles together are 120°, and each of them 60°, the triangle BCG is equiangular and equilateral. And also

$$\text{angle } DCL = \text{angle } BCG;$$

therefore G, C, and F are points on the same straight line.

And so

$$\text{ecc. rad. } GF = 60^p$$

where

$$CG = CD = 3^p,$$

CD being the straight line between the centres. And, by subtraction,

$$CF = 57^p.$$

Again, since

$$\text{angle } DCL = 60^\circ$$
$$= 120^\circ \text{ to 2 rt.},$$

therefore, on the circle about right triangle CDL,

$$\text{arc } DL = 120^\circ$$

and, by subtraction from the semicircle,

$$\text{arc } CL = 60^\circ.$$

And therefore

$$\text{chord } DL = 103^p55',$$
$$\text{chord } CL = 60^p$$

where

$$\text{hypt. } CD = 120^p.$$

And so

$$DL = 2^p36',$$
$$CL = 1^p30',$$

and, by subtraction,

$$LF = 55^p30'$$

where

$$DC = 3^p$$

and likewise

$$CF = 57^p.$$

And since

$$\text{sq. } LF + \text{sq. } DL = \text{sq. } DF,$$

therefore

$$DF = 55^p34'$$

where it was supposed

$$FH = FK = 22^p30',$$

that being the epicycle's radius.

And therefore

$$FH = FK = 48^{p}35'$$

where

$$\text{hypt. } DF = 120^{p};$$

and

$$\text{angle } FDH = \text{angle } FDK = 47°46' \text{ to 2 rt.}$$

so that, by addition,

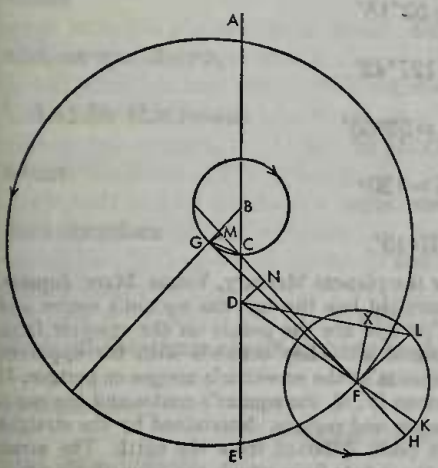
$$\text{angle } HDK = 47°46'.$$

Which it was required to prove.

10. On the Correction of Mercury's Periodic Movements

Now, the establishing of Mercury's periodic movements and its epochs follows next. And we have the longitudinal movements—that is, those carrying the epicycle regularly about *C*—immediately given from those of the sun; and the movements of anomaly—that is, those carrying the star on the epicycle about the epicycle's centre—we have taken from two sure observations, one from those recorded by us, the other from old ones.

For in the year 2 of Antonine, which was the year 886 of Nabonassar, Egyptianwise Epiphi 2-3, we observed Mercury with the astrolabe when it had not yet come to its greatest evening elongation, and being sighted with the star in the Lion's heart [Regulus] it appeared to be 17½° within the Twins. At that time it was past the moon's centre to the east by 1⅙°, and in Alexandria the time was 4½ equatorial hours before midnight of the third, since on the astrolabe the point 12° within the Virgin was culminating and the sun was about 23° within the Bull. But the sun's mean position at that hour, according to the hypotheses demonstrated by us, was 22°34′ within the Bull. The moon's mean position was 12°14′ within the Twins, and the moon was 281°20′ in anomaly from the epicycle's apogee, so that the true position of the moon's centre came to 17°10′ within the Twins, and its apparent position 16°20′ within the Twins. Therefore Mercury, since it was 1⅙° beyond the moon's centre, was 17½° within the Twins.



With this assumed, let *ABCDE* be the diameter through the apogee and perigee. And let *A* be supposed its point at the apogee; *B* the point about which the eccentric's centre moves westward; *C* the point about which the epicycle's centre moves eastward; and *D* the centre of the ecliptic. And let the epicycle's centre *F* have been moved through angle *ACF* about point *C* by the straight line *CF;* and let the eccentric's centre *G* have been moved about *B* by the straight line *BG* to the extent of angle *ABG*, of course always equal to angle *ACF* because of the equal rate of the movements. And, with the epicycle *HKL* described about *F*, let the star be

supposed at L. And let CG, GF, DF, FL, and DL be joined; and let GM and DN be drawn from G and D perpendicular to CFH produced, and FX from F perpendicular to DL.

And it is required to find the epicycle's arc from the apogee H[1] to the star at L.

Since, then, the mean sun at that time was 22°34′ within the Bull, and the star's perigee very nearly 10° within the Ram so that its mean longitudinal position was 42°34′ from the perigee, therefore

$$\text{angle } CBG = 42°34′$$
$$= 85°8′ \text{ to 2 rt.},$$

and, because always

$$BC = BG,$$
$$\text{angle } BGC = \text{angle } BCG = 137°26′ \text{ to 2 rt.}$$

And so, on the circle about triangle BCG,

$$\text{arc } GC = 85°8′$$

and

$$\text{arc } BC = 137°26′.$$

And therefore

$$\text{chord } GC = 81^{p}10′$$
$$\text{chord } BC = 111^{p}49′.$$

And so

$$GC = 2^{p}11′$$

where

$$BC = 3^{p}.$$

Again, since

$$\text{angle } BCG = 137°26′ \text{ to 2 rt.}$$

and

$$\text{angle } BCM = 85°8′ \text{ to 2 rt.},$$

therefore also, by subtraction,

$$\text{angle } GCM = 52°18′ \text{ to 2 rt.}$$

And so also, on the circle about right triangle CGM,

$$\text{arc } GM = 52°18′$$

and, by subtraction from the semicircle,

$$\text{arc } CM = 127°42′.$$

And therefore

$$\text{chord } GM = 52^{p}53′$$

where

$$\text{hypt. } CG = 120^{p}$$

and

$$CM = 107^{p}43′.$$

[1]The regular epicyclic apogee and perigee, for the planets Mercury, Venus, Mars, Jupiter, and Saturn, are determined as always by the straight line through the equant's centre and the epicycle's centre. The regular apogee and perigee are the points on the epicycle from which the anomalistic angle is regularly measured off. They coincide with the apparent apogee and perigee only when the epicycle's centre is at the eccentric's apogee or perigee. It should be recalled here that, in the case of the moon [V, 5], the equant's centre and the earth coincide, and that there is the moon's mean apogee and perigee, determined by the straight line through the epicycle's centre and another point different from the earth. The mean apogee and perigee in the case of the moon are the points from which the anomalistic angle is measured off.

And so also

$$GM = 0^{p}58'$$

and

$$CM = 1^{p}58'$$

where

$$GC = 2^{p}11'$$

and

$$GF = 60^{p},$$

GF being the radius of the eccentric bearing the epicycle. And being for this reason inappreciably less than the hypotenuse GF,

$$MF = 60^{p},$$

and, by subtraction,

$$CF = 58^{p}2'.$$

Likewise, since

$$\text{angle } DCN = 85°8' \text{ to 2 rt.},$$

therefore, on the circle about right triangle CDN,

$$\text{arc } DN = 85°8',$$

and, by subtraction from the semicircle,

$$\text{arc } CN = 94°52'.$$

And so

$$\text{chord } DN = 81^{p}10'$$

and

$$\text{chord } CN = 88^{p}23'$$

where

$$\text{hypt. } CD = 120^{p}.$$

And therefore

$$DN = 2^{p}2'$$

and

$$CN = 2^{p}13'$$

and, by subtraction,

$$NF = 55^{p}49'$$

where

$$CD = 3^{p}$$

and, as was shown,

$$CF = 58^{p}2'.$$

And for this reason

$$\text{hypt. } DF = 55^{p}51'$$

where

$$\text{epic. rad.} = 22^{p}30'.$$

And therefore

$$DN = 4^{p}22'$$

where

$$\text{hypt. } DF = 120^{p}.$$

And, on the circle about right triangle DFN,

$$\text{arc } DN = 4°11',$$

so that also

$$\text{angle } DFN = 4°11' \text{ to 2 rt.}$$

and, by addition,

$$\text{angle } EDF = 89°19' \text{ to 2 rt. [Eucl. I, 32]},$$

and, also by addition,

angle $EDL = 135°$ to 2 rt.

because the star at that time appeared to be 67°30′ from the apogee; and, by subtraction,

angle $FDL = 45°41'$ to 2 rt.

And therefore, on the circle about right triangle DFX,

arc $FX = 45°41'$;

and

chord $FX = 46^{p}35'$

where

hypt. $DF = 120^{p}$.

And so also

$FX = 21^{p}41'$

where

$DF = 55^{p}51'$

and

epic. rad. $FL = 22^{p}30'$.

And again

$FX = 115^{p}39'$

where

hypt. $FL = 120^{p}$.

And therefore, on the circle about triangle FLX,

arc $FX = 149°2'$

and

angle $FLX = 149°2'$ to 2 rt.

And it was proved that

angle $FDL = 45°41'$ to 2 rt.

and likewise

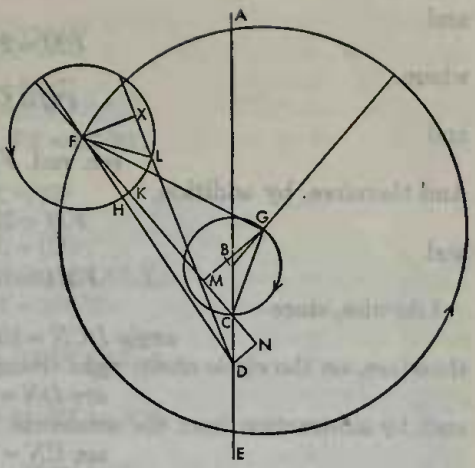
angle $HFK = 4°11'$ to 2 rt.

And so, by addition,

angle $HFL = 198°54'$ to 2 rt.
$= 99°27'$.

And therefore the arc HKL of the epicycle, which is Mercury's distance from the apogee H at the observation, is 99°27′. Which it was required to prove.

And again, in the year 21 according to Dionysius, which was the year 484 of Nabonassar, Scorpion 22, Egyptianwise Thoth 18-19, the morning Mercury was the distance of a moon east of the straight line through the northern and middle stars of the Scorpion's forehead, and was 2 moons north of the northern star of the forehead. But the middle star of the Scorpion's forehead was at that time, according to our starting points, $1\frac{2}{3}°$ within the Scorpion and the same number of degrees south of the ecliptic. And the most northern star was $2\frac{1}{3}°$ within the Scorpion and $1\frac{1}{3}°$ north of the ecliptic. Therefore Mercury was very nearly $3\frac{1}{3}°$ within the Scorpion. It is also evident that it had not yet come to its greatest morning elongation, because 4 days later, Scorpion 26, it is recorded that it was a moon and a half east of the same straight line. For the elongation was greater, since the sun had moved nearly 4°, and the star a half moon. And the mean sun, Thoth 19 at dawn, was $(20+\frac{1}{2}+\frac{1}{3})°$ within the Scorpion according to our ways of counting, and the star's apogee was 6° within the Balance because the 400 years or so between the observations made the apogee shift very nearly 4°.



Now, with these things supposed, let a figure similar to the one above be again constructed. But because of the dissimilarity of the positions, let the angles at the apogee A be drawn acute, and let the straight lines joining the star be drawn to the west of the epicycle, and let the perpendicular FX be drawn above the epicycle's radius FL.

Since, then, the mean position of the star was 44°50′ from the apogee,

angle $ABG = 44°50'$
$= 89°40'$ to 2 rt.

And so, by subtraction,

angle $CBG = 270°20'$ to 2 rt.,

and

angle BCG = angle $BGC = 44°50'$ to 2 rt.

And therefore

chord $CG = 84^{p}36'$

and

$BC = BG = 45^{p}46'$

where, in the circle about triangle BCG,

the diameter $= 120^{p}$.

And therefore

$CG = 5^{p}33'$

where

$BC = BG = 3^{p}$.

Again, since it is supposed

angle $ACF = 89°40'$ to 2 rt.

and

angle $BCG = 44°50'$ to 2 rt.,

and by addition

angle $FCG = 134°30'$ to 2 rt.,

therefore, on the circle about right triangle CGM,

arc $GM = 134°30'$

and, by subtraction from the semicircle,

arc $CM = 45°30'$.

And therefore

chord $GM = 110^{p}40'$,

and

chord $CM = 46^{p}24'$

where

hypt. $CG = 120^{p}$.

And so

$GM = 5^{p}7'$

and

$$CM = 2^{p}10',$$

where

$$CG = 5^{p}33'$$

and

$$\text{ecc. rad. } FG = 60^{p}.$$

And therefore, by addition,

$$FM = 59^{p}47'$$

and

$$FMC = 61°57'.$$

Likewise, since

$$\text{angle } DCN = 89°40' \text{ to 2 rt.},$$

therefore, on the circle about right triangle CDN,

$$\text{arc } DN = 89°40'$$

and, by subtraction from the semicircle,

$$\text{arc } CN = 90°20'.$$

And therefore

$$\text{chord } DN = 84^{p}36',$$
$$\text{chord } CN = 85^{p}6'$$

where

$$\text{hypt. } CD = 120^{p}.$$

And so also

$$DN = 2^{p}7',$$
$$CN = 2^{p}8',$$

and by addition

$$FCN = 64^{p}7'$$

where

$$CD = 3^{p}.$$

And therefore

$$DN = 3^{p}58'$$

where

$$FD = 120^{p}.$$

And, on the circle about right triangle DFN,

$$\text{arc } DN = 3°48'.$$

And so also

$$\text{angle } DFN = 3°48' \text{ to 2 rt.},$$

and, by subtraction,

$$\text{angle } ADF = 85°52' \text{ to 2 rt.}$$

But also it is supposed

$$\text{angle } ADL = 54°40' \text{ to 2 rt.}$$

because, according to the observation, the star was 27°20′ from the apogee. And so, by subtraction,

$$\text{angle } FDL = 31°12' \text{ to 2 rt.}$$

And therefore, on the circle about right triangle FDX,

$$\text{arc } FX = 31°12',$$

and

$$\text{chord } FX = 32^{p}16'$$

where

$$\text{hypt. } DF = 120^{p}.$$

And therefore

$$FX = 17^{p}15'$$

where

$$DF = 64^{p}7'$$

or

$$\text{epic. rad. } FL = 22^{p}30'.$$

And

$$FX \fallingdotseq 92^{p}$$

where

$$FL = 120^{p}.$$

And so also, on the circle about right triangle FLX,

$$\text{arc } FX = 100°8'$$

and

$$\text{angle } FLX = 100°8' \text{ to 2 rt.}$$

And it was also shown that

$$\text{angle } FDL = 31°12' \text{ to 2 rt.}$$

And

$$\text{angle } HFK = 3°48' \text{ to 2 rt.}$$

and so also, by subtraction,

$$\text{angle } KFL = 65°8' \text{ to 2 rt.}$$
$$= 32°34'.$$

Therefore, at this observation, the star was 32°34′ from K the epicycle's perigee, and of course 212°34′ from the apogee. At the time of our own observation, it was shown to be 99°27′ from the epicycle's apogee; the time from one observation to the other is 402 Egyptian years, 283 days, and very nearly 13½ hours; and that time embraces 1,268 of the star's returns in anomaly, since, with 20 Egyptian years making very nearly 63 cycles, the 400 years add up to 1,260 cycles, and the 2 years with the days added on give 8 whole cycles more. Therefore it is clear to us that, in 402 Egyptian years, 283 days, and 13½ hours, Mercury added on, over and above the 1,268 returns in anomaly, the 246°53′ by which its position at our own observation exceeds its position at the older one. And just about that number of degrees result in the tables set out by us, since we made the correction of Mercury's periodic movements from these very observations, reducing the proposed time to days and the cycles of anomaly plus the surplus to degrees. For when the number of degrees had been divided over the number of days, we got Mercury's daily movement of anomaly as we gave it.

11. On the Epoch of its Periodic Movements

In order, then, as with the sun and moon, to establish the epoch for the five planets at the year 1 of Nabonassar, Egyptianwise Thoth 1 at noon, we took the time from it to the older and nearer of the observations. The time came to 483 Egyptian years, 17 days, and very nearly 18⅓ hours. And a surplus of 190°39′ in mean movement of anomaly corresponds to this amount of time. If we subtract these from the 212°34′ representing the distance from the apogee at the observation, we shall have the epoch in the year 1 of Nabonassar, Egyptianwise Thoth 1 at noon (1) 21°55′ from the epicycle's apogee for anomaly; (2) for longitude the same as the sun, that is 45′ within the Fishes; (3) and the apogee of eccentricity 1⅙° within the Balance, since the number of years in hundreds comes to about 4+½+⅓, just the number by which the 6° within the Balance at the observation exceeds 1⅙°.

BOOK TEN

1. Demonstration of Venus' Apogee

Therefore we have ascertained in this way the hypotheses of Mercury, the magnitudes of its anomalies, and further the quantity of its periodic movements and of their epochs. In the case of Venus, we again sought, from equal greatest elongations on corresponding sides, in what parts of the ecliptic the apogee and perigee of eccentricity are. For this purpose we did not have any old observations exactly corresponding, but we have made just such an addition from our own observations.

For among those given us from Theo the mathematician, we found an observation recorded in the year 16 of Hadrian, Egyptianwise Pharmouthi 21-22, in which he says the evening Venus was at its greatest elongation from the sun, being west of the middle star of the Pleiades by as much as the length of the Pleiades. It seemed also to be passing by a little to the north. Now, since the middle star of the Pleiades was at that time, by our starting points, 3° within the Bull, and since their length is very nearly $1\frac{1}{2}°$, it is evident Venus was then $1\frac{1}{2}°$ within the Bull. And so, since the mean sun was then $14\frac{1}{4}°$ within the Fishes, the greatest evening elongation from the mean position was $47\frac{1}{4}°$.

And in the year 14 of Antonine, Egyptianwise Thoth 11-12, we observed the morning Venus at its greatest elongation from the sun; and it was north of the star in the middle of the Twins' knees, and east of it by half a full moon. At that time that fixed star, by our starting points, was $18\frac{1}{4}°$ within the Twins so that Venus was very nearly $18\frac{1}{2}°$ within the Twins, while the mean sun was $(5+\frac{1}{2}+\frac{1}{4})°$ within the Lion. Therefore, the greatest morning elongation was also $47\frac{1}{4}°$.

Then, since the mean position at the first observation was $14\frac{1}{4}°$ within the Fishes, and at the second observation $(5+\frac{1}{2}+\frac{1}{4})°$ within the Lion, and since the point midway between them falls 25° within the Bull and 25° within the Scorpion, the diameter through the apogee and perigee would be at these points.

Likewise, among the observations from Theo, we found that in the year 12 of Hadrian, Egyptianwise Athyr 21-22, the morning Venus was at its greatest elongation from the sun, being east of the star at the tip of Virgin's southern wing by the length of the Pleiades, or this length less Venus' own magnitude. And it seemed to be passing north of the star by one moon. Now, since this fixed star was at that time, according to us, $(28+\frac{1}{2}+\frac{1}{3}+\frac{1}{12})°$ within the Lion so that Venus was very nearly $\frac{1}{3}°$ within the Virgin, and since the mean sun was $(17+\frac{1}{2}+\frac{1}{3}+\frac{1}{30})°$ within the Balance, the greatest elongation from the mean position was $(47+\frac{1}{2}+\frac{1}{30})°$.

And in the year 21 of Hadrian, Egyptianwise Mechir 9-10 in the evening, we ourselves observed Venus at its greatest elongation from the sun. It was very nearly $\frac{2}{3}$ a full moon west of the northernmost star of the four in a square, following the star to the east of, and in a line with, the Water Bearer's testicles; and it seemed to outshine the star. And so since this fixed star was then, accord-

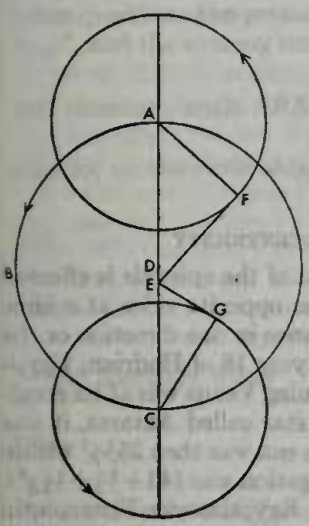
ing to us, 20° within the Water Bearer, and therefore Venus was about 19⅗° within the Water Bearer, and since the mean sun was 2$\frac{1}{15}$° within the Goat, the greatest evening elongation was $(47+\frac{1}{2}+\frac{1}{30})$°. And the points midway between the $(17+\frac{1}{2}+\frac{1}{3}+\frac{1}{30})$° within the Balance of the first observation and the 2$\frac{1}{15}$° within the Goat of the second are again very nearly 25° within the Scorpion and within the Bull.

2. On the Magnitude of its Epicycle

In this way, therefore, we took the apogee and perigee of eccentricity for our time at 25° within the Bull and within the Scorpion. And next we again examined the greatest elongations when the sun's mean position was about 25° within the Bull and about 25° within the Scorpion.

For, among the observations given us from Theo, we find that in the year 13 of Hadrian, Egyptianwise Epiphi 2-3, the morning Venus was at its greatest elongation from the sun, being 1⅖° west of the straight line through the principal star of the three in the Ram's head and through the star of the hind-leg. Its distance from the principal star in the Ram's head was very nearly double that from the star in the leg. And at that time, according to us, the principal star of the three in the Ram's head was 6⅗° within the Ram and 7⅓° north of the ecliptic; and the star in the hind-leg was $(14+\frac{1}{2}+\frac{1}{4})$° within the Ram and 5¼° south of the ecliptic. Therefore Venus was 10⅗° within the Ram and 1½° south of the ecliptic. And so, since the mean sun was then 25⅖° within the Bull, the greatest elongation from the mean position was 44¼°.

And in the year 21 of Hadrian, Egyptianwise Tybi 2-3 in the evening, we observed Venus at its greatest elongation from the sun; and, sighted with the stars in the Goat's horns, it appeared to be $(12+\frac{1}{2}+\frac{1}{3})$° within the Goat, while the mean sun was 25½° within the Scorpion. And so the greatest elongation from the mean position came to 47⅓°. It is clear that the apogee is 25° within the Bull and the perigee 25° within the Scorpion. And it is evident to us that there is only the eccentric bearing Venus' epicycle, because nowhere on the ecliptic is the sum of both the greatest elongations on either side found to be either less than the sum of both those in the Bull or greater than the sum of both those in the Scorpion.



Then, with these things assumed, let there be the eccentric circle *ABC* on which Venus' epicycle is always borne; and let this circle be about diameter *AC* on which let *D* be supposed the eccentric's centre, *E* the ecliptic's centre, and *A* the point 25° within the Bull. About points *A* and *C* let equal epicycles be described with points *F* and *G* on them. And let the tangents *EF* and *EG* be drawn through, and *AF* and *CG* joined.

Since, then, angle *AEF*, at the ecliptic's centre, subtends the greatest elongation at the star's apogee which is assumed to be 44⅘°, therefore

angle *AEF* = 44°48′
= 89°36′ to 2 rt.

And so, on the circle about right triangle AEF,

arc $AF = 89°36'$,

and

chord $AF = 84^p33'$,

where

hypt. $AE = 120^p$.

Likewise, since angle CEG subtends the greatest elongation at the perigee which is assumed to be $47\frac{1}{3}°$, therefore

angle $CEG = 47°20'$
$= 94°40'$ to 2 rt.

And so, on the circle about right triangle CEG,

arc $CG = 94°40'$,

and

chord $CG = 88^p13'$

where

hypt. $CE = 120^p$.

And therefore, where

rad. epic. CG = rad. epic. AF
$= 84^p33'$

and

$AE = 120^p$,

there

$EC = 115^p1'$

and clearly, by addition,

$AC = 235°1'$,

and, half of AC

$AD = 117^p30'$,

and, by subtraction,

1. betw. c. $DE = 2^p29'$.

And so also

1. betw. c. $DE = 1\frac{1}{4}^p$,
epic. rad. $AF = 43^p6'$,

where

ecc. rad. $AD = 60^p$.

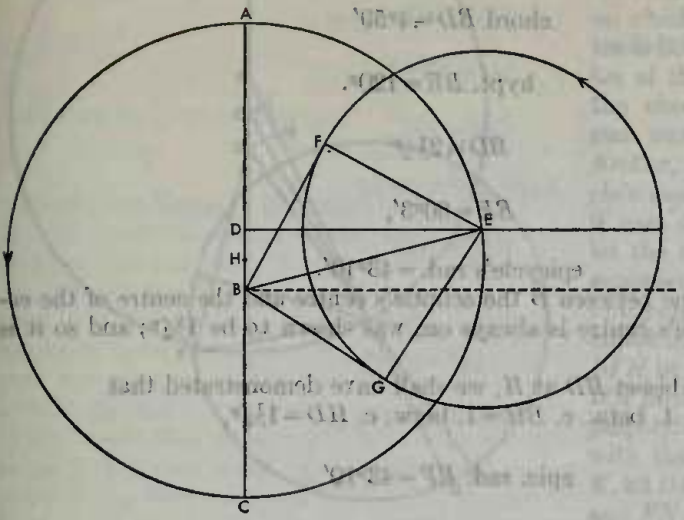
3. On the Ratios of the Star's Eccentricity

Since it is not clear whether the regular movement of the epicycle is effected about point D, we took two greatest elongations on opposite sides at a time when the sun's mean position was a quadrant's distance in one direction or the other from the apogee. One of them we made in the year 18 of Hadrian, Egyptianwise Pharmouthi 2-3, according to which the morning Venus was at its greatest elongation from the sun; and, sighted with the star called Antares, it was $(11+\frac{1}{2}+\frac{1}{3}+\frac{1}{12}°)$ within the Goat, while the mean sun was then $25\frac{1}{2}°$ within the Water Bearer. And so the greatest morning elongation was $(43+\frac{1}{2}+\frac{1}{12}°)$.

The other one we made in the year 3 of Antonine, Egyptianwise Pharmouthi 4-5 in the evening, according to which Venus was at its greatest elongation from the sun; and, sighted with the bright star of the Hyades [Aldebaran] was (13+

$\frac{1}{2}+\frac{1}{3})°$ within the Ram, while the mean sun was again $25\frac{1}{2}°$ within the Water Bearer. And so, therefore, the greatest evening elongation from the mean position was $48\frac{1}{3}°$.

With these things assumed, let there be the diameter ABC through the apogee and perigee of eccentricity; and let A be supposed the point 25° within the Bull, and B the ecliptic's centre.



Let it be required to find the centre around which we say the epicycle's regular movement is effected.

Then let it be the point D, and let DE be drawn through it perpendicular to AC in order that the epicycle's mean position may be a quadrant's distance from the apogee as in the observations. And let the epicycle's centre E be taken on DE according to the observations given. With epicycle FG described about E, let BF and BG be drawn from B tangent to it, and let BF, EF, and EG be joined. Since, then, for the proposed mean position, the greatest morning elongation is assumed to be $(43+\frac{1}{2}+\frac{1}{12})°$, and the evening one $48\frac{1}{3}°$, therefore, by addition,

$$\text{angle } FBG = 91°55';$$

and therefore, angle FBE being half of it,

$$\text{angle } FBE = 91°55' \text{ to 2 rt.}$$

And so, on the circle about right triangle BEF,

$$\text{arc } EF = 91°55',$$

and

$$\text{chord } EF = 86^{p}16'$$

where

$$\text{hypt. } BE = 120^{p}.$$

And therefore where

$$\text{epic. rad. } EF = 43^{p}10',$$

there

$$BE = 60^{p}3'.$$

Again, since the difference between these greatest elongations, which is 4°45′, embraces twice the difference of zodiacal anomaly for that time which is contained in angle BED, therefore

angle $BED = 2°22\frac{1}{2}'$
$= 4°45'$ to 2 rt.

And so also, on the circle about right triangle BDE,

arc $BD = 4°45'$,

and

chord $BD \fallingdotseq 4^p59'$

where

hypt. $BE = 120^p$.

And therefore

$BD \fallingdotseq 2\frac{1}{2}^p$

where

$BE = 60^p3'$,

and

epicycle's rad. $= 43^p10'$.

But the straight line between B the ecliptic's centre and the centre of the eccentric the epicycle's centre is always on, was shown to be $1\frac{1}{4}^p$; and so it is half BD.

If, therefore, we bisect BD at H, we shall have demonstrated that

1. betw. c. $BH = 1$. betw. c. $HD = 1\frac{1}{4}^p$,

and

epic. rad. $EF = 43^p10'$

where

$AH = 60^p$,

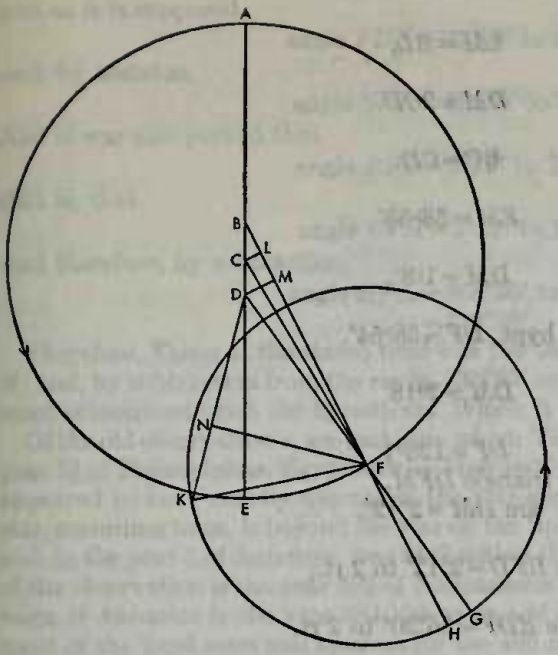
AH being radius of the eccentric bearing the epicycle.

4. On the Correction of the Star's Periodic Movements

Now, the mode of the hypothesis and the ratios of the anomalies were gotten by us in this way. But again, for the periodic movements of the star and its epochs, we took two sure observations from among our own and from among the old ones.

With the astrolabe, in the year 2 of Antonine, Egyptianwise Tybi 29-30, we observed the morning Venus with Spica, after its greatest elongation. And it appeared to be $6\frac{1}{2}°$ within the Scorpion. At that time it was between, and on a straight line with, the northernmost star in the Scorpion's forehead and the moon's apparent centre; and it was west of the moon's centre by one-and-a-half times its distance east of the northernmost star in the forehead. But this fixed star was then, by our starting points, 6°20′ within the Scorpion and 1°20′ north of the ecliptic. And the time was $4+\frac{1}{2}+\frac{1}{4}$ equatorial hours after midnight, since, with the sun about 23° within the Archer, the point on the astrolabe 2° within the Virgin was culminating. At this time the sun's mean position was 22°9′ within the Archer; and in mean longitude the moon was 11°24′ within the Scorpion, in anomaly 87°30′ from the apogee, and in latitude 12°22′ from the northern limit. Therefore, the moon's centre had its true position 5°45′ within the Scorpion, and 5° north of the ecliptic; and in Alexandria it appeared to be 6°45′ within the Scorpion in longitude, and 4°40′ north of the ecliptic. Therefore Venus, for these reasons, was 6°30′ within the Scorpion and 2°40′ north of the ecliptic.

With these things assumed, let $ABCDE$ be the diameter through the apogee.



And let A be supposed the point 25° within the Bull; B the point about which the epicycle moves regularly; C the centre of the eccentric on which the epicycle's centre is borne; and D the centre of the ecliptic. Since in the observation the mean sun was 22°9′ within the Archer, so that the epicycle's mean position was 27° 9′ east of the perigee at E, let the epicycle's centre be supposed at F; and, with epicycle GHK described about it, let DFG, CF, and BFH be joined. Let CL and DM be drawn from C and D perpendicular to BF. And, with the star put at point K, let DK and FK be joined, and FN drawn perpendicular.

It is required to find the arc HK, the distance of the star from the epicycle's apogee.

Since, then,

$$\text{angle } EBF = 27°9′$$
$$= 54°18′ \text{ to 2 rt.},$$

therefore also, on the circle about right triangle BCL,

$$\text{arc } CL = 54°18′$$

and, by subtraction from the semicircle,

$$\text{arc } BL = 125°42′.$$

And therefore

$$\text{chord } CL = 54^{p}46′,$$
$$\text{chord } BL = 106^{p}47′$$

where

$$\text{hypt. } BC = 120^{p}.$$

And so also

$$CL = 0^{p}34′,$$
$$BL = 1^{p}7′$$

where

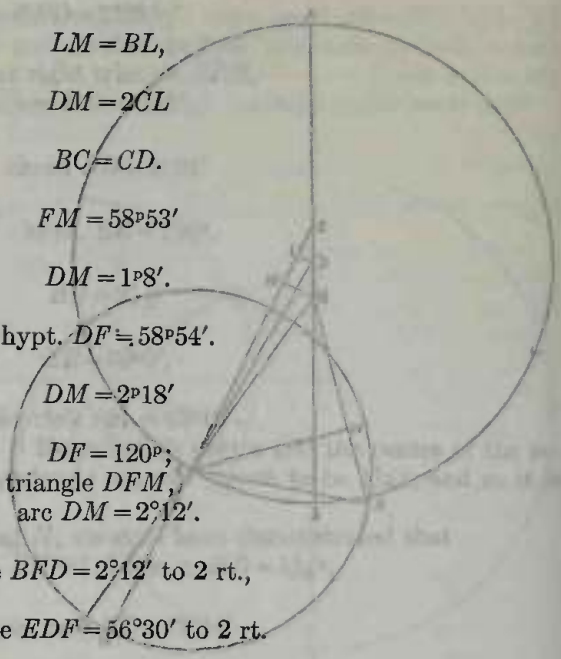
$$BC = 1^{p}15′$$

and

$$\text{ecc. rad. } CF = 60^{p}.$$

And since

$$\text{sq. } CF - \text{sq. } CL = \text{sq. } FL,$$
$$FL \fallingdotseq 60^{p}.$$



Also
$$LM = BL,$$
and
$$DM = 2CL$$
because
$$BC = CD.$$
And so also, by subtraction,
$$FM = 58^{p}53'$$
and
$$DM = 1^{p}8'.$$
And consequently
$$\text{hypt. } DF = 58^{p}54'.$$
And therefore
$$DM = 2^{p}18'$$
where
$$DF = 120^{p};$$
and, on the circle about right triangle DFM,
$$\text{arc } DM = 2°12'.$$
And so also
$$\text{angle } BFD = 2°12' \text{ to 2 rt.},$$
and, by addition,
$$\text{angle } EDF = 56°30' \text{ to 2 rt.}$$
And also
$$\text{angle } EDK = 18°30'$$
because by just so much, according to the observations, is the star west of the perigee at E—that is, west of 25° within the Scorpion; and
$$\text{angle } EDK = 37° \text{ to 2 rt.}$$
And therefore, by addition,
$$\text{angle } FDK = 93°30' \text{ to 2 rt.};$$
and, on the circle about right triangle DFN,
$$\text{arc } FN = 93°30'.$$
And therefore
$$\text{chord } FN = 87^{p}25'$$
where
$$FD = 120^{p}.$$
But
$$FN = 42^{p}54'$$
where
$$DF = 58^{p}54',$$
that is, where
$$\text{epic. rad. } FK = 43^{p}10' \text{ [p. 316].}$$
And so also
$$FN = 119^{p}18'$$
where
$$\text{hypt. } FK = 120^{p};$$
and, on the circle about right triangle FKN,
$$\text{arc } FN = 167°38'.$$
And therefore
$$\text{angle } DKF = 167°38' \text{ to 2 rt.}$$

just as it is supposed

angle $FDK = 93°30'$ to 2 rt.;

and, by addition,

angle $GFK = 261°8'$ to 2 rt.

And it was also proved that

angle $BFD = 2°12'$ to 2 rt.,

that is, that

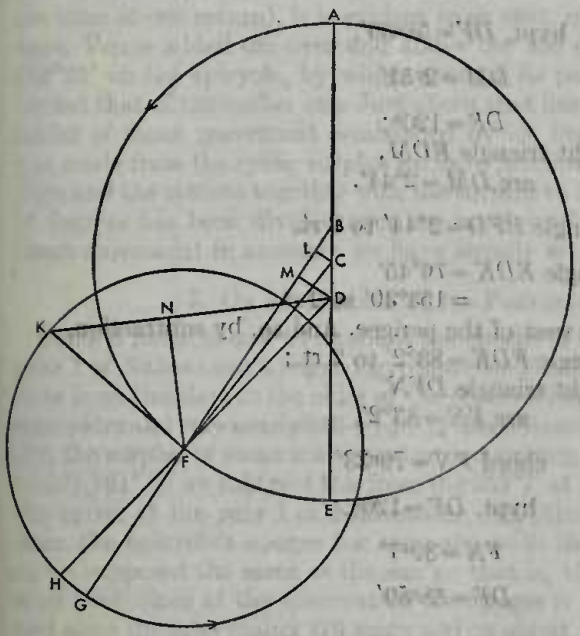
angle $GFH = 2°12'$ to 2 rt.

And therefore, by subtraction,

angle $HFK = 258°56'$ to 2 rt.
$= 129°28'$.

Therefore, Venus at the stated time was 129°28′ west of the epicycle's apogee *H;* and, by subtraction from the circle, 230°32′ east of it according to the movement consequent upon the hypothesis. Which it was required to find.

Of the old observation's, we took one which Timocharis records thus: In the year 13 of Philadelphus, Egyptianwise Mesore 17-18 at the twelfth hour, Venus appeared to have exactly overtaken the star opposite Vindemiatrix. And this star, according to us, is beyond the star on the tip of the Virgin's southern wing, and, in the year 1 of Antonine, was $8\frac{1}{4}°$ within the Virgin. Since, then, the year of the observation is the year 406 of Nabonassar, and since the beginning of the reign of Antonine is the year 884, (so as to add on very nearly a $4\frac{1}{12}°$ movement of the fixed stars and apogees for the 408 intervening years) it is evident that Venus was $4\frac{1}{6}°$ within the Virgin, and the eccentric's perigee $(20+\frac{1}{2}+\frac{1}{3}+\frac{1}{12})°$ within the Scorpion. And at this time Venus had passed its greatest morning elongation. For 4 days after this observation, Mesore 21-22, from what Timocharis says, Venus was, by our starting points, $(8+\frac{1}{2}+\frac{1}{3})°$ within the Virgin, while the sun's mean position at the first observation was 17°3′ within the Balance, and at the second 20°59′ within the Balance. And so the elongation of the first observation comes to 42°53′, and of the second to 42°9′.



Then, with these things given, let a similar figure be again set out, but having the epicycle to the west of the perigee because of the epicycle's mean position's being 17°3′ within the Balance and the eccentric's perigee's being 20°55′ within the Scorpion.

Since, then, for this reason

angle $EBF = 33°52'$
$= 67°44'$ to 2 rt.,

therefore, on the circle about right triangle BCL,

arc $CL = 67°44'$,

and, by subtraction from the semicircle,

arc $BL = 112°16'$.

And therefore

chord $CL = 66^p52'$,
chord $BL = 99^p38'$

where

hypt. $BC = 120^p$.

And so also

$CL = 0^p42'$,
$BL = 1^p2'$

where

$BC = 1^p15'$

and

ecc. rad. $CF = 60^p$.

And since

sq. CF—sq. CL = sq. FL,

in length

$FL = 60^p$.

And, for the same reason as before,

$BL = LM$

and

$DM = 2CL$.

And so, by subtraction,

$FM = 58^p58'$,
$DM = 1^p24'$.

And consequently

hypt. $DF = 58^p59'$.

And therefore

$DM = 2^p51'$

where

$DF = 120^p$;

and, on the circle about right triangle FDM,

arc $DM = 2°44'$.

And so also

angle $BFD = 2°44'$ to 2 rt.

And

angle $EDK = 76°45'$
$= 153°30'$ to 2 rt.,

which is the star's distance west of the perigee. And so, by subtraction,

angle $FDK = 83°2'$ to 2 rt.;

and, on the circle about right triangle DFN,

arc $FN = 83°2'$.

And therefore

chord $FN = 79^p33'$

where

hypt. $DF = 120^p$.

But

$FN = 39^p7'$

where

$DF = 58^p59'$,

that is, where

epic. rad. $FK = 43^p10'$ [p. 316].

And so also

$$FN = 108^{p}45'$$

where

$$\text{hypt. } FK = 120^{p};$$

and, on the circle about right triangle FKN,

$$\text{arc } FN = 130°.$$

And therefore

$$\text{angle } DKF = 130° \text{ to 2 rt.}$$

just as it is supposed

$$\text{angle } FDK = 83°2' \text{ to 2 rt.,}$$

and, by addition,

$$\text{angle } HFK = 213°2' \text{ to 2 rt.}$$

And it was also proved that

$$\text{angle } BFD = 2°42' \text{ to 2 rt.,}$$

that is, that

$$\text{angle } GFH = 2°42' \text{ to 2 rt.}$$

And therefore, by addition,

$$\text{angle } GFK = 215°46' \text{ to 2 rt.}$$
$$= 107°53'.$$

And therefore at that time Venus was, by subtraction from the whole circle, 252°7′ east of the epicycle's apogee at G. Which was to be proved.

Since, then, at the time of our own observation, it was likewise 230°32′ from the epicycle's apogee, and since the time from one observation to the other embraces 409 Egyptian years and very nearly 167 days, and 255 complete returns in anomaly (for with 8 Egyptian years making very nearly 5 cycles, 408 years add up to 255 cycles, and the year left over taken with the days does not fill out the time of one return), it is evident to us that, in 409 Egyptian years and 167 days, Venus added on, over and above the 255 complete returns in anomaly, 338°25′ on the epicycle, by which amount its position at our observation exceeded that of the earlier one. Just about that number of degrees turn up in the tables of mean movement established by us, because the correction of them was made from the cyclic surplus we have just found, the time being reduced to days and the returns together with the surplus to degrees. For when the number of degrees has been divided over the number of days, there results the daily mean movement in anomaly we have already set forth.

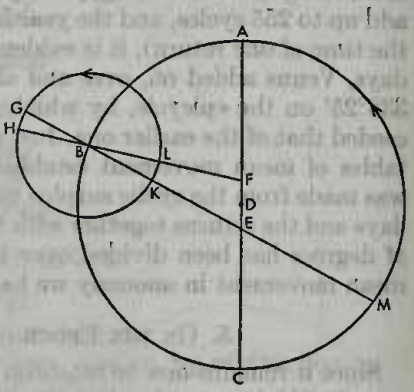
5. On the Epoch of its Periodic Movements

Since it remains now to establish the epochs of its periodic movements at the year 1 of Nabonassar's reign, Egyptianwise Thoth 1 at noon, we again took the time from this date to the older of the observations. And it comes to 475 Egyptian years and very nearly $346+\frac{1}{2}+\frac{1}{4}$ days. According to the columns of anomaly, the surplus of mean movement corresponding to this amount of time is very nearly 181°. If we subtract this from the 252°7′ at the observation, we shall have the epoch at the year 1 of Nabonassar, Egyptianwise Thoth 1 at noon; 71°7′ from the epicycle's apogee for anomaly, with the mean position in longitude again supposed the same as the sun's—that is, 45′ within the Fishes. It is evident that, since at the observation the apogee is about 20°55′ within the Bull, and since the intervening 476 years add on about $(4+\frac{1}{2}+\frac{1}{4})°$, at the proposed time of the epoch the apogee will be about 16°10′ within the Bull.

6. Preliminaries to the Demonstrations concerning the Other Stars

In the case of the two stars Mercury and Venus, we have used such methods as these for getting hold of the hypotheses and demonstrating the anomalies. In the case of the other three—Mars, Jupiter, and Saturn—we find the one and the same hypothesis as that apprehended for Venus; that is, the hypothesis according to which the eccentric circle the epicycle's centre is always borne on is described with its centre at the point bisecting the straight line between the centres of the ecliptic and of the circle effecting the epicycle's uniform revolution. For in the case of each of these stars, the difference in general found at the point of greatest difference of zodiacal anomaly turns out to be very nearly double the difference of the eccentricity calculated from the magnitude of the epicycle's regressions at the greatest and least distances.[1] And we find that demonstrations, by means of which we establish each of the magnitudes of the anomalies and of the apogees, cannot be worked out for these stars in the same way as for the other two, because they can be at any angular distance from the sun and because it is not evident from the observations—as in the case of the greatest elongations of Mercury and Venus, when the star is at the point of contact of the straight line drawn from our eye tangent to the epicycle. Since, then, this method does not work here, we have used the positions observed when diametrically opposite the sun's mean position, and from these we show the ratios of eccentricity and the apogees, since only in the positions so considered do we find the zodiacal anomaly set off by itself. For then there is no anomalistic difference with respect to the sun.

For let *ABC* be the star's eccentric circle on which the epicycle's centre is borne, and let it be about centre *D*. And let *AC* be the diameter through the apogee; and point *E* on it the ecliptic's centre; and *F* the centre of the eccentric with respect to which the mean longitudinal position of the epicycle is considered. With the epicycle *GHKL* described about *B*, let the straight lines *FLBH* and *GBKEM* be joined.



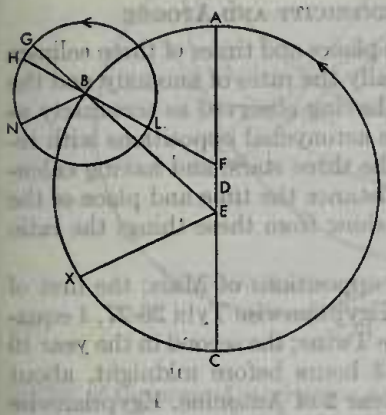
I say, first, that when the star appears along the straight line *EG* through the epicycle's centre *B* the sun's mean position will always be in the same straight line: when the star is at *G*, it is in conjunction with the sun's mean position seen at

[1]The eccentricity of the planets Mars, Jupiter, and Saturn is established by the theory of regressions in Book XII, where the angle of regression is computed for each planet at each of the three principal distances: mean, greatest, and least. The angle of regression obviously appears larger or smaller according as the star is closer or farther from the eye of the observer. It is therefore shown in Book XII that the assumption that the deferent's centre lies half way between the earth and the equant's centre saves the appearances of the angles of regression.

In the following chapters of this Book and of Book XI, this assumption concerning the position of the deferent's centre is made without more ado, and this sentence is the lone reference to the manner of its justification. For the demonstrations that follow are solely concerned with finding the centres of the equants.

G, and when it is at K, it will be diametrically opposite the sun's mean position seen at M.

For since in the case of each of these stars the mean angular distances from the apogee both in longitude and in anomaly, when added together, give the mean passage of the sun from the same starting point; and since the angle at B embracing the star's regular passage on its epicycle is always the difference between the angle at centre F which embraces the star's regular movement in longitude and the angle at E which embraces its apparent movement; therefore is it clear that, when the star is at point G, it will fall short of its return to apogee H by angle GBH, the angle which combined with angle AFB (that is, subtracted from it) gives angle AEG, the angle contained by the mean solar passage and the same as the star's apparent passage. And it is clear that, when the star is at point K, again it will have moved on the epicycle through angle HBK which, added to angle AFB, will equal the sun's mean passage from apogee A. For the sun's mean passage embraces the semicircle plus the difference between angle AFB and angle KBL—that is, angle CEM, the angle vertical to the angle of the star's apparent passage.



For this reason, in the case of these configurations, the straight line drawn from the epicycle's centre B to the star and that drawn from the point E at our eye to the sun's mean position fall both along the same straight line. And, in the case of all the other distances, they are directed to different points, but are always parallel to each other.[1]

For if, at any position on the figure just given, we draw a straight line from B to the star—for example, BN, and another from E to the sun's mean position, for instance EX—then, for the same reasons as

[1]There is a most important tacit assumption made here. In order for this theorem to apply, it is evident that we must assume that at least once in the past or future the sun, the mean sun, the earth, Mars (for example) and the centre of Mars' epicycle have been, or will be, in a straight line. Then, given the fact that the cycles of mean longitude plus the cycles of anomaly equal the cycles of the mean sun, it is proved here that those four points will always be in a straight line whenever three of them—Mars, the earth, and the mean sun—are in a straight line.

Now, Ptolemy probably referred to just this assumption and the one discussed in the previous note when, in IX, 2 , he said, "or if we are forced (2) to presuppose something without immediate foundation in the appearances, an apprehension gotten from continuous trial and adjustment." This assumption becomes a necessity in the Copernican system where the outer planet's epicycle becomes the circular orbit of the earth about the mean sun and the outer planet's eccentric circle becomes the circular orbit of the outer planet's mean longitudinal movement about the mean sun. Since, under the Copernican hypothesis, the earth continually passes between the outer planet and the mean sun, and the centre of the outer planet's epicycle has under this hypothesis become the mean sun, it is obvious that there will be a time when the earth, the planet, and the mean sun will be in a straight line; and that will be precisely when the planet is nearest the earth—that is, in the Ptolemaic system at the apparent perigee of its epicycle. Hence the above assumption of Ptolemy points almost by an inner necessity to a heliocentric theory. In Book I of this treatise Ptolemy asserts, it must be remembered, that the heliocentric hypothesis is simpler.

just given,

angle AEX = angle AFH + angle NBH

and

angle AFH = angle AEG + angle GBH.

By the subtraction of the common angle AEG,

angle GEX = angle GBN.

Therefore, straight line EX is parallel to BN.

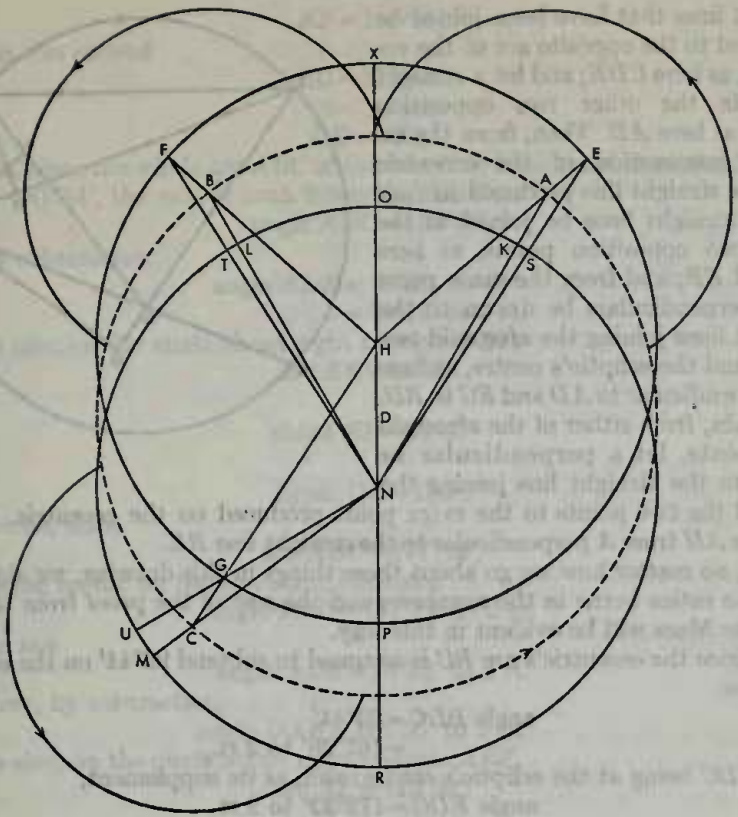
Since, then, at the configurations of conjunctions and oppositions considered in relation to the sun's mean position, we find the star to be seen in a line with the epicycle's centre as if it had no movement whatsoever on the epicycle but had its position on the circle ABC and were revolved regularly by the straight line FB in the same way as the epicycle's centre, therefore it is clear that it will be possible by means of these positions to demonstrate the ratios of eccentricity of zodiacal anomaly in themselves. But since the conjunctive configurations do not appear, it is clear that it remains only to get at the demonstrations by means of the oppositions.

7. Demonstration of Mars' Eccentricity and Apogee

Just as, in the case of the moon, we took the places and times of three eclipses at the full moon and demonstrated geometrically the ratio of anomaly and the place of the apogee, so here in the same way, having observed as accurately as possible with the astrolabe the places of three acronychal oppositions with respect to the sun's mean position for each of the three stars, and having calculated beforehand to a fine degree of angular distance the time and place of the sun's mean positions at the observations, we show from these things the ratio of eccentricity and the apogee.

In the first place, therefore, we took three oppositions of Mars: the first of which we observed in the year 15 of Hadrian, Egyptianwise Tybi 26-27, 1 equatorial hour after midnight, about 21° within the Twins; the second in the year 19 of Hadrian, Egyptianwise Pharmouthi 6-7, 3 hours before midnight, about 28°50′ within the Lion; and the third in the year 2 of Antonine, Egyptianwise Epiphi 12-13, 2 equatorial hours before midnight, about 2°34′ within the Archer. Now, the time of the interval from the first opposition to the second embraces 4 Egyptian years, 69 days, and 20 equatorial hours; and from the second to the third, likewise, 4 years, 96 days, and 1 equatorial hour. From the first interval of time there adds up 81°44′ in longitudinal movement, over and above complete circles; and from the second, 95°28′. For there will be no appreciable difference even if we calculate the mean movements for this amount of time from the periodic returns set out in the rough. And it is clear that, for the first interval, the apparent star has moved 67°50′ over and above complete circles, and, for the second, 93°44′.

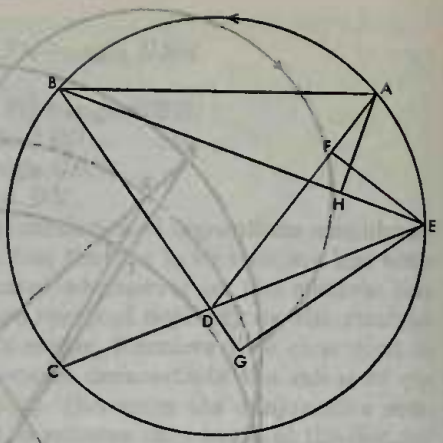
Then let three equal circles be described in the plane of the ecliptic. Let ABC about centre D be the one bearing the epicycle's centre; EFG about centre H the eccentric of regular movement; KLM about centre N, concentric with the ecliptic; and let the straight line $KOPR$ be the diameter through all the centres. Moreover, let A be supposed the point where the epicycle's centre was at the first opposition, B where it was at the second, and C where it was at the third. And let HAE, HBF, HGC, NKA, NLB, and NCM be joined, so that arc EF of the eccentric is the 81°44′ of the first periodic angular distance, and arc FG the



95°28′ of the second; and again so that arc *KL* of the ecliptic is the 67°60′ of the first apparent angular distance, and arc *LM* the 93°44′ of the second.

If, then, the eccentric's arcs *EF* and *FG* were subtended by the ecliptic's arcs *KL* and *LM*, we should need nothing more for the demonstration of the eccentricity. But since they subtend arcs *AB* and *BC* of the middle eccentric which are not given, and since, if we join *NSE*, *NTF*, and *NGU*, again arcs *ST* and *TU* of the ecliptic (clearly not given) subtend arcs *EF* and *FG* of the eccentric, it will be necessary, first, that the differences *KS*, *LT*, and *MU* be given, in order for the ratio of eccentricity to be accurately demonstrated from the corresponding arcs *EF*, *FG* and *ST*, *TU*. But since it is not possible to get these accurately prior to the ratio of eccentricity and the apogee—and yet since they will be given approximately, even if not accurately given before, because the differences are not great—we shall first make our calculation as if the arcs *ST*, *TU* did not differ appreciably from arcs *KL*, *LM*.

For let *ABC* be the eccentric circle of Mars' regular passage [the equant], and let *A* be supposed the point of the first opposition, *B* of the second, and *C* of the third. Within this circle let *D* be taken as the centre of the ecliptic where our eye is; and let all the straight lines be joined from the three opposition points to the eye—in this case *AD*, *BD*, and *CD*. And let one of the three



straight lines that have been joined be produced to the opposite arc of the eccentric, as here *CDE*; and let a straight line join the other two opposition points, as here *AB*. Then, from the resulting intersection of the eccentric with the straight line produced in (say *E*), let straight lines be joined to the other two opposition points, as here *EA* and *EB;* and from the same point *E* let perpendiculars be drawn to the straight lines joining the aforesaid two points and the ecliptic's centre, as here *EF* perpendicular to *AD* and *EG* to *BD*. Yet again, from either of the aforesaid two points, let a perpendicular be drawn to the straight line joining the other of the two points to the extra point produced on the eccentric, in this instance *AH* from *A* perpendicular to the straight line *BE*.

Now, no matter how we go about these things in this drawing, we shall find the same ratios borne in the numbers; and the rest of the proof from the arcs given for Mars will be evident in this way.

For since the eccentric's arc *BC* is assumed to subtend 93°44′ on the ecliptic, therefore

angle $BDC = 93°44'$
$= 187°28'$ to 2 rt.,

angle *BDC* being at the ecliptic's centre; and, as its supplement,

angle $EDG = 172°32'$ to 2 rt.

And so also, on the circle about right triangle *DEG*,

arc $EG = 172°32'$,

and

chord $EG = 119^{p}45'$

where

hypt. $DE = 120^{p}$.

Likewise, since

arc $BC = 95°28'$,

therefore, being an angle at the circumference,

angle $BEC = 95°28'$ to 2 rt.

And

angle $BDE = 172°32'$ to 2 rt.;

therefore, by subtraction,

angle $EBG = 92°$ to 2 rt.

And so also, on the circle about right triangle *BEG*,

arc $EG = 92°$,

and

chord $EG = 86^{p}19'$

where

hypt. $BE = 120^{p}$.

And therefore

$$BE = 166^{p}29'$$

where it was proved

$$EG = 119^{p}45'$$

and

$$DE = 120^{p}.$$

Again, since the whole arc ABC of the eccentric is assumed to subtend on the ecliptic 161°34′, the sum of both intervals, therefore

$$\text{angle } ADC = 161°34',$$

and, by subtraction,

$$\text{angle } ADE = 18°26'$$
$$= 36°52' \text{ to 2 rt.}$$

And so also, on the circle about right triangle DEF,

$$\text{arc } EF = 36°52',$$

and

$$\text{chord } EF = 37^{p}57'$$

where

$$\text{hypt. } DE = 120^{p}.$$

Likewise, since

$$\text{arc } ABC = 177°12',$$

ABC being an arc on the eccentric, therefore

$$\text{angle } AEC = 177°12' \text{ to 2 rt.}$$

And we saw

$$\text{angle } ADE = 36°52' \text{ to 2 rt.}$$

Therefore, by subtraction,

$$\text{angle } DAE = 145°56' \text{ to 2 rt.}$$

And so also, on the circle about right triangle AEF,

$$\text{arc } EF = 145°56',$$

and

$$\text{chord } EF = 114^{p}44'$$

where

$$\text{hypt. } AE = 120^{p}.$$

And therefore

$$AE = 39^{p}42'$$

where it was proved

$$EF = 37^{p}57'$$

and

$$ED = 120^{p}.$$

Again, since

$$\text{arc } AB = 81°44',$$

AB being an arc on the eccentric, therefore

$$\text{angle } AEB = 81°44' \text{ to 2 rt.}$$

And so also, on the circle about right triangle AEH,

$$\text{arc } AH = 81°44'$$

and, by subtraction from the semicircle,

$$\text{arc } EH = 98°16'.$$

And therefore

$$\text{chord } AH = 78^{p}31',$$
$$\text{chord } EH = 90^{p}45'$$

where

hypt. $AE = 120^{p}$.

And so

$AH = 25^{p}58'$,
$EH = 30^{p}2'$

where it was proved

$AE = 39^{p}42'$

supposing

$ED = 120^{p}$.

And it was also proved, with the same parts, that

whole $BE = 166^{p}29'$;

and therefore, by subtraction,

$BH = 136^{p}27'$

where

$AH = 25^{p}58'$.

And

sq. $BH = 18{,}615^{p}16'$

and likewise

sq. $AH = 674^{p}16'$,

which added together make

sq. $AB = 19{,}289^{p}32'$;

in length, therefore,

$AB = 138^{p}53'$,
$AE = 39^{p}42'$,

where

$DE = 120^{p}$.

And

$AB = 78^{p}31'$

where

ecc. diam. $= 120^{p}$;

for AB subtends an arc of 81°44′. And therefore

$DE = 67^{p}50'$,
$AE = 22^{p}44'$

where

$AB = 78^{p}31'$

and

ecc. diam. $= 120^{p}$.

And so also, AE being an arc on the eccentric,

arc $AE = 21°41'$

and, by addition,

arc $EABC = 198°53'$,

and, by subtraction,

arc $CE = 161°7'$.

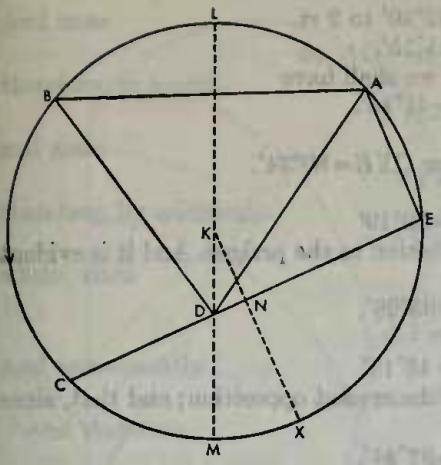
And

chord $CDE = 118^{p}22'$

where

ecc. diam. $= 120^{p}$.

If, then, the straight line CE had been found equal to the eccentric's diameter,



it is clear that the eccentric's centre would be on it, and the ratio of eccentricity would be immediately apparent. But since it has not come out equal, and has made segment *EABC* greater than a semicircle, it is evident that the eccentric's centre will fall on that side. Then let it be supposed the point *K*, and let the diameter *LKDM* through both centres be drawn through *K* and *D;* and from *K* let *KNX* be drawn perpendicular to *CE*.

Since, then, it was shown

$$CE = 118^{p}22',$$
$$DE = 67^{p}50'$$

where

$$\text{diam. } LM = 120^{p},$$

therefore also, by subtraction,

$$CD = 50^{p}32'.$$

And so, since

$$\text{rect. } ED, DC = \text{rect. } LD, DM,$$

we shall find

$$\text{rect. } LD, DM = 3{,}427^{p}51'.$$

But also

$$\text{rect. } LD, DM + \text{sq. } DK = \text{sq. } LK$$

which is the square on half the whole diameter. Therefore, if we subtract from the $3{,}600^{p}$ of the square on *LK* the $3{,}427^{p}51'$ of the rectangle *LD*, *DM*, then we shall have left the $172^{p}9'$ of the square on *DK*. And therefore we shall have in length

$$\text{1. btw. c. } DK = 13^{p}7'$$

where

$$\text{ecc. rad. } LK = 60^{p}.$$

Again, since

$$\text{half } CE \text{ or } CN = 59^{p}11',$$

and it was proved

$$DC = 50^{p}32'$$

where

$$\text{diam. } LM = 120^{p},$$

therefore, by subtraction,

$$DN = 8^{p}39'$$

where it was found

$$DK = 13^{p}7'.$$

And so also

$$DN = 79^{p}8'$$

where

$$\text{hypt. } DK = 120^{p};$$

and, on the circle about right triangle *DKN*,

$$\text{arc } DN = 82°30.$$

Therefore also

angle $DKN = 82°30'$ to 2 rt.
$= 41°15'$.

And since it is at the eccentric's centre, we shall have

arc $MX = 41°15'$.

And also, by addition,

arc CMX = half arc $CXE = 80°34'$.

Therefore also, by subtraction,

arc $CM = 39°19'$

where CM is the arc from the third opposition to the perigee. And it is evident that, since it was assumed

arc $BC = 95°28'$,

by subtraction,

arc $BL = 45°13'$

where LB is the arc from the apogee to the second opposition; and that, since it was assumed

arc $AB = 81°44'$,

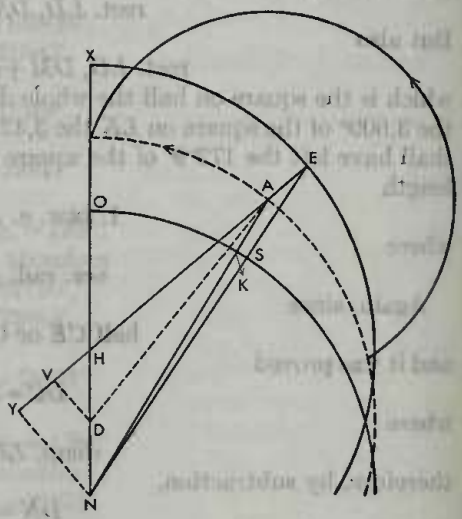
therefore, by subtraction,

arc $AL = 36°31'$

where CL is the arc from the first opposition to the apogee.

Now, with these things assumed, let us find out in the following way what differences they yield at each opopsition for the required arcs of the ecliptic.

From the figure of the three oppositions given above [p. 325], let the drawing of the first opposition alone be laid out. And after AD has been joined, let DV and NY be drawn from points D and N perpendicular to AH produced.



Since, then,

arc $EX = 36°31'$,

therefore

angle $EHX = 36°31'$,

and, being the angle vertical to it,

angle $DHV = 73°2'$ to 2 rt.

And so also, on the circle about right triangle DHV,

arc $DV = 73°2'$,

and, by subtraction from the semi-circle,

arc $HV = 106°58'$.

And therefore

chord $DV = 71^p25'$
chord $HV = 96^p27'$

where

hypt. $DH = 120^p$.

And so also

$DV = 3^p54'$,
$HV = 5^p16'$

where

$DH = 6^p33\frac{1}{2}'$

and

ecc. rad. $AD = 60^p$.

And since

sq. AD—sq. DV = sq. AV,

therefore, in length,

$AV = 59^p52'$;

and since

$VY = HV$,

therefore, by addition,

$AY = 65^p8'$

where, since

$NY = 2DV$,
$NY = 7^p48'$.

And consequently

hypt. $AN = 65^p36'$.

And therefore

$NY = 14^p16'$

where

$AN = 120^p$;

and, on the circle about right triangle ANY,

arc $NY = 13°40'$.

And so also

angle $NAY = 13°40'$ to 2 rt.

Again, since it was shown

$NY = 7^p48'$,
$HY = 10^p32'$

where

ecc. rad. $EH = 60^p$,

therefore, by addition,

$EHY = 70^p32'$

and consequently

hypt. $EN = 71^p$.

And therefore

$NY = 13^p10'$

where

$EN = 120^p$;

and, on the circle about right triangle ENY,

arc $NY = 12°36'$.

And so also

angle $NEY = 12°36'$ to 2 rt.

But

angle $NAY = 13°40'$ to 2 rt.;

and therefore, by subtraction,

angle $ANE = 1°4'$ to 2 rt.
$= 32'$.

Therefore on the ecliptic

arc $KS = 32'$.

Then let there be laid out the similar figure containing the drawing of the

second opposition. Since it is assumed

arc $FX = 45°13'$,

therefore

angle $FHX = 45°13'$,

and, being the angle vertical to it,

angle $DHV = 90°26'$ to 2 rt.

And so also, on the circle about right triangle DHV,

arc $DV = 90°26'$,

and, by subtraction from the semicircle,

arc $HV = 89°34'$.

And therefore

chord $DV = 85^p10'$,

chord $HV = 84^p32'$

where

hypt. $DH = 120^p$.

And so also

$DV = 4^p39'$,

$HV = 4^p38'$

where

$DH = 6^p33\frac{1}{2}'$

and

ecc. rad. $BD = 60^p$.

And since

sq. BD—sq. DV = sq. BV,

therefore, in length,

$BV = 59^p49'$;

and, because

$VY = HV$,

therefore, by addition,

$BY = 64^p27'$

where, since

$NY = 2DV$,

$NY = 9^p18'$.

And consequently, also,

hypt. $BN = 69^p6'$.

And therefore

$NY = 17^p9'$

where

$BN = 120^p$;

and, on the circle about right triangle BNY,

arc $NY = 16°26'$.

And so also

angle $NBY = 16°26'$ to 2 rt.

Again, since it was proved

$NY = 9^p18'$,

$HY = 9^p16'$

where

ecc. rad. $FH = 60^p$,

therefore, by addition,

$$FHY = 69^{p}16'$$

and consequently

$$\text{hypt. } FN = 69^{p}52'.$$

And therefore

$$NY = 16^{p}$$

where

$$\text{hypt. } FN = 120^{p};$$

and, on the circle about right triangle FNY,

$$\text{arc } NY = 15°20'.$$

And so also

$$\text{angle } NFY = 15°20' \text{ to 2 rt.}$$

But

$$\text{angle } NBY = 16°26' \text{ to 2 rt.}$$

And therefore, by subtraction,

$$\text{angle } BNF = 1°6' \text{ to 2 rt.}$$
$$= 33'.$$

Therefore on the ecliptic

$$\text{arc } KS = 33'.$$

Since, then, at the first opposition we found arc KS to be $32'$, it is clear that the first interval of the apparent movement considered with respect to the eccentric will be greater by the segments of both arcs, $1°5'$, and will contain $68°55'$.

Then let the drawing of the third opposition be laid out. Since then it is supposed

$$\text{arc } GP = 39°19',$$

therefore

$$\text{angle } GPH = 39°19'$$
$$= 78°38' \text{ to 2 rt.}$$

And so also, on the circle about right triangle DHV,

$$\text{arc } DV = 78°38',$$

and, by subtraction from the semicircle,

$$\text{arc } HV = 101°22'.$$

And therefore

$$\text{chord } DV = 76^{p}2',$$
$$\text{chord } HV = 92^{p}50'$$

where

$$\text{hypt. } DH = 120^{p}.$$

And so also

$$DV = 4^{p}9',$$
$$HV = 5^{p}4'$$

where

$$\text{1. btw. c. } DH = 6^{p}33\tfrac{1}{2}',$$

and

$$\text{ecc. rad. } CD = 60^{p}.$$

And since

$$\text{sq. } CD - \text{sq. } DV = \text{sq. } CV,$$

therefore

$$CV = 59^{p}51';$$

and, because

$$HV = VY,$$

by subtraction

$$CY = 54^{p}47'$$

where, since

$$NY = 2DV,$$
$$NY = 8^{p}18'.$$

And consequently also

$$\text{hypt. } CN = 55^{p}25'.$$

And therefore

$$NY = 17^{p}59'$$

where

$$CN = 120^{p};$$

and, on the circle about right triangle CNY,

$$\text{arc } NY = 17°14'.$$

And so also

$$\text{angle } NCY = 17°14' \text{ to 2 rt.}$$

Again, since it was shown

$$NY = 8^{p}18',$$
$$HY = 10^{p}8'$$

where

$$\text{ecc. rad. } GH = 60^{p},$$

therefore, by subtraction,

$$GY = 49^{p}52',$$

and therefore also

$$\text{hypt. } GN = 50^{p}33'.$$

And therefore

$$NY = 19^{p}42'$$

where

$$GN = 120^{p};$$

and, on the circle about right triangle GNY,

$$\text{arc } NY = 18°54'.$$

And so also

$$\text{angle } NGY = 18°54' \text{ to 2 rt.},$$

and it was proved

$$\text{angle } NCY = 17°14' \text{ to 2 rt.}$$

Therefore, by subtraction,

$$\text{angle } CNG = 1°40' \text{ to 2 rt.}$$
$$= 50'.$$

Therefore also on the ecliptic

$$\text{arc } MU = 50'.$$

Since, then, at the second opposition, we found the arc LT to be $33'$, it is clear that the second interval of the apparent movement, considered with respect to the eccentric, will be less by the segments of both arcs, $1°23'$, and will contain $92°21'$.

With these arcs of the ecliptic calculated for the two intervals, we again go through the theorem for demonstrating the apogee and the ratio of eccentricity, and find (not to make this treatise too long by repition)

$$1. \text{ btw. c. } DK^{1} = 11^{p}50'$$

[1] DK here refers to the fourth figure back. It is the distance between the ecliptic's centre or the earth and the equant's centre. As it has been pointed out before, the deferent's centre is assumed to be midway between the two, an assumption established and justified by the theory of regressions in Book XII.

where

ecc. rad. $= 60^{p}$;

and, on the eccentric,

arc $CM = 45°33'$,

CM being the arc from the third opposition to the perigee. And again from this

arc $BL = 38°59'$,

arc $AL = 42°45'$.

Likewise when we had used these numbers in the proofs of each opposition, we finally found the accurate magnitudes of each of the required arcs:

arc $KS = 28'$,

arc $LT = 28'$,

arc $MU = 30'$.

And putting together the differences at the first and second oppositions and adding the resulting 56′ to the 67°50′ of the first interval on the ecliptic, we had the distance of 68°46′ accurately considered with respect to the eccentric; and putting together those at the second and third oppositions and subtracting the resulting 1°8′ from the apparent 93°44′ of the second interval on the ecliptic, again we found the distance of 92°36′ accurately considered with respect to the eccentric. Finally from these results, using the same proof, we accurately ascertained the ratio of eccentricity and the apogee.

And we found

1. btw. c. $DK \fallingdotseq 12^{p}$

where

ecc. rad. $KL = 60^{p}$;

and on the eccentric

arc $CM = 44°21'$

from which, in turn, it results that

arc $BL = 40°11'$,

arc $AL = 41°33'$.

By these same means we shall also show that the observed apparent intervals of the three oppositions agree with these last magnitudes.

For let there be set out the drawing of the first opposition, containing only the eccentric circle the epicycle's centre is always borne on [the deferent]. Since then

angle $AHE = 41°33'$,

and, being the angle vertical to it,

angle $DHV = 83°6'$ to 2 rt.,

therefore, on the circle about right triangle DHV,

arc $DV = 83°6'$

and, by subtraction from the semicircle,

arc $HV = 96°54'$.

And therefore

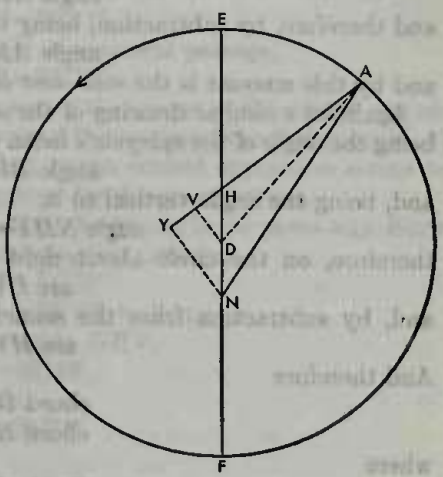
chord $DV = 79^{p}35'$,

chord $HV = 89^{p}50'$

where

hypt. $DH = 120^{p}$.

And so also



$$DV = 3^{p}58\tfrac{1}{2}',$$
$$HV = 4^{p}30'$$

where

$$DH = 6^{p},$$

and

$$AD = 60^{p}.$$

And since

$$\text{sq. } AD - \text{sq. } DV = \text{sq. } AV,$$

therefore, in length,

$$AV = 59^{p}50'.$$

Again, since

$$HV = VY$$

and

$$NY = 2DV,$$

we shall have, by addition,

$$AY = 64^{p}20'$$

where

$$NY = 7^{p}57'.$$

And consequently also

$$\text{hypt. } AN = 64^{p}52'.$$

And so also

$$NY = 14^{p}44'$$

where

$$AN = 120^{p};$$

and, on the circle about right triangle ANY,

$$\text{arc } NY = 14°6',$$

and therefore

$$\text{angle } NAY = 14°6' \text{ to 2 rt.}$$
$$= 7°3'.$$

But

$$\text{angle } AHE = 41°33',$$

and therefore, by subtraction, being the angle of apparent passage,

$$\text{angle } ANE = 34°30',$$

and by this amount is the star west of the apogee at the first opposition.

Again let a similar drawing of the second opposition be set out. Since, then, being the angle of the epicycle's mean passage

$$\text{angle } BHE = 40°11',$$

and, being the angle vertical to it,

$$\text{angle } NHY = 80°22' \text{ to 2 rt.},$$

therefore, on the circle about right triangle DHV,

$$\text{arc } DV = 80°22'$$

and, by subtraction from the semicircle,

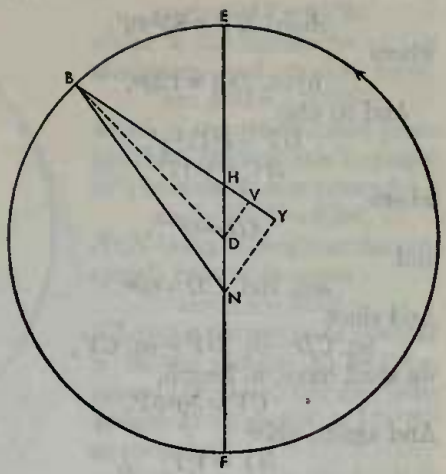
$$\text{arc } HV = 99°38'.$$

And therefore

$$\text{chord } DV = 77^{p}26',$$
$$\text{chord } HV = 91^{p}41'$$

where

$$\text{hypt. } DH = 120^{p}.$$



And also

$$DV = 3^{p}52',$$
$$HV = 4^{p}35'$$

where

$$DH = 6^{p}$$

and

$$\text{hypt. } BD = 60^{p}.$$

And since

$$\text{sq. } BD - \text{sq. } DV = \text{sq. } BV,$$

therefore, in length,

$$BV = 59^{p}53'.$$

And in the same way, since

$$HV = VY$$

and

$$NY = 2DV,$$

therefore, by addition

$$BY = 64^{p}28'$$

where

$$NY = 7^{p}44'.$$

And consequently

$$\text{hypt. } BN = 64^{p}56'.$$

And therefore

$$NY = 14^{p}19'$$

where

$$\text{hypt. } BN = 120^{p};$$

and, on the circle about right triangle BNY,

$$\text{arc } NY = 13°42'.$$

And so also

$$\text{angle } NBY = 13°42' \text{ to 2 rt.}$$
$$= 6°51'.$$

And

$$\text{angle } BHE = 40°11'.$$

Therefore, by subtraction, being the angle of apparent passage,

$$\text{angle } BNE = 33°20'.$$

By this distance the star hence appeared east of the apogee at the second opposition. It had already been proved 34°30′ west of the apogee at the first opposition; therefore the whole interval from the first to the second opposition comes to 67°50′, in agreement with the amount gotten by the observations.

Then let the drawing of the third opposition be set out in the same way. Since in this case, being the angle of the epicycle's regular passage,

$$\text{angle } CHF = 44°21'$$
$$= 88°42' \text{ to 2 rt.},$$

therefore, on the circle about right triangle DHV,

$$\text{arc } DV = 88°42'$$

and, by subtraction from the semicircle,

$$\text{arc } HV = 91°18'.$$

And therefore

$$\text{chord } DV = 83^{p}53',$$

chord $HV = 85^{p}49'$

where

hypt. $DH = 120^{p}$.

And so also

$DV = 4^{p}11\frac{1}{2}'$,

$HV = 4^{p}17'$

where

$AD = 6^{p}$

and

ecc. rad. $CD = 60^{p}$.

And since

sq. CD—sq. DV = sq. CV,

we shall have, in length,

$CV = 59^{p}51'$.

And again, since

$HV = VY$

and

$NY = 2DV$,

we shall have, by subtraction,

$CY = 55^{p}34'$

where

$NY = 8^{p}23'$.

And consequently we shall have

hypt. $CN = 56^{p}12'$.

And therefore

$NY = 17^{p}55'$

where

hypt. $CN = 120^{p}$;

and, on the circle about right triangle CNY,

arc $NY = 17°10'$.

And so also

angle $HCN = 17°10'$ to 2 rt.
$= 8°35'$.

But

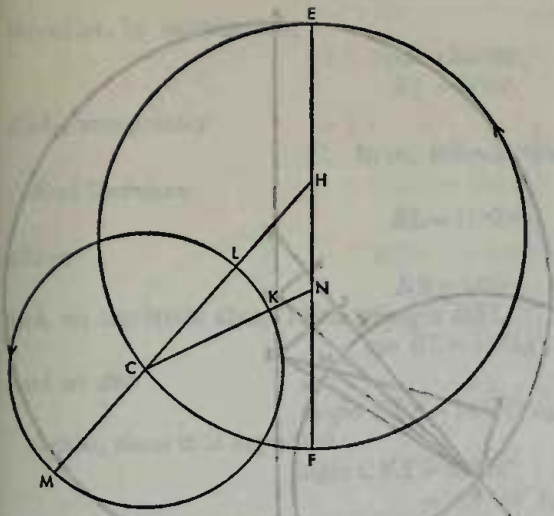
angle $CHF = 44°21'$;

and therefore, by addition,

angle $CNF = 52°56'$.

At the third opposition, therefore, the star appeared west of the perigee by just that much. And therefore, by subtraction, the resulting 93°44′ from the second to the third opposition was found to agree with the amount observed in the second interval. It is also clear that, when the star, sighted along the straight line CN, was according to observation 2°34′ within the Archer and angle CNF at the ecliptic's centre was proved to be 52°56′, then the perigee of eccentricity at point F was 25°30′ within the Goat, and the apogee diametrically opposite 25°30′ was within the Crab.

Also, if we describe the epicycle of Mars about centre C and produce straight line CN, we shall have the epicycle's mean passage of 135°39′ from the eccentric's apogee at the time of the third opposition; for it was proved



angle $CHF = 44°21'$

which is the remainder of the semicircle. And we shall have

arc $KM = 171°25'$

which is the star's mean passage from the epicycle's apogee M[1]; because it was proved

angle $HCN = 8°35'$,

HCN being an angle at the epicycle's centre; because

arc $KL = 8°35'$,

KL being the arc from the star K to the perigee L; and because, therefore, the arc from apogee M to star K is 171°25′ as proposed.

With the other results it is now clear to us that, at the time of the third opposition—that is, in the year 2 of Antonine, Egyptianwise Epiphi 12-13, 2 equatorial hours before midnight—Mars was 135°39′ in mean longitude from the eccentric's apogee, and 171°25′ in anomaly from the epicycle's apogee. Which things it was required to prove.

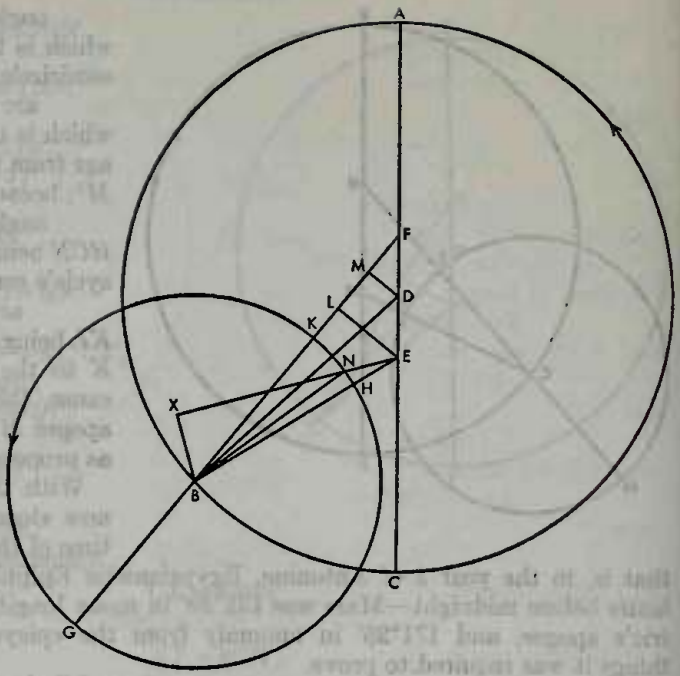
8. Demonstration of the Magnitude of Mars' Epicycle

Since the next thing was to demonstrate the ratio of the epicycle's magnitude, we took for this purpose an observation which we made very nearly 3 days after the third opposition (that is, in the year 2 of Antonine, Egyptianwise month 15-16, 3 equatorial hours before midnight), for the point on the astrolabe 20° within the Balance was culminating and the sun was then according to its mean passage 5°27′ within the Twins. In comparison with the point corresponding to Spica and sighted with its position, Mars appeared to be $1\frac{3}{5}°$ within the Archer, and at the same time appeared to be equally $1\frac{3}{5}°$ east of the moon's centre. The moon's mean position was then about 4°20′ within the Archer; and its true one was 29°20′ within the Scorpion, since in anomaly the moon was 92° from the epicycle's apogee. And the apparent moon was at the beginning of the Archer so that, consequently, Mars was then consistently, just as sighted, 1°36′ within the Archer—and, of course, 53°54′ west of the perigee. From the time of the third opposition to that of this observation there is contained 1°32′ in longitude, and very nearly 1°21′ in anomaly. And if we add these to the positions already demonstrated for the third opposition, we shall have Mars, at the time of this observation, in longitude 137°11′ from the eccentric's apogee, and in anomaly 172°46′ from the epicycle's apogee.

With these things assumed, about centre D and diameter ADC let there be the eccentric circle ABC bearing the epicycle's centre. And on the diameter let E be supposed the ecliptic's centre, and F the centre of the greater

[1]The star at K is, of course, at the apparent perigee. L and M are, respectively, the regular perigee and regular apogee, but in planetary theory they are often referred to simply as the perigee and apogee of the planet's epicycle.

eccentricity. And with epicycle *GHK* described about *B*, let *FKBG*, *EHB*, and *DB* be drawn through, and let *EL* and *DM* be drawn from points *D* and *E* perpendicular to *FB*. Let the star be supposed at point *N* on the epicycle; and, with *EN* and *BN* joined, let *BX* be drawn from *B* perpendicular to *EN* produced.



Since, then, the star is 137°11′ from the eccentric's apogee so that angle $BFC = 42°49′ = 85°38′$ to 2 rt.,

therefore, on the circle about right triangle *DFM*,

arc $DM = 85°38′$

and, by subtraction from the semicircle,

arc $FM = 94°22′$.

And therefore

chord $DM = 81^p34′$,
chord $FM = 88^p1′$

where

hypt. $DF = 120^p$.

And so also

$DM = 4^p5′$,
$FM = 4^p24′$

where

l. btw. c. $DF = 6^p$

and

ecc. rad. $BD = 60^p$.

And since

sq. BD—sq. DM = sq. BM,

therefore

$BM = 59^p52′$.

And likewise, since

$FM = LM$

and

$EL = 2\,DM$,

therefore, by subtraction,

$$BL = 55^{p}28',$$
$$EL = 8^{p}10'.$$

And consequently

$$\text{hypt. } BE = 56^{p}4'.$$

And therefore

$$EL = 17^{p}28'$$

where

$$BE = 120^{p};$$

and, on the circle about right triangle BEL,

$$\text{arc } EL = 16°44'.$$

And so also

$$\text{angle } EBF = 16°44' \text{ to 2 rt.}$$

Again, since it is supposed

$$\text{angle } CEX = 53°54',$$
$$= 107°48' \text{ to 2 rt.},$$

the angle by which Mars appeared west of the perigee C, and since

$$\text{angle } BEC = 102°22' \text{ to 2 rt.}$$

(because

$$\text{angle } BEC = \text{angle } EBF + \text{angle } BFC,$$

it having been proved

$$\text{angle } EBF = 16°42' \text{ to 2 rt.}$$

and assumed

$$\text{angle } BFC = 85°38' \text{ to 2 rt.}),$$

therefore, by subtraction,

$$\text{angle } BEX = 5°26' \text{ to 2 rt.}$$

And, on the circle about right triangle BEX,

$$\text{arc } BX = 5^{p}26'.$$

And consequently

$$\text{chord } BX = 5^{p}41'$$

where

$$\text{hypt. } BE = 120^{p}.$$

And therefore

$$BX = 2^{p}39'$$

where it was proved

$$BE = 56^{p}4'$$

and

$$\text{ecc. rad.} = 60^{p}.$$

Likewise, since the point N was 172°46′ from G the epicycle's apogee, and 7°14′ from K the perigee, therefore

$$\text{angle } KBN = 7°14'$$
$$= 14°28' \text{ to 2 rt.}$$

And

$$\text{angle } HBK = 16°44' \text{ to 2 rt.};$$

and therefore, by subtraction,

angle $HBN = 2°16'$ to 2 rt.

and, by addition,

angle $BNX = 7°42'$ to 2 rt.

And so also, on the circle about right triangle BNX,

arc $BX = 7°42'$,

and

chord $BX = 8^{p}3'$

where

hypt. $BN = 120^{p}$.

And therefore

epic. rad. $BN = 39^{p}30'$

where

$BX = 2^{p}39'$

and

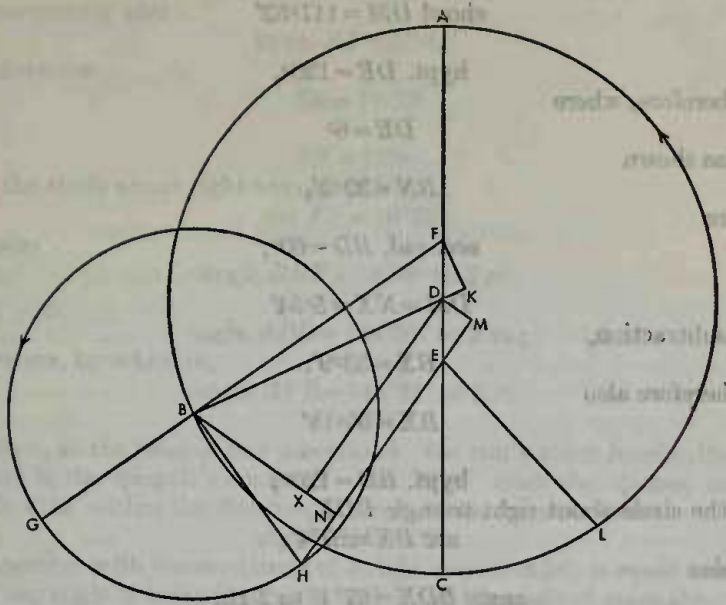
ecc. rad. $= 60^{p}$.

Therefore, the ratio of the eccentric's radius to the epicycle's radius is 60^{p} to $39^{p}30'$. Which it was required to find.

9. On the Correction of Mars' Periodic Movements

For the correction of the periodic mean movements we took one of the old observations according to which it is quite clear that in the year 13 according to Dionysius, Aigon 25 in the morning, Mars seemed to occult the Scorpion's northern forehead. Now, the date of this observation is the year 42 after the death of Alexander (that is, the year 476 of Nabonassar, Egyptianwise Athyr 20-21 in the morning) at which times we find the sun, in its mean passage, 23°54′ within the Goat. According to us, the star of the Scorpion's northern forehead was observed to be $6\frac{1}{3}°$ within the Scorpion. And so, again, since the 409 years from the observation to the reign of Antonine makes very nearly a 4°5′ change in the fixed stars and since then, at the time of the observation, this fixed star must have been $2\frac{1}{4}°$ within the Scorpion, clearly Mars must have been in the same position. And likewise, since for us—that is, at the beginning of the reign of Antonine—Mars' apogee was 25°30′ within the Crab, at the observation it must have been 21°25′ within the Crab. And it is clear that the apparent star was then 100°50′ from the apogee; the mean sun 182°29′ from the same apogee, and of course 2°29′ from the perigee.

With these things assumed, about centre D and diameter ADC let ABC be the eccentric circle bearing the epicycle's centre. On the diameter let E be supposed the ecliptic's centre; and F the centre of the greater eccentricity [of the equant]. With the epicycle GH described about centre B, let FBG and DB be drawn through, and let FK be drawn from F perpendicular to the straight line DB. Let the star be supposed at point H on the epicycle; and with BH joined, let EL be drawn from E parallel to it. Clearly, from what we have already proved, the sun's mean position will be seen along EL. And with EH joined, let DM and BN be drawn from D and B perpendicular to it; and again from D let DX be drawn perpendicular to BN, so that the figure $DMNX$ turns out to be a right parallelogram.



Since, then, being the angle of the star's apparent passage from the apogee,

angle $AEH = 100°50'$,

and, being the angle of the sun's mean passage,

angle $CEL = 2°29'$,

therefore

angle HEL = angle $BHE = 81°39'$
$= 163°18'$ to 2 rt.

And so also, on the circle about right triangle BHN,

arc $BN = 163°18'$,

and

chord $BN = 118^{p}43'$

where

hypt. $BH = 120^{p}$.

And therefore

$BN = 39^{p}3'$

where

epic. rad. $BH = 39^{p}30'$

and

1. btw. c. $DE = 6^{p}$.

Again, since

angle $AEH = 100°50'$
$= 201°40'$ to 2 rt.

and consequently, as its supplement,

angle $DEM = 158°20'$ to 2 rt.,

therefore, on the circle about right triangle DEM,

arc $DM = 158°20'$,

and chord $DM = 117^p52'$

where hypt. $DE = 120^p$,

And therefore, where $DE = 6^p$

and it was shown $BN = 39^p3'$,

and where ecc. rad. $BD = 60^p$,

there $DM = NX = 5^p54'$

and, by subtraction, $BX = 33^p9'$.

And therefore also $BX = 66^p18'$

where hypt. $BD = 120^p$;

and, on the circle about right triangle BDX, arc $BX = 67°4'$.

And so also angle $BDX = 67°4'$ to 2 rt.,

and, by addition, angle $BDM = 247°4'$ to 2 rt.

And, because it was shown angle $DEM = 158°20'$ to 2 rt.,

therefore angle $EDM = 21°40'$ to 2 rt.

And therefore, by subtraction, angle $BDE = 225°24'$ to 2 rt.,

and as its supplement angle $BDA = 134°36'$ to 2 rt.

And so also, on the circle about right triangle DFK, arc $FK = 134°36'$

and, by subtraction from the semicircle, arc $DK = 45°24'$.

And therefore chord $FK = 110^p42'$, chord $DK = 46^p18'$

where hypt. $DF = 120^p$.

And therefore $FK = 5^p32'$, $DK = 2^p19'$

and, by subtraction, $BK = 57^p41'$

where $DF = 6^p$

and ecc. rad. $DB = 60^p$.

And consequently also

hypt. $BF = 57^p57'$.

And therefore

$FK = 11^p28'$

where

$BF = 120^p$;

and, on the circle about right triangle BKF,

arc $FK = 10°58'$.

And so also

angle $DBF = 10°58'$ to 2 rt.

But also

angle $ABD = 134°36'$ to 2 rt.;

and therefore, by addition,

angle $AFB = 145°34'$ to 2 rt.
$= 72°47'$.

Therefore, at the time of this observation, the star's mean longitudinal position—that is, the epicycle's centre B—was 72°47′ from the apogee, and consequently 4°12′ within the Balance. Since it is also supposed that

angle $CEL = 2°29'$

(which together with the semicircle of 2 right angles, ABC, is equal to the sum of AFB, the angle of mean longitude, and GBH, the angle of anomaly—that is, the star's movement on the epicycle), we shall have, by subtraction,

angle $GBH = 109°42'$.

Therefore, at the time of the observation, the star was in anomaly that very 109°42′ from the epicycle's apogee. Which things it was required to find.

And we had already proved that the star, at the time of the third opposition, was in anomaly 171°25′ from the epicycle's apogee. Therefore, in the time between the observations (embracing 410 Egyptian years and very nearly 231⅔ days) it added on, over and above the 192 complete circles, 61°43′. This is very nearly the surplus we find in the tables of the star's mean movements set up by us, since we established the daily movement from these numbers, dividing the sum of the circles and surplus reduced to degrees over the total number of days in the time between the two observations.

10. On the Epoch of its Periodic Movements

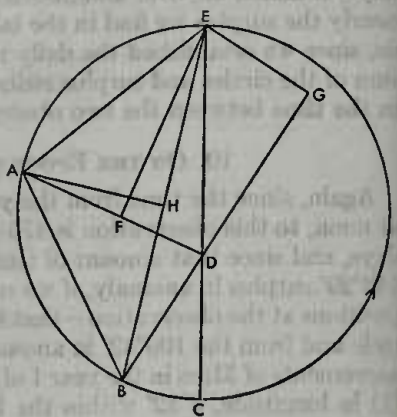
Again, since the time from the year 1 of Nabonassar, Egyptianwise Thoth 1 at noon, to this observation is 475 Egyptian years and very nearly $79+\frac{1}{2}+\frac{1}{4}$ days, and since that amount of time embraces 180°40′ surplus in longitude and 142°29′ surplus in anomaly, if we subtract them from each of the corresponding positions at the observation—that is, from the 4°12′ within the Balance in longitude and from the 109°42′ in anomaly—we shall have the epoch of the periodic movements of Mars in the year 1 of Nabonassar, Egyptianwise Thoth 1 at noon: (1) in longitude, 3°32′ within the Ram; (2) in anomaly, 327°13′ from the epicycle's apogee. And consequently, since in 475 years there comes to be a $(\frac{1}{4}+\frac{1}{2}+\frac{1}{4})°$ shift in apogees, and since Mars' apogee at the observation was about 21°25′ within the Crab, it will evidently be, at the time of the epoch, 16°40′ within the Crab.

BOOK ELEVEN

1. Demonstration of Jupiter's Eccentricity

Now that Mars' periodic movements, anomalies, and epochs have been shown, we shall next work out Jupiter's also, and in the same way, taking first again, for the demonstration of the apogee and eccentricity, 3 acronychal oppositions relative to the sun's mean passage. We observed the first of these by the astrolabe in the year 17 of Hadrian, Egyptianwise Epiphi 1-2, 1 hour before midnight, about 23°11′ within the Scorpion. The second in the year 21, Phaophi 13-14, 2 hours before midnight, about 7°54′ within the Fishes. And the third in the year 1 of Antonine, Athyr 20-21, 5 hours after midnight, about 14°23′ within the Ram. Now, of the two intervals, that from the first acronychal opposition to the second embraces 3 Egyptian years, 106 days, and 23 hours, and 104°43′ of the star's apparent passage; and that from the second to the third, 1 Egyptian year, 37 days, and 7 hours, and similarly 36°29′. And the mean longitudinal passage comes to 99°55′ for the time of the first interval, and 33°26′ for that of the second. From these intervals, in consequence of the methods propounded by us in the case of Mars, we first construct the proof of the things we are required to find, again with the one eccentric circle, in this way:

Let there be the eccentric circle *ABC;* and let *A* be supposed the point where the epicycle's centre was at the first achronychal opposition, *B* at the second, and *C* at the third. Within the eccentric *ABC* let *D* be taken as the centre of the ecliptic; and let *AD*, *BD*, and *CD* be joined. And, with *CDE* produced, let *AE*, *BE*, and *AB* be joined. Let *EF* and *EG* be drawn from *E* perpendicular to *AD* and *BD;* and *AH* from *A* perpendicular to *EB*.



Since, then, the eccentric's arc *BC* is assumed to subtend 36°39′ of the ecliptic,

angle *EDG* = angle *BDC*
= 36°29′
= 72°58′ to 2 rt.,

angle *BDC* being at the centre. And so also, on the circle about right triangle *EDG*,

arc *EG* = 72°58′,

and

chord *EG* = 71ᵖ21′

where

hypt. *DE* = 120ᵖ.

Likewise, since

arc *BC* = 33°26′,

therefore, being an angle at the circumference,

angle *BEC* = 33°26′ to 2 rt.,

and, by subtraction,

angle $EBG = 39°32'$.

And so also, on the circle about right triangle BEG,

arc $EG = 39°32'$,

and

chord $EG = 40^{p}35'$

where

hypt. $BE = 120^{p}$.

And therefore

$BE = 210^{p}58'$

where it was shown

$EG = 71^{p}21'$

and

$DE = 120^{p}$.

Again, since the eccentric's whole arc ABC is assumed to subtend on the ecliptic $141°12'$, the sum of both intervals; therefore, being at the ecliptic's centre,

angle $ADC = 141°12'$
$= 282°24'$ to 2 rt.

and, being adjacent to it,

angle $ADE = 77°36'$ to 2 rt.

And so also, on the circle about right triangle DEF,

arc $EF = 77°36'$,

and

chord $EF = 75^{p}12'$

where

hypt. $DE = 120^{p}$.

Likewise, since the eccentric's arc ABC adds up to $133°21'$, therefore, being an angle at the circumference,

angle $AEC = 133°21'$ to 2 rt.

But

angle $ADE = 77°36'$ to 2 rt.

And therefore, by subtraction,

angle $EAF = 149°3'$.

And so, on the circle about right triangle AEF,

arc $EF = 149°3'$,

and

chord $EF = 115^{p}39'$

where

hypt. $AE = 120^{p}$.

And therefore

$AE = 78^{p}2'$

where it was shown

$EF = 75^{p}12'$

and it is supposed

$DE = 120^{p}$.

Again, since the eccentric's arc AB is $99°55'$, therefore, being at the circumference,

angle $AEB = 99°55'$.

And so, on the circle about right triangle AEH,

arc $AH = 99°55'$

and as supplement

arc $EH = 80°5'$.

And therefore

chord $AH = 91^{p}52'$,

chord $EH = 77^{p}12'$

where

hypt. $AE = 120^{p}$.

And so

$AH = 59^{p}44'$

and likewise

$EH = 50^{p}12'$

where it was shown

$AE = 78^{p}2'$

and

$DE = 120^{p}$.

It had already been shown

$BE = 210^{p}58'$.

And therefore, by subtraction,

$BH = 160^{p}46'$

where

$AH = 59^{p}44'$.

And

sq. $BH = 25{,}845^{p}55'$

and likewise

sq. $AH = 3{,}568^{p}4'$,

which, added together, make

sq. $AB = 29{,}413^{p}59'$;

in length, therefore,

$AB = 171^{p}30'$

where

$DE = 120^{p}$,

and likewise

$AE = 78^{p}2'$.

And also

$AB = 91^{p}52'$

where

ecc. diam. $= 120^{p}$;

for AB subtends an arc of $99°55'$. And therefore

$DE = 64^{p}17'$,

$AE = 41^{p}47'$

where

$AB = 91^{p}52'$

and

ecc. diam. $= 120^{p}$.

And so also, AE being an arc on the eccentric,

arc $AE = 40°45'$,

and, by addition,

arc $EABC = 174°6'$.

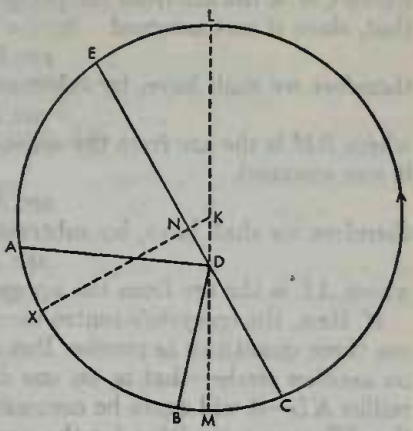
And therefore

chord $EDC = 119^{p}50'$

where

ecc. diam. = 120^p.

Since, then, segment $EABC$ is less than a semicircle, and therefore the eccentric's centre falls outside it, let it be supposed the point K. And let the diameter through both centres be drawn through it and D, and let KNX be drawn from K perpendicular to CE and produced.



Since, then, it was shown

$CE = 119^p50'$,

$DE = 64^p17'$

where

diam. $LM = 120^p$,

therefore, by subtraction, we shall have

$CD = 55^p33'$.

And so, since

rect. ED, DC = rect. LD, DM,

we shall have

rect. LD, $DM = 3{,}570^p56'$

where

diam. $LM = 120^p$.

But

rect. LD, DM + sq. DK = sq. half diam.
= sq. KL

Therefore, if we subtract the rectangle LD, DM or $3{,}570^p56'$ from the square on half the diameter or $3{,}600^p$, we shall have left the square on DK as $29^p4'$. And therefore we shall have

l. betw. c. $DK = 5^p23'$

where

ecc. rad. $KL = 60^p$.

Again, since

half CE or $CN = 59^p55'$

where

diam. $LM = 120^p$,

and it was shown also

$CD = 55^p33'$,

therefore also, by subtraction,

$DN = 4^p22'$

where

$DK = 5^p23'$.

And so also

$DN = 97^p20'$

where

$DK = 120^p$;

and, on the circle about right triangle DKN,

arc $DN = 108°24'$.

And therefore

angle $DKN = 108°24'$ to 2 rt.
$= 54°12'$.

And since it is at the eccentric's centre, we shall have also

arc $MX = 54°12'$.

And also, by addition,

arc CMX = half arc CXE = 87°3′.

Therefore also, by subtraction,

arc CM = 32°51′

where CM is the arc from the perigee to the third opposition. And it is evident that, since it was assumed

arc BC = 33°26′,

therefore we shall have, by subtraction

arc BM = 0°35′

where BM is the arc from the second opposition to the perigee; and that, since it was assumed

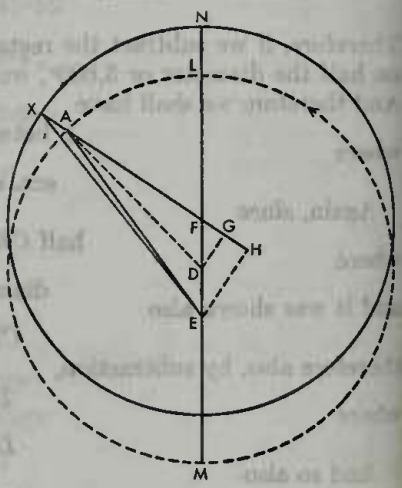
arc AB = 99°55′,

therefore we shall have, by subtraction,

arc AL = 79°30′

where AL is the arc from the apogee to the first opposition.

If, then, the epicycle's centre were borne on this eccentric, it would suffice to use these quantities as precise. But since, according to the hypothesis, it moves on another circle—that is, on one drawn with a centre bisecting DK and with radius KL—it will again be necessary, as in the case of Mars, to calculate first the differences resulting for the apparent distances, and to show of what size they would be, considering the ratios of eccentricity very nearly the same as if the epicycle's centre were borne, not on another eccentric, but on the first also embracing the zodiacal anomaly; that is, on the one drawn about centre K.



Then let the circle LM about centre D be the eccentric bearing the epicycle's centre. And let the circle NX about centre F and equal to LM be the circle of its regular movement. With the diameter NLM through the centres joined, let E be taken on it as the ecliptic's centre. And first let the epicycle's centre be supposed at point A for the first opposition. Let DA, EA, FAX, and EX be joined; and let DG and EH be drawn from points D and E perpendicular to AF produced.

Since, then, angle NFX of the regular longitudinal movement was shown to be 79°30′, therefore, being the angle vertical to it,

angle DFG = 79°30′
= 159° to 2 rt.

And so also, on the circle about right triangle DFG,

arc DG = 159°

and, as its supplement,

arc FG = 21°.

And therefore

chord DG = 117^{p}59′,
chord FG = 21^{p}52′

where

hypt. DF = 120^{p}.

And so also

$$DG = 2^{p}39'$$

and likewise

$$FG = 0^{p}30'$$

where

$$DF = \text{half } EF \fallingdotseq 2^{p}42'$$

and

$$\text{ecc. rad. } AD = 60^{p}.$$

And since

$$\text{sq. } AD - \text{sq. } DG = \text{sq. } AG,$$

therefore we shall have also

$$AG = 59^{p}56'.$$

And likewise, since

$$FG = GH$$

and

$$EH = 2\ DG,$$

therefore also, by addition,

$$AH = 60^{p}26'$$

where

$$EH = 5^{p}18',$$

and therefore

$$\text{hypt. } AE = 60^{p}40'.$$

And therefore

$$EH = 10^{p}29'$$

where

$$AE = 120^{p};$$

and, on the circle about right triangle AEH,

$$\text{arc } EH \fallingdotseq 10°1'.$$

And so also

$$\text{angle } EAH = 10°1' \text{ to 2 rt.}$$

Again, since

$$\text{rad. ecc. } FX = 60^{p},$$

$$FH = 1^{p}$$

and, by addition, clearly

$$HX = 61^{p}$$

where

$$EH = 5^{p}18',$$

therefore we shall have also

$$\text{hypt. } EX = 61^{p}14'.$$

And so also

$$EH = 10^{p}23'$$

where

$$EX = 120^{p};$$

and, on the circle about right triangle EHX,

$$\text{arc } EH = 9°55'.$$

And therefore

$$\text{angle } EXH = 9°55' \text{ to 2 rt.}$$

But it was shown

$$\text{angle } EAH = 10°1' \text{ to 2 rt.}$$

And therefore, by subtraction,

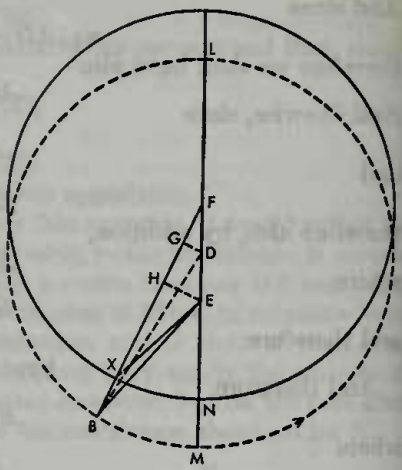
$$\text{angle } AEX = 0°6' \text{ to 2 rt.}$$

$$= 0°3',$$

which is the difference sought.

But the star, sighted along the straight line AE, appeared, at the first opposition, to be situated 23°11′ within the Scorpion. It is hence evident that, if the epicycle's centre were not borne on the eccentric LM but on NX, it would be at point X on it and the star would appear along straight line EX, differing by 3 sixtieths and being situated 23°14′ within the Scorpion.

Again, in a like figure, let also the drawing of the second opposition be laid down in a position a little west of the perigee.



Since the eccentric's arc NX was shown to be 35 sixtieths, therefore

$$\text{angle } NFX = 0°35'$$
$$= 1°10' \text{ to 2 rt.}$$

And so also, on the circle about right triangle DFG,

$$\text{arc } DG = 1°10',$$

and, as its supplement,

$$\text{arc } FG = 178°50'.$$

And therefore

$$\text{chord } DG = 1^{p}13',$$
$$\text{chord } FG = 120^{p}$$

where

$$\text{hypt. } DF = 120^{p}.$$

And so also

$$DG = 0^{p}2'$$

and likewise

$$FG = 2^{p}42'$$

where

$$DF = 2^{p}42'$$

and

$$\text{ecc. rad. } BD = 60^{p}.$$

And likewise also, since it is indistinguishable from the hypotenuse BD,

$$BG = 60^{p}.$$

And again, since

$$GH = FG,$$

and

$$EH = 2\ DG,$$

we shall also have, by subtraction,

$$BH = 57^{p}18'$$

where

$$EH = 0^{p}4',$$

and therefore also

$$\text{hypt. } BE = 57^{p}18'.$$

And so also

$$EH = 0^{p}8'$$

where

$$BE = 120^{p};$$

and, on the circle about right triangle BEH, again

$$\text{arc } EH = 0°8'.$$

And therefore

angle $EBH = 0°8'$ to 2 rt.

Likewise, since it was shown

$FH = 5^p24'$

where

ecc. rad. $FX = 60^p$,

therefore, by subtraction, we shall also have

$HX = 54^p36'$

where

$EH = 0^p4'$

and therefore

hypt. $EX = 54^p36'$.

And therefore

$EH = 0^p10'$

where

$EX = 120^p$;

and, on the circle about right triangle EHX,

arc $EH = 0°10'$.

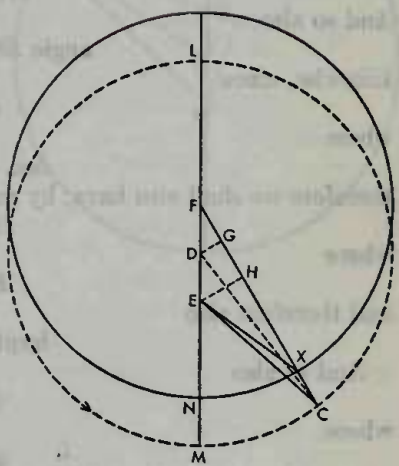
And so also

angle $EXH = 0°10'$ to 2 rt.,

and, by subtraction,

angle $BEX = 0°2'$ to 2 rt.
$= 0°1'$.

And now it is evident, therefore, that, since, at the second opposition, the star as it appeared along EB was situated $7°54'$ within the Fishes, if in turn it appeared along EX it would be only $7°53'$ within the Fishes.



And now let the drawing of the third opposition be laid out in a position east of the perigee. Since the eccentric's arc NX is assumed to be $32°51'$, therefore

angle $NFX = 32°51'$
$= 65°42'$ to 2 rt.

And so also, on the circle about right triangle DFG,

arc $DG = 65°42'$,

and as its supplement

arc $FG = 114°18'$.

And therefore

chord $DG = 65^p6'$,

chord $FG = 100^p49'$

where

hypt. $DF = 120^p$.

And so also

$DG = 1^p28'$

and likewise

$FG = 2^p16'$

where

$$DF = 2^{p}42'$$

and

$$\text{ecc. rad. } CD = 60^{p}.$$

And since

$$\text{sq. } CD - \text{sq. } DG = \text{sq. } CG,$$

we shall also have

$$CG \fallingdotseq 59^{p}59'.$$

And likewise, since

$$GH = FG$$

and

$$EH = 2\ DG,$$

we shall also have, by subtraction,

$$CH = 57^{p}43'$$

where

$$EH = 2^{p}56',$$

and therefore also

$$\text{hypt. } CE = 57^{p}47'.$$

And therefore

$$EH = 6^{p}5'$$

where

$$CE = 120^{p};$$

and, on the circle about right triangle CEH,

$$\text{arc } EH \fallingdotseq 5°48'.$$

And so also

$$\text{angle } ECH = 5°48' \text{ to 2 rt.}$$

Likewise, since

$$FH = 4^{p}32'$$

where

$$\text{ecc. rad. } FX = 60^{p},$$

therefore we shall also have, by subtraction,

$$HX = 55^{p}28',$$

where

$$EH = 2^{p}56';$$

and therefore also

$$\text{hypt. } EX = 55^{p}33'.$$

And so also

$$EH = 6^{p}20'$$

where

$$EX = 120^{p},$$

and, on the circle about right triangle EHX,

$$\text{arc } EH = 6°2'.$$

And therefore

$$\text{angle } EXH = 6°2' \text{ to 2 rt.},$$

and, by subtraction,

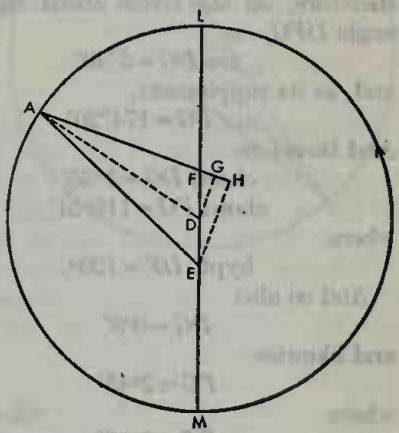
$$\text{angle } CEX = 0°14' \text{ to 2 rt.}$$
$$= 0°7'.$$

And so, since at the third opposition the star sighted along EC was situated 14°23′ within the Ram, it is evident that, if it were along the straight line EX,

it would be situated 14°30′ within the Ram. And it was shown that at the first opposition it was 23°14′ within the Scorpion, and at the second 7°53′ within the Fishes. Therefore, if the star's apparent intervals were considered—not with respect to the eccentric bearing the epicycle's centre, but with respect to that embracing its regular movement—they would come to 104°39′ from the first opposition to the second, and to 36°37′ from the second to the third. Following these with the theorem already shown, we find the intervals between the centres of the ecliptic and of the eccentric embracing the epicycle's regular movement to be very nearly 5ᵖ30′ where the eccentric's diameter is 120ᵖ. Of the eccentric's arcs, we find that from the apogee to the first opposition to be 77°15′, that from the second opposition to the perigee to be 2°50′, and that from the perigee to the third opposition to be 30°36′.

That these quantities have been here taken accurately because the differences of the intervals are very nearly the same as the first and are calculated through them—all this is evident from the fact that the star's apparent intervals are found, by means of the discovered ratios, to be the same as the observed ones, as will be clear to us from what follows.

For again let the drawing of the first opposition be set out having only the eccentric bearing the epicycle's centre.



Since it was shown

angle $LFA = 77°15'$
$= 154°30'$ to 2 rt.,

and, being vertical to it,

angle $DFG = 154°30'$ to 2 rt.,

therefore, on the circle about right triangle DFG,

arc $DG = 154°30'$,

and, as its supplement,

arc $FG = 25°30'$.

And therefore

chord $DG = 117^p2'$
chord $FG = 26^p29'$

where

hypt. $DF = 120^p$.

And so also

$$DG = 2^p41'$$

and likewise

$$FG = 0^p36'$$

where

$$DF = 2^p45'$$

and

ecc. rad. $AD = 60^p$.

And, for the same reasons as shown before,

$$AG = 59^p56',$$

and

$$AH = 60^p32'$$

where

$$EH = 2\,DG = 5^p22';$$

and so it is calculated

$$AE = 60^{p}46'.$$

And therefore

$$EH = 10^{p}36'$$

where

$$AE = 120^{p};$$

and, on the circle about right triangle AEH,

$$\text{arc } EH = 10°8'.$$

And therefore

$$\text{angle } EAH = 10°8' \text{ to 2 rt.}$$

and, by subtraction,

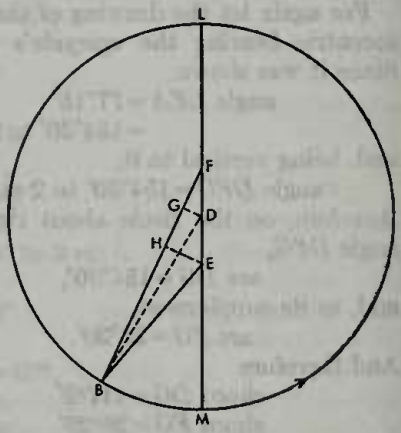
$$\text{angle } LEA = 144°22' \text{ to 2 rt.}$$
$$= 72°11'.$$

Therefore, by just so many degrees was the star distant from the ecliptic's apogee at the first opposition.

Again, let the drawing of the second opposition be set out. Since it is assumed

$$\text{angle } BFM = 2°50'$$
$$= 5°40' \text{ to 2 rt.},$$

therefore, on the circle about right triangle DFG,



$$\text{arc } DG = 5°40'$$

and, as its supplement,

$$\text{arc } FG = 174°20'.$$

And therefore

$$\text{chord } DG = 5^{p}55'$$
$$\text{chord } FG = 119^{p}51'$$

where

$$\text{hypt. } DF = 120^{p}.$$

And so also

$$DG = 0^{p}8'$$

and likewise

$$FG = 2^{p}45'$$

where

$$DF = 2^{p}45'$$

and

$$\text{ecc. rad. } BD = 60^{p}.$$

And for the same reasons also

$$BG = 60^{p},$$

and, by subtraction,

$$BH = 57^{p}15'$$

where

$$EH = 0^{p}16'.$$

And so also it is calculated

$$\text{hypt. } BE = 57^{p}15'.$$

And therefore

$$EH = 0^{p}33'$$

where

$$BE = 120^{p};$$

and, on the circle about right triangle EBH,

arc $EH = 0°32'$.

And so also

angle $EBH = 0°32'$ to 2 rt.

and, by addition,

angle $BEM = 6°12'$ to 2 rt.
$= 3°6'$.

Therefore the star at the second opposition was also $3°6'$ west of the perigee. But it was also shown to be $72°11'$ east of the apogee at the first. Therefore the apparent distance from the first opposition to the second is calculated to be the $104°43'$ of the remainder of the semicircle, in accord with the distance gotten from the observations.

Then also let the drawing of the third opposition be set out. Since it was shown

angle $MFC = 30°36'$
$= 61°12'$ to 2 rt.,

therefore, on the circle about right triangle DFG

arc $DG = 61°12'$,

and, as its supplement,

arc $FG = 118°48'$.

And therefore

chord $DG = 61^p6'$,
chord $FG = 103^p17'$

where

hypt. $DF = 120^p$.

And so also

$DG = 1^p24'$

and likewise

$FG = 2^p22'$

where

$DF = 2^p45'$

and

ecc. rad. $CD = 60^p$.

And for the same reasons also

$CG = 59^p59'$

and, by subtraction,

$CH = 57^p37'$

where it is calculated

$EH = 2^p48'$.

And so also

hypt. $CE = 57^p41'$.

And therefore

$EH = 5^p50'$

where

$CE = 120^p$;

and, on the circle about right triangle CEH,

arc $EH = 5°34'$.

And so also

angle $ECH = 5°34'$ to 2 rt.,

and, by addition,

angle $MEC = 66°46'$ to 2 rt.
$= 33°23'$.

By just as many degrees was the star east of the perigee, therefore, at the third opposition. And it was shown to be 3°6′ west of the same perigee at the second. Consequently, the apparent distance from the second to the third comes to the same result of 36°29′, again in accord with things observed.

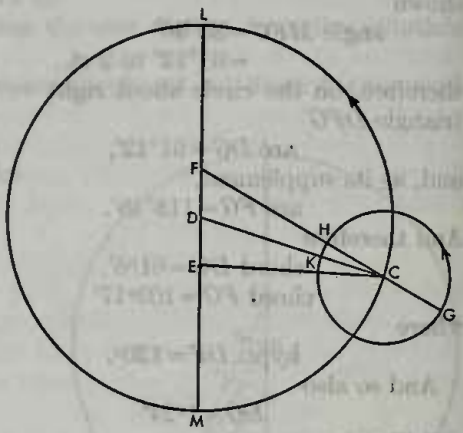
It is immediately clear that since, at the third opposition, the star was situated by observation 14°23′ within the Ram (at a distance, as was shown, of 33°23′ east of the perigee), the perigee of the eccentricity at that time was 11° within the Fishes, and the apogee exactly opposite 11° within the Virgin.

If we draw the epicycle *GHK* about centre C, we shall have immediately the mean longitudinal passage from the eccentric's apogee *L* of 210°36′, because we have shown

angle $MFC = 30°36'$;

and we shall have the epicycle's arc *HK*, from the perigee *H* to the star at *K*, equal to 2°47′, because we have shown

angle $ECF = 5°34'$ to 2 rt.
$= 2°47'$.



Therefore, at the time of the third opposition—that is, in the year 1 of Antonine, Egyptianwise Athyr 20-21, 5 hours after midnight—Jupiter considered with respect to its mean passages was in longitude 210°36′ from the eccentric's apogee (that is, it was situated 11°36′ within the Ram) and in anomaly 182°47′ from *G*, the epicycle's apogee.

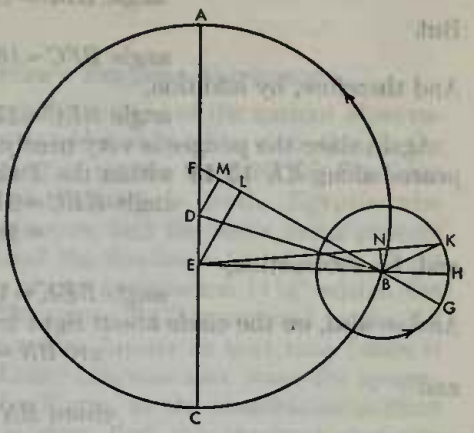
2. Demonstration of the Size of Jupiter's Epicycle

Again in what follows for the demonstration of the epicycle's size we took the observation we sighted in the year 2 of Antonine, Egyptianwise Mesore 26-27 before the rising of the sun—that is, very nearly 5 equatorial hours after midnight, when the sun's mean passage was 16°11′ within the Crab and the second degree of the Ram was culminating in the astrolabe. At that time, Jupiter, sighted relative to the bright star in the Hyades [Aldebaran], appeared to be 15¾° within the Twins, and it appeared at the same longitude as the centre of the moon which was more to the south. But at that hour, by calculations already set forth, we find the moon's mean position to be 9° within the Twins, and in anomaly we find it to be 272°5′ from the epicycle's apogee; and therefore its true passage to be about 14°50′ within the Twins, and its apparent passage in Alexandria to be about 15°45′. Therefore Jupiter was in this way situated 15¾° within the Twins.

Again, since the time from the third opposition to the given observation is 1 Egyptian year and 276 days, and since that amount of time embraces 53°17′ in longitude and 218°31′ in anomaly (for there will be no sensible difference

even if this amount is taken more or less roughly), if we add these to the demonstrated positions at the third opposition we shall have at the time of the observation very nearly 263°53′ in longitude from the same apogee and 41°18′ in anomaly from the epicycle's apogee.

Then, with these things supposed, let there be laid out again the drawing of a proof similar to that of Mars, having the epicycle's position east of the eccentric's perigee and the star's just after the epicycle's apogee in accord with the mean passages in longitude and anomaly here laid out.



Since, then, the mean passage in longitude from the eccentric's apogee is 263°53′, therefore

angle $BFC = 83°53'$
$= 167°46'$ to 2 rt.

And so also, on the circle about right triangle DFM,

arc $DM = 167°46'$

and, as its supplement,

arc $FM = 12°14'$.

And therefore

chord $DM = 119^{p}19'$,
chord $FM = 12^{p}47'$

where

hypt. $DF = 120^{p}$.

And so also

$DM = 2^{p}44'$,

and likewise

$FM = 0^{p}18'$

where

$DF = 2^{p}45'$

and

ecc. rad. $BD = 60^{p}$.

And since

sq. BD—sq. DM = sq. BM,

therefore

$BM = 59^{p}56'$.

And likewise, since

$FM = LM$

and

$EL = 2\ DM$,

therefore, by subtraction,

$BL = 59^{p}38'$

where it is calculated

$EL = 5^{p}28'$,

and therefore

hypt. $BE = 59^{p}52'$.

And therefore

$EL = 10^{p}58'$

where

$$BE = 120^{p};$$

and, on the circle about right triangle BEL,

$$\text{arc } EL = 10°30'.$$

And so also

$$\text{angle } EBF = 10°30' \text{ to 2 rt.}$$

But

$$\text{angle } BFC = 167°46' \text{ to 2 rt.}$$

And therefore, by addition,

$$\text{angle } BEC = 178°16' \text{ to 2 rt.}$$

Again since the perigee is very nearly 11° within the Fishes and the star appeared along EK 15°45′ within the Twins, therefore

$$\text{angle } KEC = 94°45' = 189°30' \text{ to 2 rt.},$$

and, by subtraction,

$$\text{angle } BEK = 11°14' \text{ to 2 rt.}$$

And so also, on the circle about right triangle BEN,

$$\text{arc } BN = 11°14',$$

and

$$\text{chord } BN = 11^{p}44'$$

where

$$\text{hypt. } BE = 120^{p}.$$

And therefore

$$BN = 5^{p}50'$$

where

$$BE = 59^{p}52'$$

and

$$\text{ecc. rad.} = 60^{p}.$$

And likewise, since

$$\text{arc } GK = 41°18',$$

therefore

$$\text{angle } GBK = 41°18' = 82°36' \text{ to 2 rt.}$$

But

$$\text{angle } GBH = \text{angle } EBF = 10°30' \text{ to 2 rt.}$$

And therefore, by subtraction,

$$\text{angle } HBK = 72°6' \text{ to 2 rt.}$$

But it was shown also

$$\text{angle } HEK = 11°14' \text{ to 2 rt.},$$

and therefore, by subtraction,

$$\text{angle } BKN = 60°52' \text{ to 2 rt.}$$

And so also, on the circle about right triangle BKN,

$$\text{arc } BN = 60°52',$$

and

$$\text{chord } BN = 60^{p}47'$$

where

$$\text{hypt. } BK = 120^{p}.$$

And therefore

epic. rad. $BK \fallingdotseq 11^{p}30'$

where

$BN = 5^{p}50'$

and

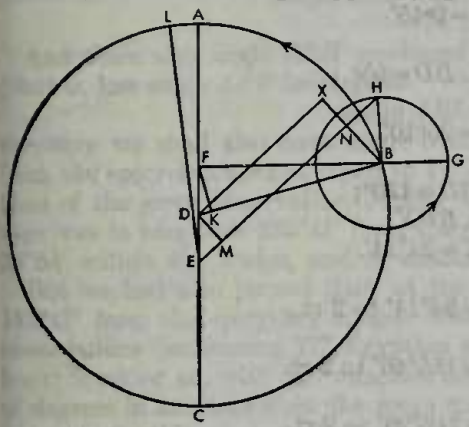
ecc. rad. $= 60^{p}$.

Which it was required to find.

3. On the Correction of Jupiter's Periodic Movements

Next, for the periodic movements, we again took one of the ancient observations very faithfully recorded, according to which it is quite clear that in the year 45 of Dionysius, Parthenon 10, Jupiter at sunrise occulted the Southern Ass. Now, this time is the year 83 after Alexander's death, Egyptianwise Epiphi 17-18 in the morning, at which time we find the sun's mean passage 9°56′ within the Virgin. But the star called the Southern Ass of those about the nebula of the Crab was at the time of our observation $11\frac{1}{3}°$ within the Crab, but clearly at that observation 7°33′, since 3°47′ correspond to the 378 years between the observations. And therefore Jupiter at that time (since it occulted the star) was 7°33′ within the Crab. Likewise also, since the apogee according to us was about 11° within the Virgin, at the observation it must have been 7°13′ within the Virgin. It is clear that the apparent star was 300°20′ from the eccentric's apogee at that time, and the mean sun was 2°43′ from the same apogee.

Now, with these things supposed, again let the drawing of the similar proof for Mars be laid out, only here according to the passages given at the observation, having the epicycle's position at B before the apogee A and that of the sun's mean situation at L a little after the same apogee. And therefore it also has the position of the star at H after the epicycle's apogee G. Likewise let FBG, DB, BH, and EH still be joined; and let FK be drawn perpendicular to DB, DM and BN to EH, and here DX to NB produced to make the rectangular parallelogram $DMNX$.



Now since, containing as it does the remainder of 300°20′ from the one circle of the ecliptic,

angle $AEH = 59°40'$

and

angle $AEL = 2°43'$,

therefore, by addition,

angle LEH or angle $BHE = 62°23'$
$= 124°46'$ to 2 rt.

And so also, on the circle about right triangle BHN,

arc $BN = 124°46'$,

and

chord $BN = 106^{p}20'$

where

hypt. $BH = 120^{p}$.

And therefore

$BN = 10^{p}12'$

where

epic. rad. $= 11^{p}30'$.

Again, since it is assumed

angle $DEM = 59°40'$
$= 119°20'$ to 2 rt.

and, by subtraction,

angle $MDE = 60°40'$ to 2 rt.,

therefore also, on the circle about right triangle DEM,

arc $DM = 119°20'$;

and

chord $DM = 103^{p}34'$

where

hypt. $DE = 120^{p}$.

And therefore

$DM = 2^{p}23'$

and, by addition,

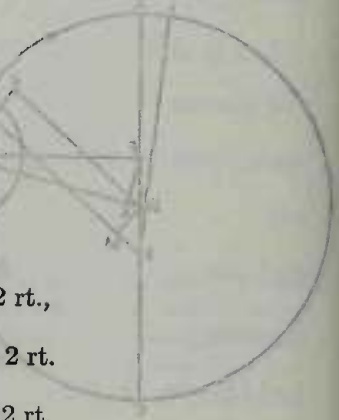
$BNX = 12^{p}35'$

where

$DE = 2^{p}45'$

and

ecc. rad. $BD = 60^{p}$.



And so also

$BX = 25^{p}10'$

where

hypt. $BD = 120^{p}$;

and, on the circle about right triangle BDX,

arc $BX = 24°14'$.

And therefore

angle $BDX = 24°14'$ to 2 rt.,

and, as its complement,

angle $BDM = 155°46'$ to 2 rt.

and likewise, by addition,

angle $BDE = 216°26'$ to 2 rt.

and again, as its supplement,

angle $BDF = 143°34'$ to 2 rt.

And so also, on the circle about right triangle FDK,

arc $FK = 143°34'$,

and, as its supplement,

arc $DK = 36°26'$.

And therefore also

chord $FK = 113^{p}59'$,
chord $DK = 37^{p}31'$

where

hypt. $DF = 120^{p}$.

And therefore

$FK = 2^{p}37'$

and likewise

$DK = 0^{p}52'$

and, by subtraction,

$BK = 59^{p}8'$

where

$DF = 2^{p}45'$

and

ecc. rad. $BD = 60^{p}$.

And therefore also

hypt. $BF = 59^{p}12'$.

And so also

$FK = 5^{p}18'$

where

$BF = 120^{p}$;

and, on the circle about right triangle BFK,

arc $FK = 5°4'$.

And therefore

angle $FBD = 5°4'$ to 2 rt.

and by addition, embracing the regular longitude,

angle $AFB = 148°38'$ to 2 rt.

$= 74°19'$.

And since also angle GBH combined with angle BFC and the semicircle (that is, less angle AFB here) gives

angle $AEL = 2°43'$,

therefore we shall also have angle GBH, which contains the star's passage from the epicycle's apogee, equal to $77°2'$. We have hence shown that, at the time of the given observation, Jupiter considered according to its mean passage was in longitude $285°41'$ from the eccentric's apogee or in mean position $22°54'$ within the Twins, and in anomaly $77°2'$ from the epicycle's apogee.

But we had also proved that, at the time of the third opposition, it was $182°47'$ from the epicycle's apogee. Therefore, in the time between the two observations (embracing 377 Egyptian years and 128 days less very nearly 1 hour) it added on, with 345 complete circles, $105°45'$ in anomaly. The surplus of degrees in anomaly from the mean movements worked out by us comes to very nearly this amount, because we established the daily movement from these, dividing the total number of degrees from the circle and surplus by the total number of days of that time.

4. On the Epoch of Jupiter's Periodic Movements

Now here again, since the time from the year 1 of Nabonassar, Egyptianwise Thoth 1 at midday, to the given ancient eclipse is 506 Egyptian years and very nearly $316\frac{3}{4}$ days, and since that amount of time embraces surpluses of $258°13'$ in longitude and $290°58'$ in anomaly, therefore, if we subtract these from the proper given positions at the observation, we shall have, at the time of the epoch the same as for the others, Jupiter's mean longitudinal position

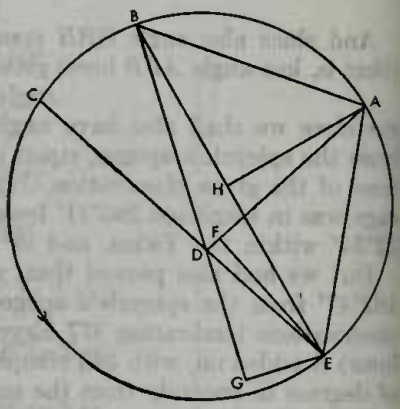
4°41′ within the Balance, and its anomalistic position 146°41′ from the epicycle's apogee. Also for the same reasons the apogee of its eccentricity will be 2°9′ within the Virgin.

5. Demonstration of Saturn's Eccentricity and Apogee

And now that it was left at this place to demonstrate the anomalies of Saturn and its positions, first for the examination of its apogee and eccentricity, we again, as for the others, took the acronychal oppositions relative to the sun's mean passage. The first of these we observed with the astrolabe in the year 11 of Hadrian, Egyptianwise, Pachons 7-8 in the evening, 1°13′ within the Balance; and the second in the year 17 of Hadrian likewise, Egyptianwise Epiphi 18; and we calculated the time and place of the true opposition, by observations on it, to be 4 hours after noon of the 18th, 9°40′ within the Archer. Observing the third opposition in the year 20 of Hadrian again, Egyptianwise Mesore 24, we likewise calculated the time of the true opposition to have been at the very noon of the 24th, and its place to have been 14°14′ within the Goat.

Now, of the two intervals, the one from the first opposition to the second embraces 6 Egyptian years, 70 days, and 22 hours, and 68°27′ of the star's apparent passage; and that from the second to the third, 3 Egyptian years, 35 days, and 20 hours, and likewise 34°34′. And 75°43′ of mean longitudinal passage are calculated in the rougher way[1] for the first interval, and 37°52′ for the second. With these intervals supposed, we again show what is required by the same theorem that is first on one eccentric in this way.

For, not to repeat, let there be laid out the drawing similar to those of the same proof. And since the eccentric's arc BC is assumed to subtend 34°34′ of the ecliptic, therefore, being at the ecliptic's centre



$$\text{angle } EDG = \text{angle } BDC$$
$$= 34°34′$$
$$= 69°8′ \text{ to 2 rt.}$$

And so also, on the circle about right triangle DEG,

$$\text{arc } EG = 69°8′;$$

and

$$\text{chord } EG = 68^{p}5′$$

where

$$\text{hypt. } DE = 120^{p}.$$

Likewise, since

$$\text{arc } BC = 37°52′;$$

therefore, being at the circumference,

$$\text{angle } BEC = 37°52′ \text{ to 2 rt.}$$

and, by subtraction,

$$\text{angle } EBG = 31°16′ \text{ to 2 rt.}$$

And so also, on the circle about right triangle EBG,

$$\text{arc } EG = 31°16′,$$

and

$$\text{chord } EG = 32^{p}20′$$

[1]That is, without correction. For such a small angular distance it would make no difference.

where

hypt. $BE = 120^{p}$.

And therefore

$BE = 252^{p}41'$

where it was shown

$EG = 68^{p}5'$

and

$DE = 120^{p}$.

Again, since the whole arc ABC subtends on the ecliptic the 103°1′ all told of both distances, therefore, being at the ecliptic's centre,

angle $ADC = 103°1'$.

And for this reason also, as adjacent to it,

angle $ADE = 76°59'$
$= 153°58'$ to 2 rt.

And so also, on the circle about right triangle DEF,

arc $EF = 153°58'$

and

chord $EF = 116^{p}55'$

where

hypt. $DE = 120^{p}$.

And likewise, since the eccentric's arc ABC totals 113°35′, therefore, being at the circumference,

angle $AEC = 113°35'$ to 2 rt.

But

angle $ADE = 153°58'$ to 2 rt.

And therefore, by subtraction,

angle $FAE = 92°27'$ to 2 rt.

And so also, on the circle about right triangle AEF,

arc $EF = 92°27'$,

and

chord $EF = 86^{p}39'$

where

hypt. $AE = 120^{p}$.

And therefore

$AE = 161^{p}55'$

where it was shown

$EF = 116^{p}55'$

and

$DE = 120^{p}$.

Again, since the eccentric's arc AB is 75°43′, therefore also, being at the circumference,

angle $AEB = 75°43'$ to 2 rt.

And so also, on the circle about right triangle AEH,

arc $AH = 75°43'$

and as its supplement

arc $EH = 104°17'$.

And therefore

chord $AH = 73^{p}39'$,
chord $EH = 94^{p}45'$

where

hypt. $AE = 120^{p}$.

And so also

$AH = 99^{p}43'$

and likewise

$EH = 127^{p}51'$

where it was shown

$AE = 161^{p}55'$

and

$DE = 120^{p}$.

And it has been shown

$BE = 252^{p}41'$;

and therefore, by subtraction,

$BH = 124^{p}50'$

where

$AH = 99^{p}43'$.

And

sq. $BH = 15{,}583^{p}22'$

and likewise

sq. $AH = 9{,}877^{p}3'$,

which, added together, make

sq. $AB = 25{,}460^{p}25'$.

Therefore, in length,

$AB = 159^{p}34'$

where

$DE = 120^{p}$

and likewise

$AE = 161^{p}55'$.

But also

$AB = 73^{p}39'$

where

ecc. diam. $= 120^{p}$,

for AB subtends an arc of 75°43′. And therefore

$DE = 55^{p}23'$

and

$AE = 74^{p}43'$

where

$AB = 73^{p}39'$

and

ecc. diam. $= 120^{p}$.

And so also

ecc. arc $AE = 77°1'$

and, by addition,

arc $EABC = 190°36'$,

and clearly, by subtraction,

arc $CE = 169°24'$.

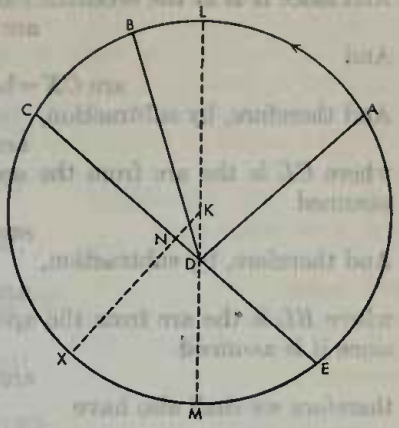
And for this reason also

chord $CDE = 119^{p}28'$

where

ecc. diam. = 120^p.

Then let the eccentric's centre be taken within the segment EAC since it is greater than a semicircle, and let it be K. Through it and D let the eccentric's diameter through both centres $LKDM$ be drawn. And let KNX be drawn perpendicular to CE and produced.



Since, then, it was shown

$CE = 119^p28'$

and

$DE = 55^p23'$

where

diam. $LM = 120^p$,

therefore we shall also have, by subtraction,

$CD = 64^p5'$.

And so, since

rect. ED, DC = rect. LD, DM,

we shall also have

rect. LD, $DM = 3{,}549^p9'$

where

diam. $LM = 120^p$.

But also

rect. LD, DM + sq. DK = sq. half diam.
= sq. LK.

If, therefore, we subtract $3{,}549^p9'$ from the square on half the diameter or $3{,}600^p$, we shall have left the square on DK as $50^p51'$. And therefore we shall have, in length,

l. betw. c. $DK = 7^p8'$

where

ecc. diam. = 120^p.

Again, since

half CE or $EN = 59^p44'$

where

diam. $LM = 120^p$

and it was shown also

$DE = 55^p23'$,

therefore we shall also have, by subtraction,

$DN = 4^p21'$

where

$DK = 7^p8'$.

And so also

$DN = 73^p11'$

where

hypt. $DK = 120^p$;

and, on the circle about right triangle DKN,

arc $DN = 75°10'$.

And therefore

angle $DKN = 75°10'$ to 2 rt.
$= 37°35'$.

And since it is at the eccentric's centre, we shall also have

arc $MX = 37°35'$.

And

arc CX = half arc $CXE = 84°42'$.

And therefore, by subtraction,

arc $CL = 57°43'$

where CL is the arc from the apogee to the third opposition. But also it is assumed

arc $BC = 37°52'$;

And therefore, by subtraction,

arc $BL = 19°51'$

where BL is the arc from the apogee to the second opposition. And likewise, since it is assumed

arc $AB = 75°43'$,

therefore we shall also have

arc $AL = 55°52'$

where AL is the arc from the first opposition to the apogee.

Now, since again the epicycle's centre is not borne on this eccentric, but on the eccentric drawn with a centre bisecting DK and with radius KL, we calculated accordingly, as for the others, the resulting differences of the apparent distances on the ecliptic, considering the ratios very nearly the same if one should change the epicycle's passage to the eccentric effecting the zodiacal anomaly.

For let the drawing of the first opposition for the similar proof be laid out in a position west of the apogee L. Since, then, angle NFX of the regular longitudinal passage, (that is, angle DFG) was shown to be $55°52'$ or $111°42'$ to 2 right angles' $360°$, therefore, on the circle about right triangle DFG,

arc $DG = 111°42'$

and, as its supplement,

arc $FG = 68°16'$.

And therefore

chord $DG = 99^p20'$,
chord $FG = 67^p20'$

where

hypt. $DF = 120^p$.

And so also

$DG = 2^p57'$

and likewise

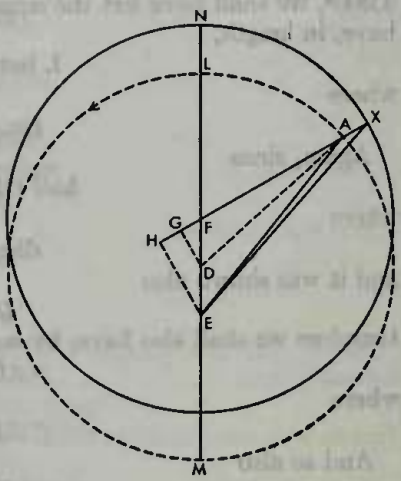
$FG = 2^p$

where

l. betw. c. $DF = 3^p34'$

and

ecc. rad. $AD = 60^p$.



And since

$$\text{sq. } AD - \text{sq. } DG = \text{sq. } AG,$$

we shall have

$$AG = 59^{p}56'.$$

And likewise, since

$$FG = GH$$

and

$$EH = 2\ DG,$$

therefore, by addition,

$$AH = 61^{p}56'$$

where

$$EH = 5^{p}54'.$$

And for this reason also

$$\text{hypt. } AE = 62^{p}13'.$$

And so also

$$EH = 11^{p}21'$$

where

$$\text{hypt. } AE = 120^{p};$$

and, on the circle about right triangle AEH,

$$\text{arc } EH = 10°51'.$$

And therefore

$$\text{angle } EAH = 10°51' \text{ to 2 rt.}$$

Again, since

$$\text{ecc. rad. } FX = 60^{p},$$
$$FH = 4^{p}$$

and, by addition, clearly

$$HX = 64^{p}$$

where

$$EH = 5^{p}54';$$

therefore we shall also have

$$\text{hypt. } EX = 64^{p}16'.$$

And therefore

$$EI = 11^{p}2'$$

where

$$\text{hypt. } EX = 120^{p};$$

and, on the circle about right triangle EHX,

$$\text{arc } EI = 10°33'.$$

And so also

$$\text{angle } EXH = 10°33' \text{ to 2 rt.}$$

But it was also shown

$$\text{angle } EAH = 10°51' \text{ to 2 rt.};$$

and therefore, by subtraction,

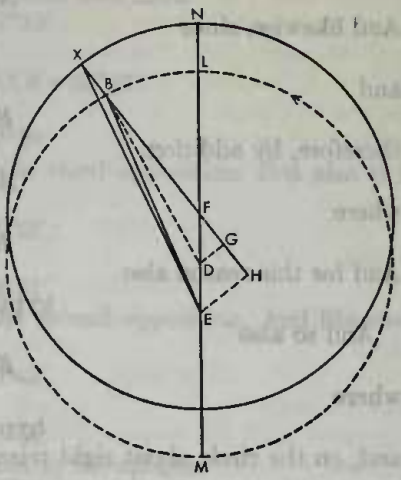
$$\text{angle } AEX = 0°18' \text{ to 2 rt.}$$
$$= 0°9'$$

which is the difference sought.

But at the first opposition the star appeared along straight line AE, situated 1°13′ within the Balance. Now, it is clear that, if the epicycle's centre were not borne on the epicycle AL but on NX, it would be at point X on it, and the star

would appear along straight line EX, 9 sixtieths west of the position at A, and it would be situated 1°4′ within the Balance.

Again, for the same proof, let the drawing of the second opposition be laid out east of the apogee.



Since the eccentric's arc NX was shown to be 19°51′, therefore angle NFX and angle DFG, vertical to it, would each equal 19°51′, or 39°42′ to 2 right angles' 360°. And so also, on the circle about right triangle DFG,

arc $DG = 39°42'$

and, as its supplement,

arc $FG = 140°18'$.

And therefore

chord $DG = 40^{p}45'$,

chord $FG = 112^{p}52'$

where

hypt. $DF = 120^{p}$.

And so also

$DG = 1^{p}13'$

and likewise

$FG = 3^{p}21'$

where

$DF = 3^{p}34'$

and

ecc. rad. $BD = 60^{p}$.

And since

sq. BD − sq. DG = sq. BG,

therefore

$BG = 59^{p}59'$.

And likewise, since

$FG = GH$

and

$EH = 2\ DG$,

therefore, by addition, we shall have

$BH = 63^{p}20'$

where

$EH = 2^{p}26'$;

and therefore also

hypt. $BE = 63^{p}23'$.

And therefore

$EH = 4^{p}36'$

where

hypt. $BE = 120^{p}$;

and, on the circle about right triangle BEH,

arc $EH = 4°24'$.

And so also

angle $EBH = 4°24'$ to 2 rt.

Likewise, since

$$FH = 6^{p}42'$$

where

$$\text{ecc. rad. } FX = 60^{p},$$

therefore we shall have, by addition,

$$HX = 66^{p}42'$$

where it was supposed

$$EH = 2^{p}26',$$

and therefore

$$\text{hypt. } EX = 66^{p}45'.$$

And so also

$$EH = 4^{p}23'$$

where

$$\text{hypt. } EX = 120^{p};$$

and, on the circle about right triangle EHX,

$$\text{arc } EH = 4°12'.$$

And therefore

$$\text{angle } EXH = 4°12' \text{ to 2 rt.}$$

But it had already been shown also

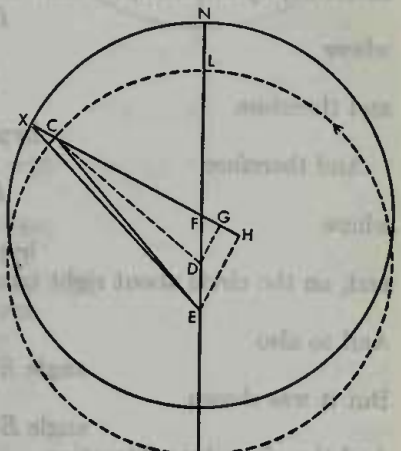
$$\text{angle } EBH = 4°24' \text{ to 2 rt.}$$

And therefore, by subtraction,

$$\text{angle } BEX = 0°12' \text{ to 2 rt.}$$
$$= 0°6'.$$

Now, it is clear here that, since at the second opposition the star appearing along EB was situated 9°40′ within the Archer, if in turn it appeared along EX it would be situated 9°46′ within the Archer. But it was shown that, at the first opposition, it was situated likewise 1°4′ within the Balance. Hence it is evident that the apparent distance from the first opposition to the second would amount to 68°42′ on the ecliptic if it were considered with respect to the eccentric NX.

And likewise let the drawing of the third opposition be laid out according to the same general figure as that of the second.



Since arc NX was shown to be 57°43′, therefore angle NFX or angle DFG is 57°43′, or 115°26′ to 2 right angles' 360°. And so also, on the circle about right triangle DFG,

$$\text{arc } DG = 115°26'$$

and as its supplement

$$\text{arc } FG = 64°34'.$$

And therefore

$$\text{chord } DG = 101^{p}27',$$
$$\text{chord } FG = 64^{p}6'$$

where

$$\text{hypt. } DF = 120^{p}.$$

And so also

$$DG = 3^{p}1'$$

and likewise

$$FG = 1^{p}54'$$

where

$$DF = 3^{p}34'$$

and

$$\text{ecc. rad. } CD = 60^{p}.$$

And again, since

$$\text{sq. } CD - \text{sq. } DG = \text{sq. } CG,$$

we shall also have

$$CG = 59^{p}56'.$$

And likewise, since

$$FG = GH$$

and

$$EH = 2\ DG,$$

therefore, by addition, we shall have

$$CH = 61^{p}50'$$

where

$$EH = 6^{p}2',$$

and therefore

$$\text{hypt. } CE = 62^{p}8'.$$

And therefore

$$EH = 11^{p}39'$$

where

$$\text{hypt. } CE = 120^{p};$$

and, on the circle about right triangle CEH,

$$\text{arc } EH = 11°9'.$$

And so also

$$\text{angle } ECH = 11°9' \text{ to 2 rt.}$$

Likewise, since

$$FH = 3^{p}48'$$

where

$$\text{ecc. rad. } FX = 60^{p},$$

therefore, by addition, we shall have

$$HX = 63^{p}48'$$

where

$$EH = 6^{p}2',$$

and therefore

$$\text{hypt. } EX = 64^{p}5'.$$

And therefore

$$EH = 11^{p}18'$$

where

$$\text{hypt. } EX = 120^{p};$$

and, on the circle about right triangle EHX,

$$\text{arc } EH = 10°49'.$$

And so also

$$\text{angle } EXH = 10°49' \text{ to 2 rt.}$$

But it was shown

$$\text{angle } ECH = 11°9' \text{ to 2 rt.}$$

And therefore, by subtraction,

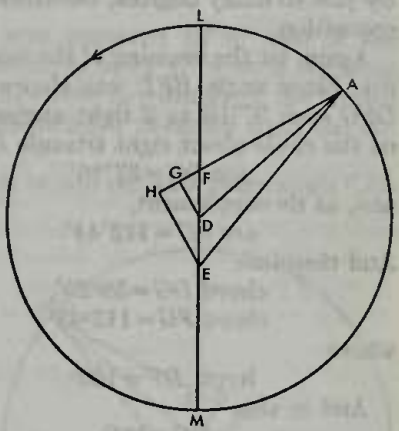
$$\text{angle } CEX = 0°20' \text{ to 2 rt.}$$
$$= 0°10'.$$

And so, since, at the third opposition, the star appearing along EC was situated 14°14′ within the Goat, it is evident that, if it were on straight line EX, it would be 14°24′ within the Goat, and in turn the apparent distance from the second opposition to the third, considered relative to the eccentric NX, would be 34°38′.

Working with these distances in the same theorem, we find the distance between the centres of the ecliptic and of the eccentric embracing the epicycle's regular movement (or the distance equal to EF) to be very nearly 6ᵖ50′ where the eccentric's diameter is 120ᵖ. And, of the same eccentric's arcs, we find that from the first opposition to the apogee is 57°5′; that from the apogee to the second opposition is 18°38′; and that from the apogee to the third opposition is 56°30′.

Here again these quantities have been accurately taken because the differences of the ecliptic's arcs amount to very nearly the same as the former ones, by using these arcs, and because the star's apparent distances are found to be in accord with those observed, as will be clear to us from similar proofs.

For let the figure of the first opposition be laid out for only the eccentricity bearing the epicycle's centre.



Since, then, angle AFL, subtending as it does 57°5′ of the eccentric, is 57°5′, and it and the vertical angle DFG are each 114°10′ to 2 right angles' 360°, therefore, on the circle about right triangle DFG,

arc $DG = 114°10′$,

and as its supplement

arc $FG = 65°50′$.

And therefore

chord $DG = 100ᵖ44′$,

chord $FG = 65ᵖ13′$

where

hypt. $DF = 120ᵖ$.

And so also

$DG = 2ᵖ52′$

and likewise

$FG = 1ᵖ51′$

where

1. betw. c. $DF = 3ᵖ25′$

and

ecc. rad. $AD = 60ᵖ$.

And again, since

sq. AD − sq. DG = sq. AG,

we shall also have

$AG = 59ᵖ56′$.

And likewise, since

$FG = GH$

and

$EH = 2\ DG$,

therefore, by addition, we shall have

$AH = 61ᵖ47′$

where

$$EH = 5^{p}44',$$

and therefore also

$$\text{hypt. } AE = 62^{p}3'.$$

And therefore

$$EH = 11^{p}5'$$

where

$$\text{hypt. } AE = 120^{p};$$

and, on the circle about right triangle AEH,

$$\text{arc } EH = 10°36'.$$

And so also

$$\text{angle } EAF = 10°36' \text{ to 2 rt.}$$

But it was supposed

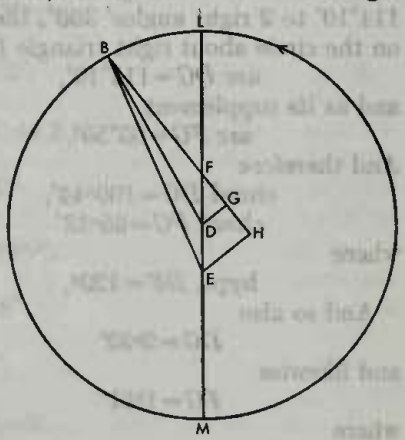
$$\text{angle } AFL = 114°10' \text{ to 2 rt.}$$

And therefore, by subtraction,

$$\text{angle } AEL = 103°34' \text{ to 2 rt.}$$
$$= 51°47'.$$

By just so many degrees, therefore, the star was west of the apogee at the first opposition.

Again, let the drawing of the second opposition be laid out in a similar manner. Since angle BFL was shown to be 18°38′, and it and the vertical angle DFG each 37°16′ to 2 right angles' 360°,



on the circle about right triangle DFG,

$$\text{arc } DG = 37°16'$$

and, as its supplement,

$$\text{arc } FG = 142°44'.$$

And therefore

$$\text{chord } DG = 38^{p}20',$$
$$\text{chord } FG = 113^{p}43'$$

where

$$\text{hypt. } DF = 120^{p}.$$

And so also

$$DG = 1^{p}5'$$

and likewise

$$FG = 3^{p}14'$$

where

$$DF = 3^{p}25'$$

and

$$\text{ecc. rad. } BD = 60^{p}.$$

And since

$$\text{sq. } BD - \text{sq. } DG = \text{sq. } BG,$$

therefore we shall have

$$BG = 59^{p}59'.$$

And likewise, since

$$FG = GH$$

and

$$EH = 2\ DG,$$

therefore we shall have, by addition,

$$BH = 63^{p}13'$$

where

$$EH = 2^{p}10',$$

and therefore also

$$\text{hypt. } BE = 63^{p}15'.$$

And therefore

$$EH = 4^{p}7'$$

where

$$\text{hypt. } BE = 120^{p};$$

and, on the circle about right triangle BEH,

$$\text{arc } EH = 3°56'.$$

And so also

$$\text{angle } EBF = 3°56' \text{ to 2 rt.}$$

But it was supposed

$$\text{angle } BFL = 37°16' \text{ to 2 rt.}$$

And therefore, by subtraction,

$$\text{angle } BEL = 33°20' \text{ to 2 rt.}$$
$$= 16°40'.$$

Therefore at the second opposition the star appeared 16°40′ east of the apogee. But it was shown, at the first opposition, to be 51°47′ west of the same apogee. Hence the apparent distance from the first opposition to the second comes to a total of 68°27′ in accord with the number obtained from the observations.

Then let the drawing of the third opposition be laid out also. Since angle CFL was shown to be 56°30′, and it and the vertical angle DFG each 113° to 2 right angles' 360°, therefore, on the circle about right triangle DFG,

$$\text{arc } DG = 113°$$

and as its supplement

$$\text{arc } FG = 67°.$$

And therefore

$$\text{chord } DG = 100^{p}4',$$
$$\text{chord } FG = 66^{p}14'$$

where

$$\text{hypt. } DF = 120^{p}.$$

And so also

$$DG = 2^{p}51'$$

and likewise

$$FG = 1^{p}53'$$

where

$$DF = 3^{p}25'$$

and

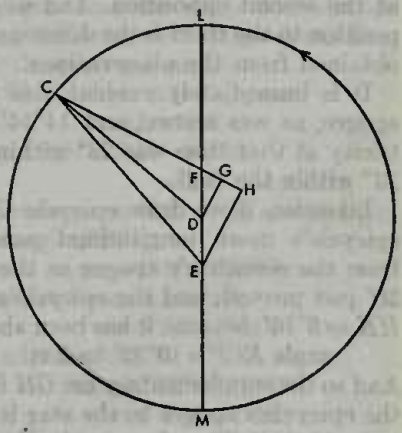
$$\text{ecc. rad. } CD = 60^{p}.$$

And since again

$$\text{sq. } CD - \text{sq. } DG = \text{sq. } CG,$$

we shall also have

$$CG = 59^{p}56'.$$



And likewise, since

$$FG = GH$$

and

$$EH = 2\ DG,$$

therefore we shall also have, by addition,

$$CH = 61^{p}49'$$

where

$$EH = 5^{p}42',$$

and therefore also

$$\text{hypt. } CE = 62^{p}5'.$$

And therefore

$$EH = 11^{p}10'$$

where

$$\text{hypt. } CE = 120^{p};$$

and, on the circle about right triangle CEH,

$$\text{arc } EH = 10°32'.$$

And so also

$$\text{angle } ECH = 10°32' \text{ to 2 rt.}$$

But it is supposed

$$\text{angle } CFA = 113° \text{ to 2 rt.}$$

And therefore, by subtraction,

$$\text{angle } CEL = 102°28' \text{ to 2 rt.}$$
$$= 51°14'.$$

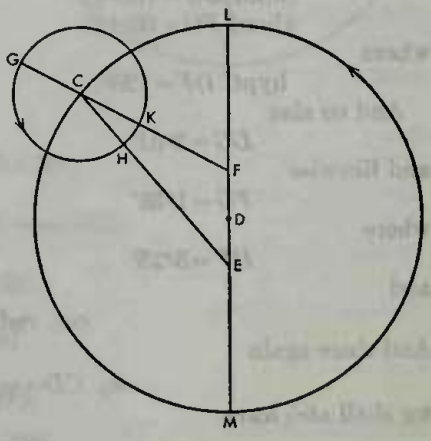
Therefore by just so many degrees, at the third opposition, did the star appear east of the apogee. But it was also shown to be 16°40′ east of the apogee at the second opposition. And so the apparent distance from the second opposition to the third is the difference of 34°34′, again in accord with the number obtained from the observations.

It is immediately evident also that, since the star was 51°14′ east of the apogee, as was shown, and 14°14′ within the Goat, the apogee of its eccentricity at that time was 23° within the Scorpion, and its perigee just opposite 23° within the Bull.

Likewise, if we draw epicycle GH about centre C, then we shall have the epicycle's mean longitudinal passage from the eccentric's apogee as the 56° 30′ just proved; and the epicycle's arc HK as 5°16′, because it has been shown

$$\text{angle } ECF = 10°32' \text{ to 2 rt.}$$

And so the supplementary arc GH from the epicycle's apogee to the star is left to be 174°44′. Therefore, at the time of the third opposition—that is, in the year 20 of Hadrian, Egyptianwise Mesore 24 at midday—Saturn considered with respect to its mean passages was 56°30′ in longitude from the eccentric's apogee or 19°30′ within the Goat, and 174°44′ in anomaly from the epicycle's apogee. Which it was required to find.

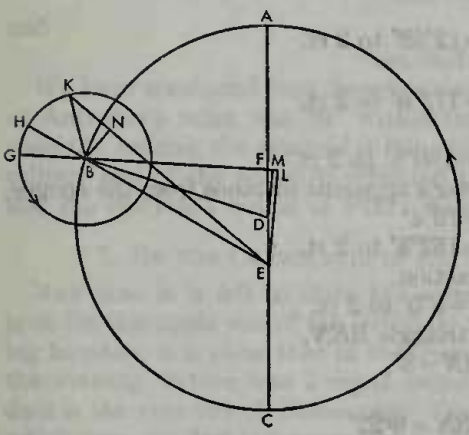


6. Demonstration of the Size of Saturn's Epicycle

And then again for showing the epicycle's size we took an observation we made in the year 2 of Antonine, Egyptianwise Mechir 6-7, 4 equatorial hours before midnight, when the last degree of the Ram was culminating in the astrolabe, with the mean sun 28°41′ within the Archer. At that time Saturn, sighted relative to the bright star of the Hyades [Aldebaran], appeared to be $9\frac{1}{15}°$ within the Water Bearer and was very nearly $\frac{1}{2}°$ east of the moon's centre. For it was just that far from its northern horn. But at that hour the moon, according to its mean passage, was 8°55′ within the Water Bearer, and in anomaly was 174°15′ from the epicycle's apogee. Therefore its true passage must have been 9°40′ within the Water Bearer, and its apparent passage in Alexandria 8°34′. Consequently, since it was very nearly $\frac{1}{2}°$ east of its centre Saturn must have been $9\frac{1}{15}°$ within the Water Bearer. And it was 76°4′ from the eccentric's same apogee because in that amount of time it did not change appreciably.

Since the time from the third opposition to this observation is 2 Egyptian years, 167 days, and 8 hours, and since again Saturn, in that amount of time, moves approximately in longitude 30°3′ and in anomaly 134°24′, it follows that if we add these to the position given at the third opposition we shall have, for the time of the given observation, in longitude 86°33′ from the eccentric's apogee and in anomaly 309°8′ from the epicycle's apogee.

Now, with these things supposed, let the drawing of the similar proof be laid out, having the epicycle's position to the east of the eccentric's apogee, and the star's position before the epicycle's apogee according to the passages supposed.



Since, then, it is supposed

angle AFB or
angle $DFM = 86°33′$
$= 173°6′$ to 2 rt.,

therefore, on the circle about right triangle DFM,

arc $DM = 173°6′$

and, as its supplement,

arc $FM = 6°54′$.

And therefore

chord $DM = 119^p47′$,
chord $FM = 7^p13′$

where

hypt. $DF = 120^p$.

And so also

$DM = 3^p25′$

and likewise

$FM = 0^p12′$

where

l. betw. c. $DF = 3^p25′$

and

ecc. rad. $BD = 60^{p}$.

And since

sq. BD − sq. DM = sq. BM,

we shall also have

$BM = 59^{p}54'$.

And likewise, since

$FM = LM$,

and

$EL = 2\ DM$,

therefore also we shall have, by addition,

$BL = 60^{p}6'$

where

$EL = 6^{p}50'$,

and therefore also

hypt. $BE = 60^{p}29'$.

And therefore

$EL = 13^{p}33'$

where

hypt. $BE = 120^{p}$;

and, on the circle about right triangle BEL,

arc $EL = 12°58'$.

And so also

angle $EBF = 12°58'$ to 2 rt.

And it is also supposed

angle $AFB = 173°6'$ to 2 rt.

And therefore, by subtraction,

angle $AEB = 160°8'$ to 2 rt.

But also, as the angle containing the star's apparent distance from the apogee,

angle $AEK = 76°4'$
$= 152°8'$ to 2 rt.

And therefore we shall have, by subtraction,

angle $KEB = 8°0'$ to 2 rt.

And so also, on the circle about right triangle BEN,

arc $BN = 8°$,

and

chord $BN = 8^{p}22'$

where

hypt. $BE = 120^{p}$.

And therefore

$BN = 4^{p}13'$

where

$BE = 60^{p}29'$

and

ecc. rad. $= 60^{p}$.

Again, since the star was 309°8′ from the epicycle's apogee G, therefore, by subtraction,

arc $GK = 50°52'$.

And therefore

angle $GBK = 50°52'$
$= 101°44'$ to 2 rt.

But

angle EBF = angle $GBH = 12°58'$ to 2 rt.

And therefore, by subtraction,

angle $HBK = 88°46'$ to 2 rt.

where it was shown

angle $KEB = 8°$ to 2 rt.

And therefore, by subtraction, we shall have

angle $BKN = 80°46'$ to 2 rt.

And so also, on the circle about right triangle BKN,

arc $BN = 80°46'$

and

chord $BN = 77^{p}45'$

where

hypt. $BK = 120^{p}$.

And therefore we shall have

epic. rad. $BK = 6\frac{1}{2}^{p}$

where it was shown

$BN = 4^{p}13'$

and

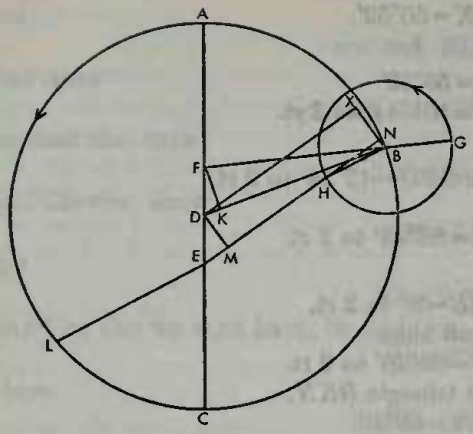
ecc. rad. $= 60^{p}$.

We have concluded that Saturn's apogee at these dates near the beginning of Antonine's reign was 23° within the Scorpion. Where the radius of the eccentric bearing the epicycle is 60^{p}, there the line between the centres of the ecliptic and of the eccentric effecting the regular movement comes to $6^{p}50'$, and the epicycle's radius to $6^{p}30'$. Which it was required to find.

7. On the Correction of Saturn's Periodic Movements

Now that it is left to show the correction of the periodic movements, we took for this again one of the faithfully recorded ancient observations, according to which it is clear that in the year 82 of the Chaldeans, Xanthicus 5, in the evening, Saturn was 2 digits below the Virgin's southern shoulder. This date is the year 519 of Nabonassar, Egyptianwise Tybi 14 in the evening, at which time we find the mean sun 6°10′ within the Fishes. But the fixed star of the Virgin's northern shoulder at the time of our observation was $13\frac{1}{6}°$ within the Virgin. At the time of the given observation, since very nearly $3\frac{2}{3}°$ of the fixed star's movement correspond to the intervening 366 years, it was clearly $9\frac{1}{2}°$ within the Virgin; and also Saturn, since it was south of the fixed star by 2 digits. Likewise, since its apogee was shown in our time to be nearly 23° within the Scorpion, at that time it must have been $19\frac{1}{3}°$ within the Scorpion. By means of these things it is concluded that at the given time the apparent star was 290°10′ along the ecliptic from the apogee at that time, and the mean sun was 106°50′ from the same apogee.

With these things supposed, again let there be set out the drawing for a



similar proof, having the epicycle's position west of the eccentric's apogee and the sun's west of the perigee and parallel to it the line from the epicycle's centre to the star.

Now, since Saturn appeared west of the apogee by the 69°50′ failing one complete circle, therefore, being at the ecliptic's centre,

angle $AEH = 69°50'$
$= 139°40'$ to 2 rt.

But also the angle AEL of the solar distance is assumed to be 106°50′ or 213°40′ to 2 right angles' 360°. And therefore, by addition,

angle HEL or angle $BHE = 353°20'$ to 2 rt.

because BH is parallel to EL, And by subtraction

angle $BHN = 6°40'$ to 2 rt.

And so also, on the circle about right triangle BHN,

arc $BN = 6°40'$,

and

chord $BN = 6^{p}58'$

where

hypt. $BH = 120^{p}$.

And therefore

$BN = 0^{p}23'$

where

epic. rad. $BH = 6^{p}30'$.

Likewise, since

angle $AEH = 139°40'$ to 2 rt.

and

angle $EDM = 40°20'$ to 2 rt.

therefore also, on the circle about right triangle DEM,

arc $DM = 139°40'$,

and

chord $DM = 112^{p}39'$

where

hypt. $DE = 120^{p}$.

And therefore

$DM = XN = 3^{p}12'$

where

l. betw. c. $DE = 3^{p}25'$

and

ecc. rad. $BD = 60^{p}$;

and, by addition,

$BNX = 3^{p}35'$

where

hypt. $BD = 60^{p}$.

And therefore

$$BX = 7^{p}10'$$

where

$$BD = 120^{p};$$

and, on the circle about right triangle BDX,

$$\text{arc } BX = 6°52'.$$

And so also

$$\text{angle } BDX = 6°52' \text{ to 2 rt.},$$

and as its complement

$$\text{angle } BDM = 173°8' \text{ to 2 rt.},$$

and likewise by addition

$$\text{angle } BDE = 213°28' \text{ to 2 rt.},$$

and as its supplement

$$\text{angle } BDA = 146°32' \text{ to 2 rt.}$$

And so also, on the circle about right triangle DFK,

$$\text{arc } FK = 146°32'$$

and, as its supplement,

$$\text{arc } DK = 33°28'.$$

And therefore

$$\text{chord } FK = 114^{p}55',$$
$$\text{chord } DK = 34^{p}33'$$

where

$$\text{hypt. } DF = 120^{p}.$$

And therefore

$$FK = 3^{p}17'$$

and likewise

$$DK = 0^{p}59'$$

where

$$\text{l. betw. c. } DF = 3^{p}25'$$

and

$$\text{ecc. rad. } BD = 60^{p};$$

and, by subtraction,

$$BK = 59^{p}1'$$

where

$$FK = 3^{p}17',$$

and therefore

$$\text{hypt. } BF = 59^{p}6'.$$

And so also

$$FK = 6^{p}40'$$

where

$$\text{hypt. } BF = 120^{p};$$

and, on the circle about right triangle BFK,

$$\text{arc } FK = 6°22'.$$

And therefore

$$\text{angle } FBK = 6°22' \text{ to 2 rt.}$$

But

$$\text{angle } ADB = 146°32' \text{ to 2 rt.}$$

And therefore, by addition, we shall have

$$\text{angle } AFB = 152°54' \text{ to 2 rt.}$$
$$= 76°27',$$

the angle which embraces the regular longitudinal passage.

Therefore, at the time of the given observation, Saturn, according to its mean longitudinal passage, was 283°33′ from the apogee, or situated 2°53′

within the Virgin. Since, moreover, the sun's mean passage is supposed 106° 50′, if we add to this the 360° of one circle and from the resulting 466°50′ subtract the 283°33′ in longitude, we shall have, for that same time, the 183°17′ of anomaly from the epicycle's apogee.

At the time of the given observation, which was in the year 519 of Nabonassar, Tybi 14 in the evening, Saturn was shown to be 183°17′ from the epicycle's apogee, and at the time of the third opposition, which was in the year 883 of Nabonassar, Mesore 24 at midday, it was 174°44′; therefore it is evident that in the time between the two observations, which embraces 364 Egyptian years and 219¾ days, Saturn has moved 351°27′ above 351 complete circles of anomaly. And again the surplus from the mean movements worked out by us comes to very nearly this much. For the daily mean movement was established by dividing the total number of degrees resulting from the number of circles and the surplus by the total number of days of the time.

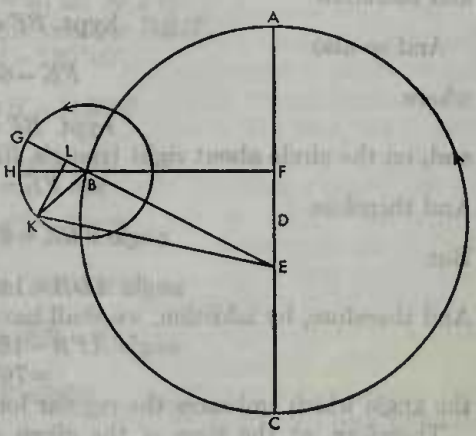
8. On the Epoch of Saturn's Periodic Movements

But since also the time from the year 1 of Nabonassar, Thoth 1, at midday to the given ancient observation is 518 Egyptian years and 133¼ days, and since this amount of time embraces surpluses of 216°9′ in longitude and 149° 15′ in anomaly, if we subtract these from the given positions at the observations, we shall have, again for this time of the epoch, Saturn's mean longitudinal position 26°44′ within the Goat, its anomalistic position 34°2′ from the epicycle's apogee, and therefore the apogee of its eccentricity about 14°10′ within the Scorpion. Which it was required to find.

9. How the True Passages are Gotten Geometrically from the Periodic Movements

By means of the same things it will be clear to us that conversely, if the periodic arcs of the eccentric embracing the regular movement and of the epicycle are given, then also the stars' apparent passages are easily obtained by geometry.

For if, in the simple drawing of the eccentric and epicycle, we join *FBH* and *EBG*, then, given the mean longitudinal passage (that is, angle *AFB*) angle *AEB*, and angle *EBF* or angle *GBH* are also given according to both hypotheses, from things already shown. Furthermore, the ratio of the straight line *BE* to the epicycle's radius is given.



Let the star be supposed, for instance, at point *K* on the epicycle, and *EK* and *BK* be joined. And let arc *HK* be given. Now, if we no longer (as in the converse proof) draw a straight line from the epicycle's centre *B* perpendicular to *EK* but rather from the star at *K* perpendicular to *EB* (as here *KL*) then the

whole angle *GBK* is given, and therefore also the ratio of *KL* and *LB* to *BK* and *EB;* and, accordingly, the ratio of the whole *EBL* to *LK*. And so also, given the angle *LEK*, we get the angle *AEK* containing the star's apparent distance from the apogee.

10. Treatment of the Construction of Tables for the Anomalies

Yet in order that we may not always calculate the apparent passages geometrically (although this way alone accurately obtains what is required, but is too difficult for ease in researches), we have constructed tables as convenient and as nearly accurate as possible, for each of the five stars, and containing their particular anomalies combined in such a way that, when the periodic movements from the proper apogees are given, we may readily calculate from them the apparent passages for any time.

Each of these tables we have arranged again in 45 rows, for symmetry's sake, and in 8 columns. Of these columns, the first two contain the numbers of the mean passages, as in the case of the sun and the moon, with the 180° from the apogee arranged downwards in the first and the remaining 180° of the semicircle arranged upwards in the second, so that the number 180 has been put in the last row in both, and their increase for the first 15 rows downwards is by intervals of 6° and for the remaining 30 under them by intervals of 3°. For the differences of the sections of anomaly with respect to each other are less appreciable at the apogees, but they change more rapidly at the perigees.

Of the next two columns, the third contains the addition-subtractions resulting for the corresponding number of mean longitudinal passage through the greatest eccentricity, yet taken simply as if the epicycle's centre were borne on the eccentric embracing the regular movement. The fourth column contains the differences of the addition-subtractions resulting from the fact that the epicycle's centre is not borne on that circle but on another. And the way in which each of these two is obtained geometrically, at the same time and separately, can be readily understood through the many theorems already propounded by us. It was fitting that at this point in this treatise such an analysis of the zodiacal anomaly be displayed and therefore set forth in two columns; but, for actual use, one column combined from the addition-subtraction of them both would suffice.

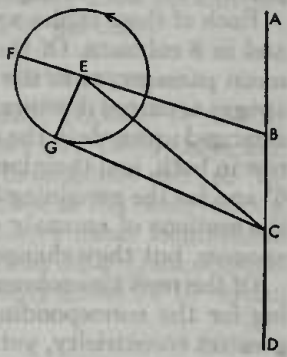
Of the next three columns, each contains the addition-subtractions due to the epicycle, again taken simply, with the apogees and perigees considered relative to the distance from our eye. For the method of this kind of proof can be easily understood by us with the help of theorems already set out. The middle one of these three columns, or the sixth from the first, contains the addition-subtractions calculated according to the ratios of the mean distances. But the fifth contains, for the same sections, the differences of the addition-subtractions at the greatest distance with respect to those at the mean; and the seventh contains the differences of the addition-subtractions at the least distance with respect to those at the mean.

For we have shown that, where the epicycle's radius is $6^{p}30'$ in the case of Saturn (for it would be well finally to begin from the top down), $11^{p}30'$ for Jupiter, $39^{p}30'$ for Mars, $43^{p}10'$ for Venus, and $22^{p}30'$ for Mercury, there the mean distance of all is 60^{p}—that is, the distance considered with respect to the radius of the eccentric bearing the epicycle; and the greatest distance with respect to the ecliptic's centre is for Saturn $63^{p}25'$, for Jupiter $62^{p}45'$, for Mars 66^{p}, for

Venus 61ᵖ15′, and for Mercury 69ᵖ; and likewise the least distance is for Saturn 56ᵖ35′, for Jupiter 57ᵖ15′, for Mars 54ᵖ, for Venus 58ᵖ45′, and for Mercury 55ᵖ34′.

We have arranged the eighth and last column for obtaining the corresponding parts of the difference, set out when the stars' epicycles are neither at the mean, greatest, or least distances, but in the passages between. We have arranged the calculation of this correction for each distance between, only with respect to the greatest addition-subtractions at the tangents from our eye to the epicycle, since there is no appreciable deviation in the proportion of the differences for the other sections of the epicycle relative to those for the greatest addition-subtractions.

To make what is here said still clearer, and to have the method of proportions stand out in evidence, let there be laid out the straight line *ABCD* through the centres of the ecliptic and of the eccentric embracing the regular movement of the epicycle. Let *C* be supposed the ecliptic's centre, and *B* the centre of the epicycle's regular movement. With *BEF* produced, let the epicycle *FG* be drawn about centre *E*; and let straight line *CG* be drawn from *C* tangent to it, letting *CE* and the perpendicular *EG* be joined. For an example, let the epicycle's centre for each of the five stars be supposed 30° in regular movement from the apogee of eccentricity.



Now (not to draw out the calculations by proving the same things again) it was shown by many theorems preceding this, both for the hypothesis of Mercury and the rest, that, when angle *ABE* is given, the ratio of *CE* to the epicycle's radius or *GE* is also given; and, when angle *ABE* is assumed to be 30°, by calculations for each case, the ratio becomes 63ᵖ2′ to 6ᵖ30′ for Saturn, 62ᵖ26′ to 11ᵖ30′ for Jupiter, 65ᵖ24′ to 39ᵖ30′ for Mars, 61ᵖ26′ to 43ᵖ10′ for Venus, and 66ᵖ35′ to 22ᵖ30′ for Mercury. Therefore we shall have angle *ECG* which contains the greatest addition-subtraction due to the epicycle at that time: 5°55½′ for Saturn, 10°36½′ for Jupiter, 37°9′ for Mars, 44°56½′ for Venus, and 19°45′ for Mercury.

According to the ratios just expounded, the greatest addition-subtractions at the mean distances, become 6°13′, 11°3′, 41°10′, 46°, and 22°2′, in the order of the stars just given, to avoid repetition. At the greatest distances they become 5°53′, 10°34′, 36°45′, 44°48′, and 19°2′; and at least distances they become 6°36′, 11°35′, 47°1′, 47°17′, and 23°53′. Those at the greatest distances hence differ from those at the mean by 0°20′, 0°29′, 4°25′, 1°12′, and 3°; and those at the least differ from them by 0°23′, 0°32′, 5°51′, 1°17′, and 1°51′.

Now, the addition-subtractions of the distances sought are less than those of the mean distances and differ from them by 0°17½′, 0°26½′, 4°1′, 1°3½′, and 2°17′. These are, for Saturn, 52 and 30′ sixtieths of the given whole differences between those at the mean distances and those at the greatest; for Jupiter, 54 and 50′ sixtieths; for Mars, 54 and 34′; for Venus 52 and 55′; and for Mercury, 45 and 40′. And therefore in the eighth column in each table we have placed just that number of sixtieths in the row containing the number 30 of periodic longi-

tude. And in the case of distances having addition-subtractions greater than those at the mean distances, we likewise analysed their differences into sixtieths again, but relative to whole differences with those at the least distances and no longer with those at the greatest. In the same way we calculated, for the other positions at intervals of 6° mean longitude, the resulting sixtieths of the whole differences, and we put them beside the proper numbers. For, as we said, the proportion of the differences is sensibly the same, even if the passages of the stars are not at the epicycle's greatest addition-subtractions, but at other parts of it. And here is the exposition presented in the following five tables [pp. 386–390].

12. On Calculating the Longitudes of the Five Planets

Now, whenever we wish to know the apparent passages of each of the stars, from their periodic movements in longitude and anomaly, by means of the foregoing tables, we shall make the calculation, one and the same for the five stars, in this way:

Calculating, from the tables of mean movement, the regular positions of longitude and anomaly above whole circles produced in the time required, we first carry the degrees from the eccentric's apogee at that time to the mean longitudinal passage, to the table of anomaly proper to the star. If the longitudinal number set out is in the first column, we subtract from the degrees of longitude the quantities in the third column of the truly determined longitude corresponding to the number, along with the addition-subtraction of sixtieths in the fourth column; and we add them to the degrees of anomaly. But if the longitudinal number is in the second column, we add them to the longitudinal and subtract them from the anomalistic, so that we may have both passages truly determined.

Then we carry the number of degrees from the anomaly's apogee, analysed out, again to the first two tables, and we record the corresponding addition-subtraction of the mean distance in the sixth column. Likewise carrying the original number of regular longitude, brought over, to the same numbers, if it is in the first rows more apogee than that of the mean distance (which becomes clear from the sixtieths in the eighth column), then, whatever number of sixtieths in the eighth column correspond to it, we take just that many of the difference, corresponding to the recorded mean addition-subtraction, in the fifth column of the greatest distance, and we subtract the result from what we recorded. But if the number of the aforesaid longitude is in the lower rows more perigee than that at the mean distance, then likewise, whatever number of sixtieths correspond to it in the eighth column, we take just that many of the difference, corresponding to the recorded mean addition-subtraction, in the seventh column of the least distance, and we add the results to what we recorded.

The total number of degrees of the addition-subtraction so determined we add to the degrees of longitude, truly determined, if the truly determined anomalistic number is in the first column; and we subtract them from the longitudinal degrees, if it is in the second column. And counting the resulting number of degrees from the star's apogee at that time, we come to its apparent passage.

11. Tables for the Anomalies

SATURN, Apogee, 14°10′ within the Scorpion

1	2	3	4	5	6	7	8
Common Numbers		Longitudinal Addition-Subtraction	Differences of Addition-Subtraction	Differences of Subtraction	Addition-Subtraction of Anomaly	Differences of Addition	Sixtieths of Subtraction
6	354	0 37	0 2	0 2	0 36	0 2	60 0
12	348	1 13	0 4	0 4	1 11	0 4	58 30
18	342	1 49	0 6	0 5	1 45	0 7	57 0
24	336	2 23	0 8	0 7	2 18	0 9	55 30
30	360	2 57	0 9	0 8	2 50	0 11	52 30
36	324	3 29	0 10	0 10	3 20	0 13	49 30
42	318	3 59	0 11	0 11	3 49	0 15	46 30
48	312	4 28	0 11	0 12	4 17	0 17	43 30
54	306	4 55	0 10	0 14	4 42	0 19	39 0
60	300	5 20	0 9	0 15	5 4	0 20	34 30
66	294	5 42	0 8	0 17	5 25	0 20	30 0
72	288	6 0	0 7	0 18	5 42	0 21	24 0
78	282	6 14	0 5	0 18	5 55	0 21	18 0
84	276	6 24	0 3	0 19	6 5	0 22	12 0
90	270	6 30	0 1	0 19	6 12	0 22	4 30
93	267	6 31	0 0	0 20	6 12	0 23	0 45
			Addition				Subtraction
96	264	6 32	0 2	0 20	6 13	0 23	2 32
99	261	6 31	0 3	0 20	6 12	0 24	5 51
102	258	6 30	0 4	0 21	6 12	0 24	9 8
105	255	6 27	0 5	0 21	6 9	0 24	11 45
108	252	6 23	0 6	0 20	6 5	0 25	14 21
111	249	6 19	0 7	0 20	6 0	0 25	16 58
114	246	6 14	0 8	0 20	5 55	0 24	19 31
117	243	6 7	0 9	0 19	5 48	0 24	22 11
120	240	5 59	0 10	0 19	5 40	0 23	24 47
123	237	5 50	0 10	0 19	5 31	0 23	27 24
126	234	5 39	0 11	0 18	5 21	0 22	30 0
129	231	5 27	0 11	0 18	5 10	0 22	32 37
132	228	5 14	0 12	0 17	4 58	0 21	35 13
135	225	5 0	0 12	0 17	4 45	0 20	37 50
138	222	4 45	0 12	0 16	4 31	0 19	40 26
141	219	4 29	0 12	0 15	4 16	0 18	43 3
144	216	4 12	0 12	0 14	4 0	0 17	45 39
147	213	3 54	0 12	0 14	3 43	0 15	47 37
150	210	3 35	0 11	0 12	3 25	0 14	49 34
153	207	3 16	0 11	0 11	3 7	0 13	51 32
156	204	2 56	0 10	0 10	2 48	0 12	53 29
159	201	2 36	0 9	0 9	2 29	0 11	54 49
162	198	2 15	0 8	0 7	2 9	0 10	56 6
165	195	1 53	0 7	0 6	1 48	0 8	57 24
168	192	1 31	0 6	0 5	1 27	0 7	58 42
171	189	1 9	0 5	0 5	1 6	0 5	59 21
174	186	0 47	0 3	0 4	0 45	0 4	60 0
177	183	0 24	0 2	0 2	0 23	0 2	60 0
180	180	0 0	0 0	0 0	0 0	0 0	60 0

11. Tables for the Anomalies

JUPITER, Apogee, 2°9′ within the Virgin

1	2	3	4	5	6	7	8
Common Numbers		*Longitudinal Addition-Subtraction*	*Differences of Addition-Subtraction*	*Differences of Subtraction*	*Addition-Subtraction of Anomaly*	*Differences of Addition*	*Sixtieths of Subtraction*
6	354	0 30	0 1	0 2	0 58	0 2	60 0
12	348	1 0	0 2	0 5	1 56	0 5	58 58
18	342	1 30	0 3	0 7	2 52	0 7	57 56
24	336	1 58	0 4	0 9	3 48	0 9	56 54
30	330	2 26	0 5	0 11	4 42	0 11	54 50
36	324	2 52	0 6	0 13	5 34	0 13	51 43
42	318	3 17	0 7	0 15	6 25	0 15	47 35
48	312	3 40	0 7	0 17	7 12	0 18	43 27
54	306	4 1	0 7	0 19	7 57	0 20	39 19
60	300	4 20	0 6	0 21	8 37	0 22	35 8
66	294	4 37	0 5	0 23	9 14	0 24	28 58
72	288	4 51	0 4	0 24	9 46	0 26	22 45
78	282	5 2	0 3	0 25	10 13	0 28	17 35
84	276	5 9	0 2	0 26	10 35	0 30	11 23
90	270	5 14	0 1	0 26	10 51	0 31	4 40
93	267	5 15	0 0	0 27	10 57	0 31	1 8
			Subtraction				*Addition*
96	264	5 16	0 1	0 27	11 0	0 32	1 52
99	261	5 15	0 1	0 27	11 2	0 32	5 9
102	258	5 14	0 2	0 28	11 3	0 32	8 26
105	255	5 12	0 2	0 28	11 1	0 33	11 43
108	252	5 9	0 3	0 29	10 59	0 33	15 0
111	249	5 5	0 4	0 29	10 53	0 33	17 49
114	246	5 0	0 5	0 30	10 45	0 34	20 37
117	243	4 54	0 5	0 30	10 35	0 34	23 26
120	240	4 47	0 6	0 30	10 24	0 34	26 15
123	237	4 39	0 6	0 29	10 10	0 33	29 4
126	234	4 10	0 7	0 29	9 54	0 33	31 52
129	231	4 20	0 7	0 28	9 36	0 32	34 41
132	228	4 9	0 8	0 28	9 16	0 32	37 30
135	225	3 58	0 8	0 27	8 54	0 31	40 19
138	222	3 46	0 8	0 26	8 30	0 30	43 7
141	219	3 33	0 8	0 25	8 4	0 28	45 28
144	216	3 20	0 7	0 23	7 36	0 26	47 49
147	213	3 6	0 7	0 22	7 6	0 25	49 42
150	210	2 51	0 6	0 21	6 34	0 23	51 31
153	207	2 36	0 6	0 19	6 0	0 21	52 58
156	204	2 20	0 5	0 17	5 24	0 19	54 22
159	201	2 4	0 5	0 15	4 47	0 17	55 47
162	198	1 47	0 4	0 13	4 9	0 15	57 11
165	195	1 30	0 3	0 11	3 29	0 13	57 40
168	192	1 13	0 2	0 9	2 49	0 10	58 13
171	189	0 55	0 2	0 7	2 7	0 8	58 40
174	186	0 37	0 1	0 5	1 25	0 5	59 4
177	183	0 18	0 1	0 3	0 43	0 3	59 32
180	180	0 0	0 0	0 0	0 0	0 0	60 0

11. Tables for the Anomalies

MARS, Apogee, 16°40′ within the Crab

1	2	3		4		5		6		7		8	
Common Numbers		Longitudinal Addition-Subtraction		Differences of Addition-Subtraction		Differences of Subtraction		Addition-Subtraction of Anomaly		Differences of Addition		Sixtieths of Subtraction	
6	354	1	0	0	5	0	8	2	24	0	9	59	53
12	348	2	0	0	10	0	16	4	46	0	18	58	59
18	342	2	58	0	15	0	24	7	8	0	28	57	51
24	336	3	56	0	20	0	33	9	30	0	37	56	36
30	330	4	52	0	24	0	42	11	51	0	46	54	34
36	324	5	46	0	27	0	51	14	11	0	56	52	11
42	318	6	39	0	28	1	0	16	29	1	6	49	28
48	312	7	28	0	29	1	9	18	46	1	16	46	17
54	306	8	14	0	28	1	18	21	0	1	28	42	38
60	300	8	57	0	27	1	27	23	13	1	40	38	8
66	294	9	36	0	24	1	37	25	22	1	53	33	26
72	288	10	9	0	20	1	49	27	29	2	6	28	20
78	282	10	38	0	15	2	1	29	32	2	19	22	47
84	276	11	2	0	10	2	14	31	30	2	33	16	33
90	270	11	19	0	4	2	28	33	22	2	45	10	5
93	267	11	25	0	0	2	35	34	15	2	57	6	34
				Subtraction									
96	264	11	29	0	4	2	42	35	6	3	6	3	3
												Addition	
99	261	11	32	0	8	2	49	35	56	3	15	0	5
102	258	11	32	0	12	2	56	36	43	3	25	3	13
105	255	11	31	0	16	3	4	37	27	3	36	6	1
108	252	11	28	0	19	3	13	38	9	3	47	8	49
111	249	11	22	0	22	3	22	38	48	3	58	11	44
114	246	11	14	0	25	3	32	39	24	4	9	14	38
117	243	11	5	0	28	3	43	39	56	4	21	17	33
120	240	10	53	0	31	3	54	40	23	4	35	20	27
123	237	10	39	0	33	4	4	40	44	4	50	23	35
126	234	10	23	0	35	4	14	40	59	5	5	26	42
129	231	10	4	0	37	4	24	41	7	5	21	29	31
132	228	9	42	0	39	4	35	41	9	5	37	32	20
135	225	9	21	0	40	4	45	41	2	5	55	35	9
138	222	8	55	0	41	4	56	40	45	6	14	37	58
141	219	8	27	0	41	5	7	40	16	6	34	40	35
144	216	7	59	0	41	5	18	39	37	6	53	43	12
147	213	7	27	0	41	5	28	38	40	7	12	45	26
150	210	6	54	0	38	5	34	37	25	7	30	47	39
153	207	6	19	0	36	5	38	35	52	7	45	49	50
156	204	5	41	0	33	5	38	33	53	7	58	52	1
159	201	5	3	0	30	5	34	31	30	8	3	53	47
162	198	4	22	0	27	5	18	28	35	7	58	55	32
165	195	3	41	0	23	4	52	25	3	7	47	56	44
168	192	2	58	0	19	4	18	21	0	7	6	57	55
171	189	2	14	0	15	3	32	16	25	5	59	58	49
174	186	1	30	0	10	2	27	11	19	4	26	59	43
180	183	0	45	0	5	1	16	5	45	2	20	59	52
	180	0	0	0	0	0	0	0	0	0	0	60	0

11. Tables for the Anomalies

VENUS, Apogee, 16°10′ within the Bull

1	2	3		4		5		6		7		8	
Common Numbers		Longitudinal Addition-Subtraction		Differences of Addition-Subtraction		Differences of Subtraction		Addition-Subtraction of Anomaly		Differences of Addition		Sixtieths of Subtraction	
6	354	0	14	0	1	0	1	2	31	0	2	59	10
12	348	0	28	0	1	0	3	5	1	0	4	57	55
18	342	0	42	0	1	0	5	7	31	0	6	56	40
24	336	0	56	0	2	0	7	10	1	0	8	55	0
30	330	1	9	0	2	0	9	12	30	0	10	52	55
36	324	1	21	0	2	0	11	14	58	0	12	49	35
42	318	1	32	0	3	0	13	17	25	0	14	45	50
48	312	1	43	0	3	0	15	19	51	0	16	42	5
54	306	1	53	0	3	0	18	22	15	0	18	37	5
60	300	2	1	0	2	0	20	24	38	0	20	31	40
66	294	2	8	0	2	0	22	26	37	0	23	26	15
72	288	2	14	0	2	0	24	29	14	0	25	20	25
78	282	2	18	0	1	0	27	31	27	0	28	14	35
84	276	2	21	0	1	0	29	33	38	0	30	8	20
90	270	2	23	0	1	0	31	35	44	0	33	1	40
				Subtraction								*Addition*	
93	267	2	23	0	0	0	33	36	40	0	36	1	31
96	264	2	23	0	1	0	35	37	43	0	38	4	42
99	261	2	22	0	1	0	38	38	40	0	40	7	39
102	258	2	21	0	1	0	40	39	35	0	43	10	35
105	255	2	20	0	1	0	42	40	29	0	45	13	32
108	252	2	18	0	1	0	45	41	20	0	47	16	28
111	249	2	16	0	1	0	47	42	9	0	50	19	25
114	246	2	13	0	2	0	49	42	54	0	52	22	21
117	243	2	10	0	2	0	52	43	35	0	55	25	18
120	240	2	6	0	2	0	54	44	12	0	58	28	14
123	237	2	2	0	2	0	57	44	45	1	1	31	0
126	234	1	58	0	2	1	0	45	14	1	4	33	44
129	231	1	51	0	2	1	3	45	36	1	8	36	18
132	228	1	49	0	3	1	6	45	51	1	11	38	50
135	225	1	44	0	3	1	10	45	55	1	14	41	11
138	222	1	39	0	3	1	14	45	57	1	18	43	32
141	219	1	33	0	3	1	19	45	45	1	22	45	42
144	216	1	27	0	2	1	24	45	20	1	27	47	51
147	213	1	21	0	2	1	29	44	40	1	32	49	37
150	210	1	14	0	2	1	33	43	39	1	38	51	23
153	207	1	7	0	2	1	37	42	18	1	43	52	46
156	204	1	0	0	2	1	39	40	28	1	58	54	8
159	201	0	53	0	2	1	41	38	7	1	51	55	18
162	198	0	46	0	1	1	42	35	7	1	52	56	26
165	195	0	39	0	1	1	38	31	24	1	50	57	28
168	192	0	32	0	1	1	31	26	46	1	43	58	26
171	189	0	24	0	1	1	19	21	15	1	27	59	1
174	186	0	16	0	1	0	58	14	47	1	5	59	36
177	183	0	8	0	1	0	31	7	38	0	35	59	58
180	180	0	0	0	0	0	0	0	0	0	0	60	0

11. Tables for the Anomalies

MERCURY, Apogee, 1°10′ within the Balance

1	2	3		4		5		6		7		8	
Common Numbers		Longitudinal Addition-Subtraction		Differences of Addition-Subtraction		Differences of Subtraction		Addition-Subtraction of Anomaly		Differences of Addition		Sixtieths of Subtraction	
6	354	0	18	0	1	0	10	1	38	0	5	59	20
12	348	0	34	0	2	0	20	3	16	0	11	57	20
18	342	0	51	0	4	0	22	4	53	0	17	54	40
24	336	1	7	0	5	0	39	6	29	0	23	50	40
30	330	1	22	0	5	0	49	8	4	0	28	45	40
36	324	1	37	0	4	0	59	9	36	0	34	39	40
42	318	1	51	0	4	1	8	11	6	0	40	33	0
48	312	2	4	0	3	1	18	12	33	0	45	25	40
54	306	2	15	0	1	1	28	13	58	0	50	18	0
60	300	2	25	0	0	1	39	15	18	0	56	10	20
				Addition									
66	294	2	34	0	2	1	49	16	33	1	4	2	20
												Addition	
72	288	2	41	0	4	1	59	17	43	1	11	9	14
78	282	2	46	0	6	2	9	18	47	1	17	20	0
84	276	2	50	0	7	2	19	19	44	1	23	29	44
90	270	2	52	0	9	2	29	20	33	1	29	39	28
93	267	2	52	0	10	2	34	20	54	1	32	43	31
96	264	2	52	0	10	2	39	21	14	1	35	47	34
99	261	2	51	0	11	2	44	21	29	1	38	50	0
102	258	2	50	0	10	2	48	21	42	1	41	52	26
105	255	2	48	0	10	2	53	21	52	1	44	54	52
108	252	2	46	0	10	2	58	21	59	1	46	57	18
111	249	2	44	0	9	3	2	22	2	1	49	58	23
114	246	2	41	0	9	3	4	22	1	1	52	59	28
117	243	2	37	0	9	3	6	21	56	1	55	59	44
120	240	2	33	0	8	3	8	21	47	1	57	60	0
123	237	2	28	0	7	3	9	21	33	1	59	59	44
126	234	2	23	0	7	3	10	21	15	2	0	59	23
129	231	2	18	0	6	3	12	20	53	2	0	58	39
132	228	2	12	0	6	3	12	20	25	2	1	57	50
135	225	2	6	0	5	3	9	19	50	2	1	56	46
138	222	2	0	0	4	3	6	19	10	2	0	55	41
141	219	1	53	0	4	3	2	18	24	2	0	54	3
144	216	1	46	0	3	2	57	17	32	1	58	52	26
147	213	1	38	0	3	2	51	16	35	1	53	50	48
150	210	1	30	0	2	2	42	15	31	1	47	49	11
153	207	1	22	0	2	2	32	14	20	1	41	47	34
156	204	1	13	0	2	2	21	13	3	1	34	45	57
159	201	1	5	0	1	2	9	11	41	1	26	44	36
162	198	0	56	0	1	1	55	10	13	1	17	43	15
165	195	0	46	0	1	1	38	8	40	1	7	42	26
168	192	0	38	0	0	1	19	7	1	0	56	41	37
171	189	0	28	0	0	1	1	5	19	0	43	40	48
174	186	0	19	0	0	0	42	3	35	0	28	40	0
177	183	0	9	0	0	0	21	1	48	0	14	39	44
180	180	0	0	0	0	0	0	0	0	0	0	39	28

BOOK TWELVE

1. On the Preliminaries to the Regressions

Now that these things have been demonstrated, it would follow that the greatest and least regressions for each of the five planets be examined and that, from the given hypotheses, their sizes be shown as much in accord as possible with those gotten from observations.

For the understanding of this, the other mathematicians and Apollonius of Perga first demonstrate, as in the case of the one anomaly relative to the sun, that if it is taken care of according to the epicyclic hypothesis (with the epicycle making its longitudinal passage eastward about the circle concentric with the ecliptic and with the star on the epicycle making its anomalistic passage about its centre eastward from the apogee), and if a straight line is drawn from our eye cutting the epicycle in such a way that the half of its segment intercepted within the epicycle has to the straight line from our eye to its intersection at the epicycle's perigee the ratio which the speed of the epicycle has to the speed of the star, then the point produced by the straight line so drawn on the epicycle's perigeal arc divides the progressions and regressions so that the star at the point itself makes the appearance of a station.

If the anomaly relative to the sun is taken care of by the hypothesis of eccentricity (this can be done only in the case of the three stars capable of any elongation from the sun), with the eccentric's centre carried eastward about the ecliptic's centre at a speed equal to the sun's and with the star moving on the eccentric about its centre westward at a speed equal to the anomalistic passage, and if a straight line is drawn to the eccentric circle through the ecliptic's centre (that is, through our eye), in such a way that one-half of the whole line has to the lesser of the segments produced by our eye the ratio which the eccentric's speed has to the star's speed, then the star at the point at which the straight line cuts the eccentric's perigeal arc will make the appearance of stations.

Editorial Note. This second theory for the three planets capable of any elongation from the sun is not mentioned anywhere else in the *Almagest*. It is of greater importance than it might at first seem. For it is almost equivalent to the conversion theorem by which one passes from the Ptolemaic theory to the Copernican theory of the outer planets. The ratio of the radius of the circle the eccentric's centre moves on to the eccentric's radius is the same as that of the epicycle's radius to the radius of the deferent in the epicyclic theory. This is apparent from the theorem which follows in the text.

Since this equivalence is so important we shall proceed to give its proof, and then show the immediate resemblance to the conversion theorem. We shall ignore the anomaly of the equant and deferent in this.

Let us consider the epicyclic theory first. Suppose the planet first at A, and then at C. Then angle AEB is the angle of longitude, and angle DBC the angle of anomaly. And angle AEC is the angle of apparent distance between the two positions of the planet.

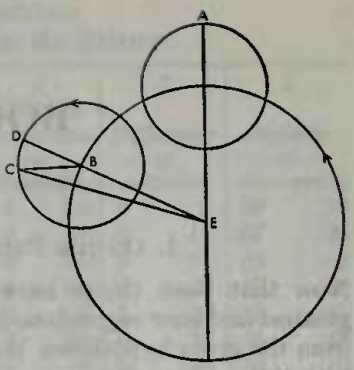
Now

angle AEC = angle AEB = angle BEC

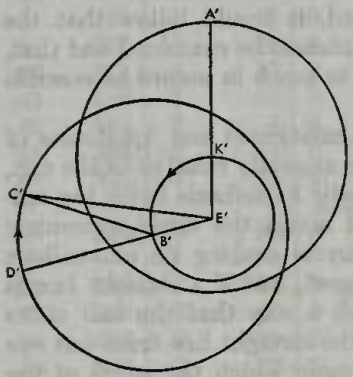
angle AEB = angle CBD — angle BCE

since

angle CBD = angle BCE + angle BEC.



Let us next consider the eccentric hypothesis. Let $K'E'$, the radius of the circle on which the eccentric's centre moves, be equal to the epicycle's radius; and let $A'K'$, the eccentric's radius, be equal to the deferent's radius. Let the planet be first at A' and then at C'. Now, according to the hypothesis, angle $A'E'D'$ is equal to the angle of the mean sun's motion, angle $C'B'D'$ is equal to the angle of anomaly, and angle $A'E'C'$ is the angle of apparent distance.



But

angle $A'E'C'$ = angle $A'E'D'$ — angle $B'C'E'$.

And, since angle of mean sun's movement minus the angle of anomaly is equal to the angle of longitude, therefore

angle of longitude = angle $A'E'D'$ — angle $C'E'B'$ — angle $B'C'E'$.

Hence

angle $A'E'C'$ = angle $A'E'D'$ — angle $C'E'B'$
= angle of longitude + angle $C'B'D'$ — angle $C'E'B'$.

But because triangle $E'B'C'$ is congruent with triangle EBC in the epicyclic hypothesis, therefore

angle $C'E'B'$ = angle BCE,

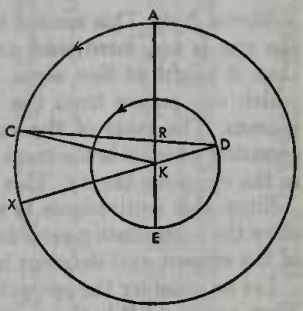
and

angle $A'E'C'$ = angle AEC,

because angle AEB represents the angle of longitude in the epicyclic hypothesis. Hence the angle of apparent distance is the same in both hypotheses.

The step from this last hypothesis to the Copernican is fairly immediate. The centre of the planet's eccentric moves eastward about the earth at the rate of the mean sun. Suppose, then, that the earth moves about the centre of the planet's eccentric at the same rate: nothing will be changed in the appearances. And then the centre of the planet's eccentric would be the mean sun. Then suppose the planet to move about the eccentric's centre at the rate of the longitudinal movement. We can now demonstrate the equivalence of these last two hypotheses.

Thus suppose the earth first at E and the planet first at A; then the earth at D moving about K at the rate of the mean sun and the planet at C moving about K at the mean longitudinal rate of the planet. Thus angle EKD is the movement of the mean sun, angle AKC the longitudinal movement of the planet, and angle ARC the apparent distance between the two positions of the planet.



Now

angle EKD — angle XKC = angle AKC.

But since angle EKD is the mean sun's movement and angle AKC the longitudinal movement, and since the sun's mean movement minus the anomalistic movement is equal to the longitudinal, therefore angle XKC is the

angle of anomaly. Hence angle CKD equals $C'B'E'$ of the hypothesis of the stationary earth, since in both cases these angles are 2 right angles minus the anomalistic angle. Hence the two triangles CKD and $C'B'E'$ are congruent and

$$\text{angle } CDK = \text{angle } C'E'B'.$$

But

$$\begin{aligned}\text{angle } ARC &= \text{angle } AKC + \text{angle } DCK \\ &= \text{angle } AKC + \text{angle } CKX - \text{angle } CDK\end{aligned}$$

And, since angle AKC is the angle of longitude and angle CKX that of anomaly, therefore

$$\text{angle } ARC = \text{angle } A'E'C'$$

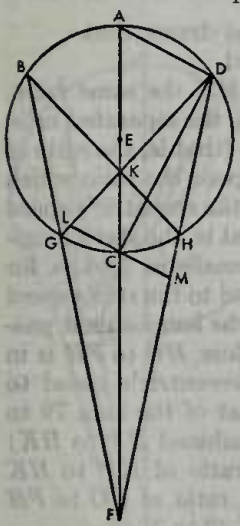
of the former hypothesis. And so in both hypotheses the apparent distances are the same, and they are equivalent for saving the appearances.

The equant and deferent of the Ptolemaic hypothesis is taken care of in the Copernican by making the planet's orbit eccentric to the mean sun and putting the planet on a small epicycle. The proportions are given without explanation by Copernicus in Book V of *The Revolutions of the Celestial Spheres*. Their equivalence will be explained in Appendix B to this book.

The passage from the Ptolemaic to Copernican hypothesis for Venus and Mercury is immediately evident. Hence it is not too daring to say that this hypothesis of eccentricity for the other planets indicates how the Greeks handled their heliocentric theories, no different from the manner of Copernicus. It is not a vague statement Ptolemy is making when he says in Book I of this treatise that the heliocentric theory is simpler for saving astronomical appearances. R. C. T.

Nevertheless we also shall set out briefly and more carefully the required point, using a common proof combined from both hypotheses for showing the accord and similarity in their ratios.

Let there be the epicycle $ABCD$ about centre E, and let AEC be its diameter drawn to F, the ecliptic's centre or our eye. With arcs CG and CH cut off equal on either side of the perigee C, let FGB and FHD be drawn from F through points G and H. And let DG and BH be joined, cutting each other at point K which will clearly fall on the diameter AC.



We say first that

$$AF : CF :: AK : CK.$$

For let AD and DC be joined. And through C parallel to AD let LCM be drawn, clearly at right angles to DC, since angle ADC is also right. Now since

$$\text{angle } CDG = \text{angle } CDH,$$

therefore

$$CL = CM.$$

And therefore

$$AD : CL :: AD : CM.$$

But

$$AD : CM :: AF : CF,$$

and

$$AD : CL :: AK : CK.$$

And therefore

$$AF : CF :: AK : CK.$$

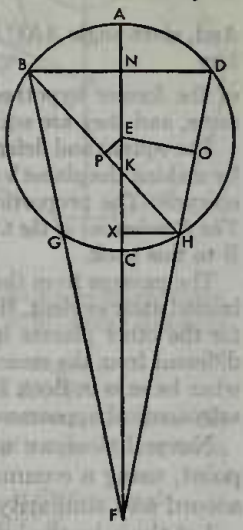
Therefore, if we conceive the epicycle $ABCD$ as the eccentric in the hypothesis of eccentricity, the point K will be the ecliptic's centre and the diameter AC will be divided by it in the same ratio of the epicyclic hypothesis, since we showed that, for the epicycle, whatever ratio the greatest distance AF has

to the least distance CF, that ratio, for the eccentric, the greatest distance AK has to the least distance CK.

And we say that also

$$DF : FH :: BK : HK.$$

For in a similar drawing let there be joined the straight line BND clearly at right angles to diameter AC; and from H let HX be drawn parallel to it.



Since, then,

$$BN = DN,$$

therefore

$$BN : HX :: DN : HX.$$

But

$$DN : HX :: DF : FH,$$

and

$$BN : HX :: BK : HK.$$

And therefore

$$DF : FH :: BK : HK.$$

And *componendo*

$$DF + FH : FH :: BH : HK,$$

and *dividendo*, with the perpendiculars EO and EP drawn,

$$FO : FH :: HP : HK.$$

And further, *separando*,

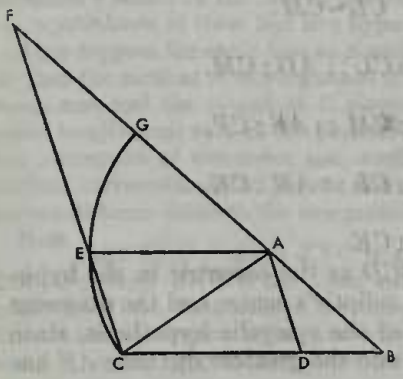
$$HO : FH :: KP : HK.$$

If therefore, in the epicyclic hypothesis, DF has been so drawn that

$$HO : FH :: \text{epicycle's speed} : \text{star's speed},$$

then, in the hypothesis of eccentricity, KP will have to HK the same ratio.

And the reason for not having used here, for the stations, the separated ratio (that is, the ratio of KP to HK) but the unseparated ratio (that is, the ratio of HP to HK) is that the epicycle's speed has to the star's speed the ratio which the longitudinal passage alone has to the anomalistic; but the eccentric's speed has to the star's the ratio which the sun's mean passage (that is, the star's longitudinal and anomalistic passages combined) has to the anomalistic. And so, for example, in the case of Mars, the ratio of the epicycle's speed to the star's speed is very nearly 42 to 37. For we demonstrated the ratio of the longitudinal passage to the anomalistic to be very nearly that. And, therefore, HO to FH is in that ratio. But the eccentric's speed to the star's speed is that of the sum 79 to 37 (that is, of the combined HP to HK) since the separated ratio of KP to HK was the same as the ratio of HO to FH that is, as the ratio of 42 to 37.



Let us consider the things up to this point as preliminary theorems. It is left to show that, when the straight lines taken in either hypothesis are divided in this ratio, then the points G and H will embrace the appearances of stations, and the arc GCH is necessarily regressive and the rest progressive. And Apollonius first

takes this little lemma that, if, in the triangle ABC having BC greater than AC, CD is cut off not less than AC, then

$$CD : BD > \text{angle } ABC \text{ angle } BCA.$$

And he proves it in this way. For, he says, let the parallelogram $ADCE$ be completed, and let AB and CE produced meet at point F. Since AE is not less than AC, the circle drawn about centre A and with radius AE will pass either through C or beyond C. Then let the circle GEC be drawn through C.

And since

$$\text{trgl. } AEF > \text{sect. } AEG,$$

and

$$\text{trgl. } AEC < \text{sect. } AEC,$$

therefore

$$\text{trgl. } AEF : \text{trgl. } AEC > \text{sect. } AEG : \text{sect. } AEC.$$

But

$$\text{sect. } AEG : \text{sect. } AEC :: \text{angle } EAF : \text{angle } EAC,$$

and

$$\text{trgl. } AEF : \text{trgl. } AEC :: \text{base } EF : \text{base } CE;$$

therefore

$$EF : CE > \text{angle } EAF : \text{angle } EAC.$$

But

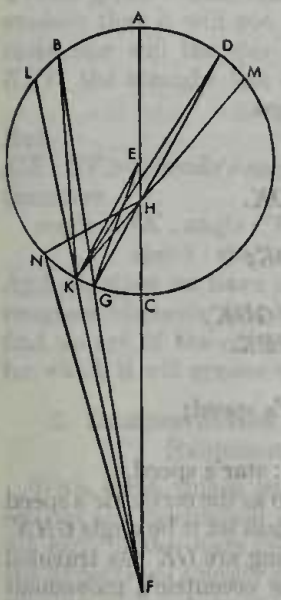
$$EF : CE :: CD : BD,$$

and

$$\text{angle } EAF = \text{angle } ABC,$$
$$\text{angle } EAC = \text{angle } ACB.$$

Therefore

$$CD : BD > \text{angle } ABC : \text{angle } ACB.$$



And it is evident that the ratio will be much greater if CD or AE is not supposed equal to AC, but greater.

Now, with this first understood, let there be the epicycle $ABCD$ about centre E and diameter AEC. And let the diameter be extended to point F our eye in such a way that

$$CE : CF > \text{epicycle's speed : star's speed.}$$

Therefore it is possible to draw through straight line FGB in such a way that

$$\text{half } BG : FG :: \text{epicycle's speed : star's speed.}^1$$

If, because of things already proved, we take arc AD equal to arc AB and join DHG, then point H in the hypothesis of eccentricity will be conceived as our eye and

$$\text{half } DG : HG :: \text{eccentric's speed : star's speed.}$$

Then we say that the star at point G, in either hypothesis, will effect the appearance of a station; and, whatever arc we take on either side of G, we shall find the arc taken towards the apogee progressive and that towards the perigee regressive.

For first let there be cut off an arc at random KG

[1]For BG takes on every magnitude between CA and nothing, and FG increases from FC to the length of the tangent from F to the epicycle. Therefore the ratio of half BG to FG can be any ratio less than CE to CF. This can be proved on Euclidean principles similar to, and yet different from, those of Dedekind.

towards the apogee. And let *FKL* and *KHM* be drawn through; and let *BK* and *DK*, and also *EK* and *EG*, be joined. Since, then, in triangle *BKF*,

$$BG > BK,$$

therefore

$$BG : FG > \text{angle } GFK : \text{angle } GBK.$$

And so also

$$\text{half } BG : FG > \text{angle } GFK : 2 \text{ angle } GBK,$$
$$> \text{angle } GFK : \text{angle } GEK.$$

But

$$\text{half } BG : FG :: \text{epicycle's speed} : \text{star's speed};$$

therefore

$$\text{angle } GFK : \text{angle } GEK < \text{epicycle's speed} : \text{star's speed}.$$

Therefore the angle, having to angle *GEK* the same ratio as the epicycle's speed to the star's speed, is greater than angle *GFK*. Then let it be angle *GFN*.

Since, then, while the star moves through the epicycle's arc *KG*, the epicycle's centre has made a passage in the contrary direction equal to the distance from *FG* to *FN*, it is evident that in the same time the epicycle's arc *KG* has carried the star westward through an angle at our eye *GFK* less than that through which the epicycle has moved it eastward—that is, angle *GFN*. And so the star has progressed through angle *KFN*.

Likewise also, if we reason it out with the eccentric circle, since

$$BG : FG > \text{angle } GFK : \text{angle } GBK,$$

therefore *componendo*

$$BF : FG > \text{angle } BKL : \text{angle } GBK.$$

But

$$BF : FG :: DH : GH,$$

and

$$\text{angle } BKL = \text{angle } DKM,^{1}$$
$$\text{angle } GBK = \text{angle } GDK.$$

Therefore

$$DH : GH > \text{angle } DKM : \text{angle } GDK.$$

And so also *componendo*

$$DG : GH > \text{angle } GHK : \text{angle } GDK;$$

and therefore *dividendo*

$$\text{half } DG : GH > \text{angle } GHK : 2 \text{ angle } GDK,$$
$$> \text{angle } GHK : \text{angle } GEK.$$

But

$$\text{half } DG : GH :: \text{eccentric's speed} : \text{star's speed};$$

therefore

$$\text{angle } GHK : \text{angle } GEK < \text{eccentric's speed} : \text{star's speed}.$$

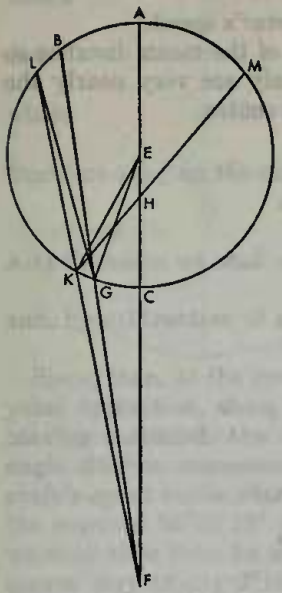
Therefore the angle, having to angle *GEK* the same ratio as the eccentric's speed to the star's speed, is greater than angle *GHK*. Then again let it be angle *GHN*.

Since, then, in the same time, the star itself describing arc *GK* has traveled westward angle *GEK*, and is carried eastward by the eccentric's movement through angle *GHN* which is greater than angle *GHK*, it is evident that the star will thus appear to progress through *KHN*.

[1]For arc *LB* is equal to arc *DM*, since, in the eccentric theory, the star moves on the eccentric according to the anomalistic speed—that is, as the star moves on the epicycle in the epicyclic theory.

It can be seen at once that by the same means the contrary will also be shown, if in the same drawing we suppose

half $KL : FK$: : epicycle's speed : star's speed,



so that

half $KM : HK$: : eccentric's speed : star's speed,

if we conceive arc GK as having been cut off in the direction of the perigee from straight line FL. For, with LG joined and producing triangle LFG in which FK is cut off greater than FG,

$KL : FK <$ angle GFK : angle GLK.

And so also

half $KL : FK <$ angle GFK : 2 angle GLK
$<$ angle GFK : angle GEK,

conversely to what was shown before. By the same means the contrary can be concluded that

angle GEK : angle $GFK <$ star's speed : epicycle's speed,

and

angle GEK : angle $GHK <$ star's speed : eccentric's speed.

And so, since an angle greater than angle GEK would have the same ratio, therefore the regressive shift would turn out greater than the progressive.

For those distances where CE does not have to CF a ratio greater than the ratio of the epicycle's speed to the star's speed, it is evident that it will not be possible to draw another line through in the same ratio, nor will the star appear to pause or to regress. For since, in triangle EKF, the straight line CE is not cut off less than EK, therefore

angle CFK : angle $CEK < CE : CF$.

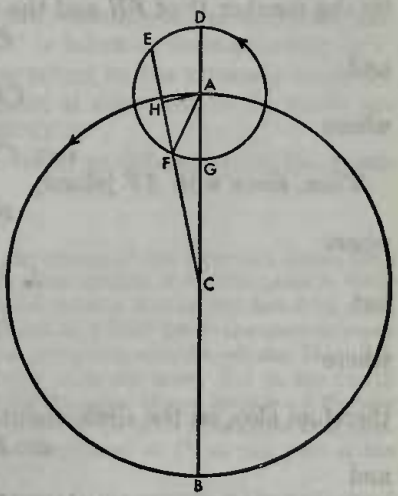
But

$CE : CF >$ epicycle's speed : star's speed;

therefore

angle CFK : angle $CEK <$ epicycle's speed : star's speed.

And so, since we have shown the star to progress wherever this happens, we shall find no arc of the epicycle and eccentric for which it will appear to regress.



2. Demonstration of Saturn's Regressions

With these things so, we shall finally set out the calculation of the regressions for each of the stars in accordance with the hypotheses already demonstrated, beginning with Saturn in this way.

Let the circle bearing the epicycle's centre be the circle AB about diameter ACB, on which let the ecliptic's centre

or our eye be supposed. And with the epicycle *DEFG* drawn about centre *A*, let the straight line *CFE* be drawn through in such a way that, if *AH* is drawn perpendicular to it, then

half *EF* or *FH* : *CF* : : epicycle's speed : star's speed.

And first let the epicycle be supposed in the position of the mean distance so that the periodic movements of longitude and anomaly are very nearly the same as those considered with respect to the ecliptic's centre.

Now since, in the case of Saturn, we showed

epic. rad. $AD = 6\frac{1}{2}^{p}$

where the mean distance *AC* equals 60^{p},

so that, by addition,

$$CD = 66^{p}30',$$

and, by subtraction,

$$CG = 53^{p}30',$$

and

$$\text{rect. } CD, CG = 3{,}557^{p}45',$$

and

$$\text{rect. } CD, CG = \text{rect. } CE, CF,$$

therefore we shall also have

$$\text{rect. } CE, CF = 3{,}557^{p}45'.$$

Again, since, according to the mean passages,

star's speed or $CF \fallingdotseq 28^{p}25'\ 46''$

where

epicycle's speed or $FH = 1^{p}$,

so that, by addition,

$$CE = 30^{p}25'46''$$

and

$$\text{rect. } CE, CF = 865^{p}5'32'',$$

therefore, if we apply [divide] $3{,}557^{p}45'$ to [by] $865^{p}5'32''$, take the square root $2^{p}1'40''$ of the $4^{p}6'45''$ resulting from the application, and multiply it by the number 1^{p} of *FH* and the number $28^{p}25'46''$ of *CF*, then we shall have

$$FH = 2^{p}1'40''$$

and

$$CF = 57^{p}38'55''$$

where

$$\text{rect. } CE, CF = 3{,}557^{p}45'.$$

Then, since with *AF* joined,

$$FH = 2^{p}1'40''$$

where

$$AF = 6^{p}30',$$

but

$$FH = 37^{p}26'9''$$

where

$$AF = 120^{p},$$

therefore also, on the circle about right triangle *AFH*,

$$\text{arc } FH = 36°21'15'',$$

and

$$\text{angle } FAH = 36°21'15'' \text{ to 2 rt.} \fallingdotseq 18°10'38''.$$

Again, since by addition

$$CFH = 59^{p}40'35''$$

where

$$\text{hypt. } CGA = 60^{p},$$

and

$$CFH = 119^{p}21'10''$$

where

$$\text{hypt. } CGA = 120^{p},$$

therefore also, on the circle about right triangle ACH,

$$\text{angle } CAH = 168°5'39'' \text{ to 2 rt.}$$
$$\fallingdotseq 84°2'50''.$$

And therefore we shall also have, by subtraction, from a right angle,

$$\text{angle } ACH = 5°57'10'',$$

and, by subtraction of angle FAH,

$$\text{angle } FAG = 65°52'12''.$$

Since, then, at the first station, the star appears along CF and, at the acronychal opposition, along CG, it is clear that, if the epicycle's centre were not moving eastward, the 65°52′12″ of arc FG would embrace the 5°57′10″ of angle ACF in regression. But since, according to the given ratio of the epicycle's speed to the star's speed, very nearly 2°19′ of longitude corresponds to the required 65°52′12″ of anomaly, from one of the stations to the opposition we shall thus have by subtraction 3°38′10″ and the 69 days in which the star moves very nearly 2°19′ of periodic longitude; and we shall have the whole regression of 7°16′20″ and 138 days.

And next we shall examine with the same means the quantities about the greatest distance; that is, when the acronychal opposition midway the stations puts the epicycle's centre at the eccentric's apogee and when each of the stations is clearly at a distance somewhere near the 2°19′ demonstrated for the mean distance, from the opposition or the apogee of the longitude truly determined.[1] At this position the straight line AC of the distance for that time is taken as indistinguishable from the greatest, because of theorems already given by us; and an addition-subtraction of very nearly 0°6′30″ is taken as corresponding to 1° of longitude. And so the longitude truly determined to the anomaly truly determined—that is, the epicycle's apparent speed at that time to the star's apparent speed[2] is in the ratio of 0°53′30″ to 28°32′16″.

Now, with the same drawing set out, since, being no different from the greatest distance,

$$AC = 63^{p}25'$$

[1]To find the longitude truly determined—that is, the change of the epicycle's centre for a particular part of the ecliptic—Ptolemy here uses the same method as for the moon in Book VI, Chap. 4. Going to the Tables of Anomaly in Book XI, we find that for the first 6° in mean longitude there is, in the third column, an addition-subtraction of 0°37′ due to the epicycle's centre being moved regularly with respect to an equant not concentric with the ecliptic. This addition-subtraction is taken as if the equant and deferent were the same. But in the fourth column we find an addition of 0°2′ due to their not being the same. Hence there is a difference of 0°39′ for 6° of the mean longitude, or a difference of 0°6½′ for each 1° of mean longitude. Therefore, while the epicycle's centre makes a mean movement of 1°, at this part of the ecliptic it makes an apparent movement of only 0°53½′.

[2]The anomaly truly determined or star's apparent speed on the epicycle is the star's speed taken over a certain distance relative to the epicycle's apparent apogee and perigee rather than its regular apogee and perigee. In planetary theory, the apparent apogee is that defined

where

ecc. rad. $AD = 6^{p}30'$;

and since for that reason, by addition,

$CD = 69^{p}55'$

and, by subtraction,

$CG = 56^{p}55'$

and

rect. CE, CF = rect. CD, CG
$= 3{,}979^{p}25'25''$;

and since

star's speed or $CF = 28^{p}32'16''$

where it is supposed

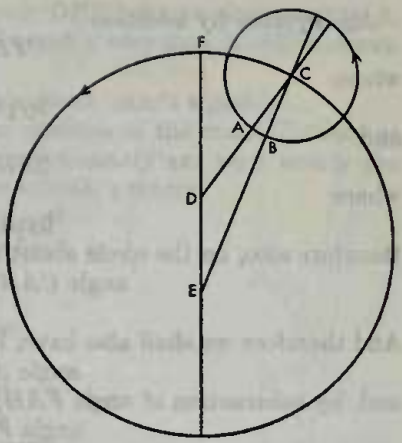
epicycle's speed or $FH = 0^{p}53'30''$

and, by addition,

$CE = 30^{p}19'16''$

and

rect. $CE, CF = 865^{p}17'50''$



therefore, if again we apply [divide] $3{,}979^{p}25'25''$ to [by] $865^{p}17'50''$, and if we multiply the square root $2^{p}8'40''$ of the $4^{p}35'56''$ resulting from the application by the $0^{p}53'30''$ of straight line FH and the $28^{p}32'16''$ of CF each separately, then we shall have

$FH = 1^{p}54'44''$,
$AC = 63^{p}25'$,
$CF = 61^{p}11'52''$,

and, by addition,

$CH = 63^{p}6'36''$

where

$AF = 6^{p}30'$.

And therefore

$FH = 35^{p}18'9''$

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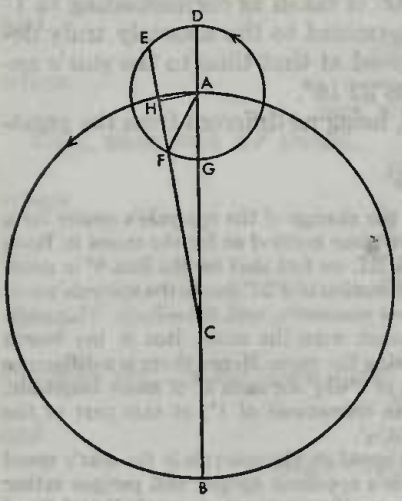
with respect to the ecliptic's centre or the earth; the regular apogee is that defined with respect to the equant's centre.



If, now, we wish to get the star's speed from a station preceding the perigee to an acronychal opposition at the greatest distance, consider the figure where E is the ecliptic's centre, D the equant's centre, and F the apogee of eccentricity. Furthermore, A is the epicycle's regular perigee and B its apparent perigee. Now consider the star moving on the epicycle toward A from the side opposite B, while the epicycle's centre C moves toward F. But as C moves towards F, the apparent apogee B moves towards the regular apogee A. Hence the star moves faster toward B than it does toward A. But its regular anomalistic movement is with respect to A, and therefore we add the difference of angle ACB to the regular anomalistic movement for the period of longitudinal movement from C to F, or, as we found above for Saturn, an average of $0°6\frac{1}{2}'$ for each 1° of mean longitude and for each $0°53\frac{1}{2}'$ of longitude truly determined.

About the least distance, the difference of apparent anomaly is subtractive for the same reasons.

where

hypt. $AF = 120^{p}$;

and

$CH = 119^{p}25'11''$

where

hypt. $AC = 120^{p}$.

And therefore, on the circle about right triangle AFH,

arc $FH = 34°13'4''$;

and, on the circle about right triangle ACH,

arc $CH = 168°43'38''$.

And therefore

angle $FAH = 34°13'4''$ to 2 rt.
$= 17°6'32''$;

and

angle $CAH = 168°43'38''$ to 2 rt.
$= 84°21'49''$.

And so, by subtraction, we would have

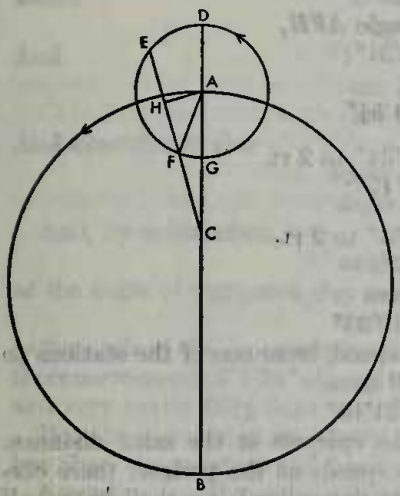
angle $ACH = 5°38'11''$

as the angle from one of the stations to the opposition, if the epicycle did not lag behind the regression; and, by subtraction, we would have

angle $FAG = 67°15'17''$

as the apparent passage on the epicycle for that distance. Since to these angles, according to the ratios of the speeds at the apogee, there corresponds 2°6′6″ of truly determined longitude, we shall have, by subtraction, half of the whole regression as 3°32′5″ and 70⅓ days in which the star very nearly moves the periodic 2°21′25″ corresponding to the 2°6′6″ of truly determined longitude required; and we shall have the whole regression as 7°4′10″ and 140⅔ days.

Again, through similar means and with the same drawing, we shall examine the quantities about the least distance, when the acronychal opposition midway the stations is at the eccentric's perigee and each of the stations is somewhere near the given longitudinal distance from the opposition or perigee. And AC, the distance at that time, is taken in this position as not differing from the least distance, and the addition-subtraction corresponding to 1° of longitude is taken as very nearly 0°7′20″. And so here the epicycle's apparent speed to the star's apparent speed is in the ratio of 1°7′20″ to 28°18′26″, and therefore



$CF = 28^{p}18'26''$

where

$FH = 1^{p}7'20''$,

and by addition

$CE = 30^{p}33'6''$,

and

rect. $CE, CF = 864^{p}49'50''$.

Now since also, not differing from the least distance,

$$AC = 56^{p}35'$$

where

$$\text{epic. rad. } AD = 6^{p}30';$$

and therefore by addition

$$CD = 63^{p}5'$$

and by subtraction

$$CG = 50^{p}5'$$

and

$$\text{rect. } CE, CF = \text{rect. } CD, CG = 3{,}159^{p}25'25'',$$

therefore, if in the same way we apply [divide] $3{,}159^{p}25'25''$ to [by] $864^{p}49'58''$ and, taking the square root $1^{p}54'42''$ of the $3^{p}39'12''$ resulting from the application, multiply by the $1^{p}7'20''$ of the straight line FH and the $28^{p}18'26''$ of CF, each separately, then we shall have

$$FH = 2^{p}8'43'',$$

$$CF = 54^{p}6'22'',$$

and by addition

$$CH = 56^{p}15'5''$$

where

$$\text{epic. rad. } AF = 6^{p}30'$$

and, as the distance at that time,

$$AC = 56^{p}35'.$$

And therefore

$$FH = 39^{p}36'18''$$

where

$$\text{hypt. } HF = 120^{p};$$

and

$$CH = 119^{p}17'46''$$

where

$$\text{hypt. } AC = 120^{p}.$$

And therefore, on the circle about right triangle AFH,

$$\text{arc } FH = 38°32'34'';$$

and, on the circle about right triangle ACH,

$$\text{arc } CH = 167°34'54''.$$

And so also

$$\text{angle } FAH = 38°32'34'' \text{ to 2 rt.} = 19°16'17'';$$

and

$$\text{angle } CAH = 167°34'54'' \text{ to 2 rt.} = 83°47'27''.$$

And therefore, by subtraction, we shall have

$$\text{angle } ACH = 6°12'33''$$

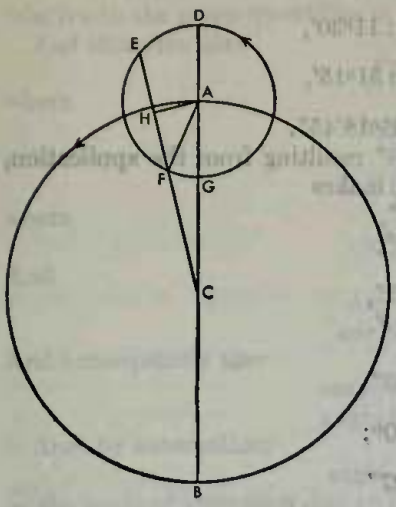
as the angle of regression, due to the star's speed, from one of the stations to the opposition; and, by subtraction, we shall have

$$\text{angle } FAG = 64°31'10''$$

as the angle of the apparent passage on the epicycle at the same distance. Since to this, according to the ratio of the speeds at the perigee, there corresponds the 2°33′28″ of the longitude truly determined, we shall have half the whole regression of 3°39′5″ and 68 days in which the star very nearly makes a mean movement of 2°16′45″ corresponding to the given 2°33′28″ of the longitude truly determined; and we shall have the whole regression of 7°18′10″ and 136 days.

3. Demonstration of Jupiter's Regressions

In the case of Jupiter, according to the calculations for the mean distance,



$$FH : CF :: 1^{p} : 10^{p}51'29'',$$

and

$$CE : CF :: 12^{p}51'29'' : 10^{p}51'29'',$$

and

$$\text{rect. } CE, CF = 139^{p}37'39'';$$

and again

$$AC : AD :: 60^{p} : 11^{p}30',$$

and

$$CD : CG :: 71^{p}30' : 48^{p}30',$$

and

$$\text{rect. } CD, CG = 3{,}467^{p}45'.$$

And the square root $4^{p}59'1''$ of the $24^{p}50'9''$ resulting from the application, multiplied by the given ratio of FH and CF, makes

$$FH = 4^{p}59'1'',$$

$$CF = 54^{p}6'44'',$$

and by addition

$$CH = 59^{p}5'45'',$$

relative to the given quantities of AC and AF.

And therefore

$$FH = 52^{p}0'10''$$

where

$$\text{hypt. } AF = 120^{p};$$

and

$$CH = 118^{p}11'30''$$

where

$$\text{hypt. } AC = 120^{p}.$$

And

$$\text{arc } FH = 51°21'41'',$$

$$\text{arc } CH = 160°4'55''.$$

And accordingly also

$$\text{angle } FAH = 25°40'50'',$$

$$\text{angle } CAH = 80°2'28''.$$

And, by subtraction,

$$\text{angle } FCA = 9°57'32''$$

as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 54°21'38''$$

as the angle of the apparent anomaly. Since to this, according to the given ratio, there corresponds $5°1'24''$ of longitudinal passage, half of the regression is $4°56'8''$ and very nearly $60\frac{1}{2}$ days; and the whole regression is $9°52'16''$ and 121 days.

But the distance at an interval of 5° from the apogee and perigee is hardly less than the greatest and hardly greater than the least.

According to the calculations near the greatest distance, the addition-subtraction of true determination is found to be $0°5'6''$. And therefore

$$FH : CF :: 0^{p}54'50'' : 10^{p}56'39'',$$

and

$$CE : CF :: 12^{p}46'19'' : 10^{p}56'39'',$$

and

$$\text{rect. } CE, CF = 139^{p}46'42''.$$

And again

$$AC : AD :: 62^{p}45' : 11^{p}30',$$

and

$$CD : CG :: 74^{p}15' : 51^{p}15',$$

and

$$\text{rect. } CD, CG = 3{,}805^{p}18'45''.$$

And the square root $5^{p}13'4''$ of the $27^{p}13'26''$ resulting from the application, multiplied by the given ratio of FH and CF, makes

$$FH = 4^{p}46'6'',$$
$$CF = 57^{p}6'19'',$$

and, by addition,

$$CH = 61^{p}52'25'',$$

relative to the given quantities of AC and AF.

And therefore also

$$FH = 49^{p}45'23''$$

where

$$\text{hypt. } AF = 120^{p};$$

and

$$CH = 118^{p}19'27''$$

where

$$\text{hypt. } AC = 120^{p}.$$

and

$$\text{arc } FH = 48°59'34'',$$
$$\text{arc } CH = 160°49'36''.$$

And consequently

$$\text{angle } FAH = 24°29'47'',$$
$$\text{angle } CAH = 80°24'48''.$$

And, by subtraction,

$$\text{angle } FCA = 9°35'12''$$

as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 55°55'1''$$

as the angle of apparent anomaly. Since to this, according to the apogeal ratios, there corresponds $4°40'35''$ of longitude truly determined and $5°6'35''$ of periodic longitude, half of the regression is $4°54'37''$ and very nearly $61\frac{1}{2}$ days, and the whole regression is $9°49'14''$ and 123 days.

According to the calculations near the least distance, the addition-subtraction of true determination is found to be $0°5\frac{2}{3}'$. And therefore

$$FH : CF :: 1^{p}5'40'' : 10^{p}45'49'',$$

and

$$CE : CF :: 12^{p}57'9'' : 10^{p}45'49'',$$

and

$$\text{rect. } CE, CF = 139^{p}24'56''.$$

And again

$$AC : AD :: 57^{p}15' : 11^{p}30',$$

and

$$CD : CG :: 68^{p}45' : 45^{p}45',$$

and

$$\text{rect. } CD, CG = 3{,}145^{p}18'45''.$$

And the square root $4^{p}45'0''$ of the $22^{p}33'39''$ resulting from the application, multiplied by the given ratio of FH and CF, makes

$$FH = 5^{p}11'55'',$$
$$CF = 51^{p}7'38'',$$

and, by addition,

$$CH = 56^{p}19'33'',$$

relative to the given quantities of AC and AF.

And therefore also

$$FH = 54^{p}14'47''$$

where

$$\text{hypt. } AF = 120^{p};$$

and

$$CH = 118^{p}3'46''$$

where

$$\text{hypt. } AC = 120^{p}.$$

And

$$\text{arc } FH = 53°45'4'',$$
$$\text{arc } CH = 159°22'40''.$$

And consequently also

$$\text{angle } FAH = 26°52'32'',$$
$$\text{angle } CAH = 79°41'20''.$$

And, by subtraction,

$$\text{angle } FCA = 10°18'40''$$

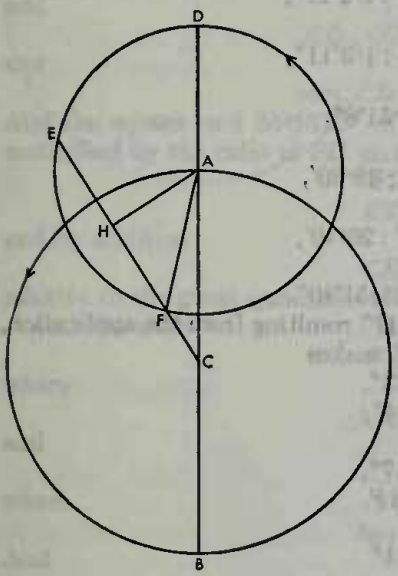
as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 52°48'48''$$

as the angle of the apparent anomaly. Since to this, according to the ratios at the perigee, there corresponds 5°21′20″ of longitude truly determined and 4°54′20″ of periodic longitude, half of the regression comes to 4°57′20″ and very nearly 59 days; and the whole regression comes to 9°54′40″ and 118 days.

4. Demonstration of Mars' Regressions

Again, in the case of Mars, according to the calculations for the mean distance



$$FH : CF :: 1^{p} : 0^{p}52'51'',$$

and

$$CE : CF :: 2^{p}52'51'' : 0^{p}52'51'',$$

and

$$\text{rect. } CE, CF = 2^{p}32'15''.$$

And again

$$AC : AG :: 60^{p} : 39^{p}30',$$

and

$$CD : CG :: 99^{p}30' : 20^{p}30',$$

and

$$\text{rect. } CD, CG = 2{,}039^{p}45'.$$

And the square root 28ᵖ21′8″ of the 803ᵖ 50′50″ resulting from the application, multiplied by the given ratio of FH and CF, makes

$$FH = 28^{p}21'8'',$$
$$CF = 24^{p}58'25'',$$

and, by addition,

$$CH = 53^{p}19'33'',$$

relative to the given quantities of AC and AF.

And therefore also

$$FH = 86^{p}8'0''$$

where

$$\text{hypt. } AF = 120^{p};$$

and

$$CH = 106^{p}39'6''$$

where

$$\text{hypt. } AC = 120^{p}.$$

And

$$\text{arc } FH = 91°44'34'',$$
$$\text{arc } CH = 125°26'10''.$$

And consequently also

$$\text{angle } FAH = 45°52'17'',$$
$$\text{angle } CAH = 62°43'5''.$$

And, by subtraction,

$$\text{angle } FCA = 27°16'55''$$

as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 16°50'48''$$

as the angle of the anomaly. Since to this, according to the given ratio, there corresponds 19°7′33″ of longitudinal passage, half of the regression is 8°9′22″ and very nearly 36½ days; and the whole regression is 16°18′44″ and 73 days.

But the distance, at the interval of the stations from the apogee and perigee, is less than the greatest and greater than the least by very nearly 20 sixtieths.

And according to the calculations near the greatest distance, the addition-subtraction of true determination is found to be 0°10⅓′ in proportion to 1°. And therefore

$$FH : CF :: 0^{p}49'40'' : 1^{p}3'11'',$$

and

$$CE : CF :: 2^{p}42'31'' : 1^{p}3'11'',$$

and

$$\text{rect. } CE, CF = 2^{p}51'8''.$$

And again

$$AC : AG :: 65^{p}40' : 39^{p}30',$$

and

$$CD : CG :: 105^{p}10' : 26^{p}10',$$

and

$$\text{rect. } CD, CG = 2{,}751^{p}51'40''.$$

And the square root 31ᵖ3′41″ of the 964ᵖ48′47″ resulting from the application, multiplied by the given ratio of FH and CF, makes

$$FH = 25^{p}42'43'',$$
$$CF = 32^{p}42'34'',$$

and, by addition,

$$CH = 58^{p}25'17'',$$

relative to the given quantities of AC and AF.

And therefore also

$$FH = 78^{p}6'44''$$

where

$$\text{hypt. } AF = 120^{p};$$

and

$$CH = 106^{p}45'36''$$

where

$$\text{hypt. } AC = 120^{p}.$$

And

$$\text{arc } FH = 81°13'8'',$$
$$\text{arc } CH = 125°39'46''.$$

And consequently

$$\text{angle } FAH = 40°36'34'',$$
$$\text{angle } CAH = 62°49'53''.$$

And, by subtraction,

$$\text{angle } FCA = 27°10'7''$$

as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 22°13'19''$$

as the angle of the apparent anomaly. Since to this, according to the apogee's ratios, there corresponds 17°13′21″ of longitude truly determined and 20°58′21″ of periodic longitude, half of the regression comes to 9°56′46″ and very nearly 40 days; and the whole regression comes to 19°53′32″ and 80 days.

According to the calculations near the least distance, the addition-subtraction of true determination is found to be $0°12\frac{2}{3}'$.

And therefore

$$FH : CF :: 1^{p}12'40'' : 0^{p}40'11'',$$

and

$$CE : CF :: 3^{p}5'31'' : 0^{p}40'11'',$$

and

$$\text{rect. } CE, CF = 2^{p}4'14''.$$

And again

$$AC : AG :: 54^{p}20' : 39^{p}30',$$

and

$$CD : CG :: 93^{p}50' : 14^{p}50',$$

and

$$\text{rect. } CD, CG = 1{,}391^{p}51'40''.$$

And the square root $25^{p}55'38''$ of the $672^{p}12'$ resulting from the application, multiplied by the ratio of FH and CF, makes

$$FH = 31^{p}24'3'',$$
$$CF = 17^{p}21'51'',$$

and by addition

$$CH = 48^{p}45'54'',$$

relative to the given quantities of AC and AF.

And therefore

$$FH = 95^{p}23'42''$$

where

$$\text{hypt. } AF = 120^{p};$$

and

$$CH = 107^{p}42'7''$$

where

$$\text{hypt. } AC = 120^{p}.$$

And

$$\text{arc } FH = 105°18'10'',$$
$$\text{arc } CH = 127°40'22''.$$

And consequently

angle $FAH = 52°39'5''$
angle $CAH = 63°50'11''$.

And, by subtraction,

angle $FCA = 26°9'49''$

as the angle of regression due to the star's speed; and

angle $FAG = 11°11'6''$

as the angle of the apparent anomaly. Since to this, according to the ratios at the perigee, there corresponds $20°33'42''$ of longitude truly determined and $16°52'52''$ of periodic longitude, half of the regression comes to $5°36'7''$ and very nearly $32\frac{1}{4}$ days; and the whole regression comes to $11°12'14''$ and $64\frac{1}{2}$ days.

5. Demonstration of Venus' Regressions

Again, in the case of Venus, according to calculations near the mean distance,

$FH : CF :: 1^p : 0^p37'31''$,

and

$CE : CF :: 2^p37'31'' : 0^p37'31''$,

and

rect. CE, $CF = 1^p38'30''$.

And again

$AC : AG :: 60^p : 43^p10'$,

and

$CD : CG :: 103^p10' : 16^p50'$,

and

rect. CD, $CG = 1{,}736^p38'20''$.

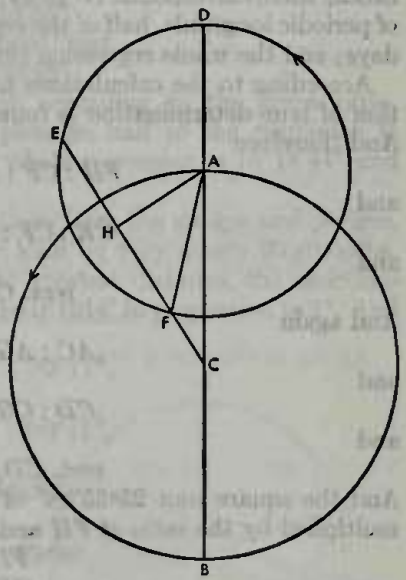
And the square root $32^p31'29''$ of the $105^p50'6''$ resulting from the application, multiplied by the given ratio of FH and CF, makes

$FH = 32^p31'29''$,
$CF = 20^p20'11''$,

and, by addition,

$CH = 52^p51'40''$,

relative to the given quantities of AC and AF.



And therefore also

$FH = 90^p24'58''$,

where

hypt. $AF = 120^p$;

and

$CH = 105^p43'20''$

where

hypt. $AC = 120^p$.

And

arc $FH = 97°47'0''$,
arc $CH = 123°31'49''$.

And consequently also

angle $FAH = 48°53'30''$,
angle $CAH = 61°45'54''$.

And, by subtraction,

angle $FCA = 28°14'6''$

as the angle of regression due to the star's speed; and

angle $FAG = 12°52'24''$

as the angle of the anomaly. Since to this, according to the given mean ratio, there corresponds $20°35'19''$ of the longitudinal passage, half of the regression comes to $7°38'47''$ and very nearly $20+\frac{1}{2}+\frac{1}{3}$ days; and the whole regression comes to $15°17'34''$ and $41\frac{2}{3}$ days.

And the distance at the interval of the stations from the apogee and perigee is less than the greatest and greater than the least, by very nearly 5 sixtieths of the mean distance.

According to the calculations near the greatest distance, the addition-subtraction of true determination is found to be $0°2\frac{1}{3}'$. And therefore

$$FH : CF :: 0^p57'40'' : 0^p39'51'',$$
$$CE : CF :: 2^p35'11'' : 0^p39'51'',$$

and

rect. CE, $CF = 1^p43'4''$.

And again

$$AC : AG :: 61^p10' : 43^p10',$$

and

$$CD : CG :: 104^p20' : 18^p0',$$

and

rect. CD, $CG = 1{,}878^p0'$.

And the square root $33^p3'53''$ of the $1{,}093^p16'23''$ resulting from the application, multiplied by the given ratio of FH and CF, makes

$$FH = 31^p46'44'',$$
$$CF = 21^p57'38'',$$

and, by addition,

$$CH = 53^p44'22'',$$

relative to the given quantities of AC and AF.

And therefore also

$$FH = 88^p20'34''$$

where

hypt. $AF = 120^p$;

and

$$CH = 105^p25'44''$$

where

hypt. $AC = 120^p$.

And

arc $FH = 94°48'54''$,
arc $CH = 122°56'27''$.

And consequently

angle $FAH = 47°24'27''$,
angle $CAH = 61°28'14''$.

And, by subtraction,

angle $FCA = 28°31'46''$

as the angle of regression due to the star's speed; and

angle $FAG = 14°3'47''$

as the angle of the apparent anomaly. Since to this, according to the ratios at the apogee, there corresponds $20°19'3''$ of longitude truly determined and $21°9'$

3″ of periodic longitude, half of the regression comes to 8°12′43″ and very nearly 21½ days; and the whole regression comes to 16°25′26″ and 43 days.

According to the calculations near the least distance, the addition-subtraction of true determination is found to be 0°2⅓′.

And therefore

$$FH : CF :: 1^{p}2'20'' : 0^{p}35'11'',$$

and

$$CE : CF :: 2^{p}39'51'' : 0^{p}35'11'',$$

and

$$\text{rect. } CE, CF = 1^{p}33'44''.$$

And again

$$AC : AD :: 58^{p}50' : 43^{p}10',$$

and

$$CD : CG :: 102^{p}0' : 15^{p}40',$$

and

$$\text{rect. } CD, CG = 1{,}598^{p}0'.$$

And the square root 31ᵖ58′58″ of the 1,022ᵖ54′7″ resulting from the application, multiplied by the given ratio of FH and CF, makes

$$FH = 33^{p}13'36'',$$
$$CF = 18^{p}45'16'',$$

and, by addition,

$$CH = 51^{p}58'52'',$$

relative to the supposed quantities of AC and AF.

And therefore also

$$FH = 92^{p}22'3''$$

where

$$\text{hypt. } AF = 120^{p};$$

and

$$CH = 106^{p}1'23''$$

where

$$\text{hypt. } AC = 120^{p}.$$

And

$$\text{arc } FH = 100°39'34'',$$
$$\text{arc } CH = 124°8'22''.$$

And consequently

$$\text{angle } FAH = 50°19'47'',$$
$$\text{angle } CAH = 62°4'11''.$$

And, by subtraction,

$$\text{angle } FCA = 27°55'49''$$

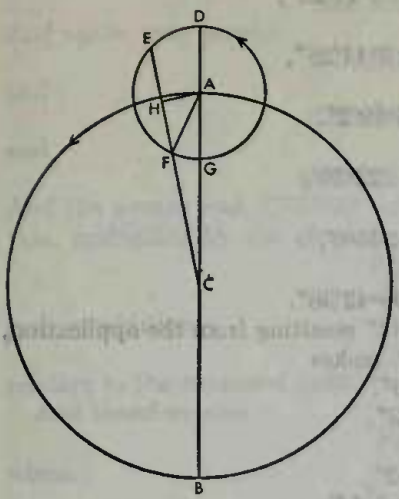
as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 11°44'24''$$

as the angle of the apparent anomaly. Since to this, according to the ratios at the perigee, there corresponds 20°53′30″ of longitude truly determined and 20°4′30″ of periodic longitude, half of the regression consequently comes to 7°2′19″ and very nearly 20⅓ days; and the whole regression comes to 14°4′38″ and 40⅔ days.

6. Demonstration of Mercury's Regressions

And again, in the case of Mercury, according to the calculations near the mean distance,



$$FH : CF :: 1^p : 3^p9'8'',$$

and

$$CE : CF :: 5^p9'8'' : 3^p9'8'',$$

and

$$\text{rect. } CE, CF = 16^p14'27''.$$

And again

$$AC : AG :: 60^p : 22\tfrac{1}{2}{}^p,$$

and

$$CD : CG :: 82^p30' : 37^p30',$$

and

$$\text{rect. } CD, CG = 3{,}093^p45'.$$

And the square root $13^p48'7''$ of the $190^p29'31''$ resulting from the application, multiplied by the give ratio of FH and CF, makes

$$FH = 13^p48'7'',$$
$$CF = 43^p30'24'',$$

and, by addition,

$$CH = 57^p18'31'',$$

relative to the supposed quantities of AC and AF.

And therefore also

$$FH = 73^p36'37''$$

where

$$\text{hypt. } AF = 120^p;$$

and

$$CH = 114^p37'2''$$

where

$$\text{hypt. } AC = 120^p.$$

And

$$\text{arc } FH = 75°40'28'',$$
$$\text{arc } CH = 145°32'52''.$$

And consequently

$$\text{angle } FAH = 37°50'14'',$$
$$\text{angle } CAH = 72°46'26''.$$

And, by subtraction,

$$\text{angle } FCA = 17°13'34''$$

as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 34°56'12''$$

as the angle of anomaly. Since to this, according to the given ratio, there corresponds $11°4'59''$ of longitudinal passage, half of the regression is left to be $6°8'35''$ and very nearly $11\frac{1}{4}$ days; and the whole regression is left to be $12°17'10''$ and $22\frac{1}{2}$ days.

According to the calculations near the greatest distance, that is, when the longitude truly determined is about 11° from the apogee (to which there corre-

sponds very nearly $11\frac{1}{2}°$ of regular longitude), the addition-subtraction of true determination is found to be very nearly $0°2\frac{1}{3}'$ in proportion to 1°. And therefore

$$FH : CF :: 0^p57'40'' : 3^p11'28'',$$

and

$$CE : CF :: 5^p6'48'' : 3^p11'28'',$$

and

$$\text{rect. } CE, CF = 16^p19'2''.$$

And again

$$AC : AG :: 68^p36' : 22^p30',$$

and

$$CD : CG :: 91^p6' : 46^p6',$$

and

$$\text{rect. } CD, CG = 4{,}199^p42'36''.$$

And the square root $16^p2'35''$ of the $257^p22'44''$ resulting from the application, multiplied by the given ratio of FH and CF, makes

$$FH = 15^p25'9'',$$
$$CF = 51^p11'43'',$$

and, by addition,

$$CH = 66^p36'52'',$$

relative to the supposed quantities of AC and AF.

And therefore also

$$FH = 82^p14'8''$$

where

$$\text{hypt. } AF = 120^p;$$

and

$$CH = 116^p31'36''$$

where

$$\text{hypt. } AC = 120^p.$$

And

$$\text{arc } FH = 86°31'4'',$$
$$\text{arc } CH = 152°27'56''.$$

And consequently

$$\text{angle } FAH = 43°15'32'',$$
$$\text{angle } CAH = 76°13'58''.$$

And, by subtraction,

$$\text{angle } FCA = 13°46'2''$$

as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 32°52'26''$$

as the angle of apparent anomaly. And since to this, according to the ratios at the apogee, there corresponds $9°48'51''$ of longitude truly determined, and $10°16'51''$ of periodic longitude, half of the regression is left to be $3°57'11''$ and very nearly $10\frac{1}{2}$ days; and the whole regression to be $7°54'22''$ and 21 days.

According to the calculations near the least distances, which are at the intervals of 120° in periodic longitude from the apogee, the addition-subtraction of true determination for the rate 11° either side of the perigees is found to be very nearly $0°1\frac{1}{2}'$. And therefore

$$FH : CF :: 1^p1'30'' : 3^p7'38'',$$

and

$$CE : CF :: 5^{p}10'38'' : 3^{p}7'38'',$$

and

$$\text{rect. } CE, CF = 16^{p}11'25''.$$

And again, very nearly,

$$AC : AG :: 55^{p}42' : 22^{p}30',$$

and

$$CD : CG :: 78^{p}12' : 33^{p}12',$$

and

$$\text{rect. } CD, CG = 2{,}596^{p}14'24''.$$

And the square root $12^{p}39'48''$ of the $160^{p}21'29''$ resulting from the application, multiplied by the given ratio of FH and CF separately, makes

$$FH = 12^{p}58'47'',$$
$$FC = 39^{p}36'4'',$$

and by addition

$$CH = 52^{p}34'51'',$$

relative to the supposed quantities of AC and AF.

And therefore also

$$FH = 69^{p}13'31''$$

where

$$\text{hypt. } AF = 120^{p};$$

and

$$CH = 113^{p}16'48''$$

where

$$\text{hypt. } AC = 120^{p}.$$

And

$$\text{arc } FH = 70°27'44'',$$
$$\text{arc } CH = 141°28'14''.$$

And consequently

$$\text{angle } FAH = 35°13'52'',$$
$$\text{angle } CAH = 70°44'7''.$$

And, by subtraction,

$$\text{angle } FCA = 19°15'53''$$

as the angle of regression due to the star's speed; and

$$\text{angle } FAG = 35°30'15''$$

as the angle of the apparent anomaly. Since to this, according to the given ratios, there corresponds 11°39′30″ of longitude truly determined, and 11° 21′30″ of periodic longitude, half of the regression is left to be 7°36′23″ and very nearly 11½ days; and the whole regression to be 15°12′46″ and 23 days.

The quantities which have been shown are in each case very nearly in accord with those taken from the appearances.

And we got the rates of longitudinal passage about the greatest and least distances in this way: for example, in the case of those about the greatest distance of Mars we showed the epicycle's apparent arc from one of the stations to the opposition (that is, the arc considered with respect to the ecliptic's centre) to be 22°13′19″. The approximately 21°10′ of periodic longitude corresponding to these in the ratio of 1^{p} to $1^{p}3'11''$, although they are not exact because the speeds given at the stations do not remain unchanged throughout the whole regressions, yet do not differ enough to make the cor-

responding addition-subtraction differ appreciably from very nearly 3°45′. And we subtracted these from the 22°13′19″ of the epicycle, since at the greatest distances the apparent passages on the epicycle are greater than the periodic, and we found the corresponding periodic passage of anomaly from one of the stations to the opposition to be 18°28′19″. Since, by the ratio of the mean movements, 20°58′21″ of periodic longitude correspond to them, we used these as exact instead of the 21°10′. Subtracting from them the 3°45′ of addition-subtraction which remained almost the same here (for at the greatest distances the apparent passages in longitude are less than the periodic) we found the apparent passage in longitude for the given interval to be 17°13′21″.

7. Construction of a Table for the Stations

And again, in order that we may be able easily to see at what sections of the epicycle each of the stars makes the appearance of stations for the distances between the mean distance and the greatest and least distances, we arrange this table of 31 rows and 12 columns. The first two columns contain the numbers of periodic longitude at intervals of 6° according to the developments of the other tables. And the ten columns following contain, for each of the five stars, the intervals of anomaly truly determined from the epicycles' apparent apogees—the first one in each case containing those of the first stations and the second those of the second. We took the quantities of these from what was demonstrated above for the mean, least, and greatest distances, and from those differences in the distances between which we have already explained for the comparison by sixtieths in the eighth column of the Tables of Anomalies, Book XI, chap. 11. The epicycles' distances, relative to which the difference of the stations is most especially considered, are there demonstrated for each passage of periodic longitude with the quantity of greatest anomalistic difference.

Since the regressions shown about the apogees and perigees do not contain the stations resulting when the epicycles' centres are at the apogees and perigees but only when they are a certain distance away, for each star we first of all got from these quantities the quantities corresponding to the apogees and perigees in this way.

Now, in the case of Saturn and Jupiter—since the epicycles' distances at the apogees and perigees differ hardly at all from those at the given intervals from them—we placed, in the proper rows, the anomalistic numbers from the epicycles' apparent apogees, obtained in these cases; that is, those of the apogees in the rows containing the number 360, and those of the perigees in the rows containing the number 180. In the case of Saturn the distance from the epicycle's perigee, at the apogee of eccentricity, was shown to be very nearly 67°15′, and at the perigee 64°31′; in the case of Jupiter, the distance at the apogee 55°55′, and at the perigee 52°49′. For convenience we arranged the numbers from the epicycles' apogees corresponding to these in the subsequent four columns along the proper rows: along the row containing the apogee's number of 360°, in the third column, the 112°45′ of Saturn's first station; in the fourth, the 247°15′ of its second station; likewise, in the fifth, the 124°5′ of Jupiter's first station; and, in the sixth, the 235°55′ of its second station. And along the row containing the perigee's number of 180°, according to the same order, 115°29′ and 244°31′; and likewise 127°11′ and 232°49′.

In the case of Mars, we showed that, whenever the epicycle's centre is 20°58′ of periodic movement from the eccentric's apogee, the star makes its stations at a distance of 22°13′ from the epicycle's apparent perigee. But the passage at the mean distance contains 16°51′, so that there is a difference of 5°22′. And where the mean distance is 60^{p}, the greatest is 66^{p}, and the difference relative to the mean is 6^{p}. But at the given angular distance from the apogee, it is 65^{p}40′, and the difference relative to the mean 5^{p}40′. Now, multiplying the 6^{p} by the 5°22′, and applying [dividing] the result to [by] the 5^{p}40′, we found the distance at the apogee over the mean distance to be very nearly 5°41′. And so the angular distance from the epicycle's apparent perigee comes to 22°32′, and the angular distance of the first station from the apogee comes to 157°28′ which we place in the seventh column in the row of 360°; and of the second station to 202°32′ which we place in the eighth column in the same row.

Likewise, when the epicycle's centre is 16°53′ in mean movement from the perigee, the star makes its stations at a distance of 11°11′ from the epicycle's apparent perigee, so that the difference relative to the mean distance comes to 5°40′. But of the linear distances, the least is 54^{p} according to the difference of 6^{p} relative to the mean, and that of the given angular distance from the eccentric's perigee 54^{p}20′; and the difference relative to the mean 5^{p}40′. Therefore we shall have the whole difference at the perigee to be 6°, and the passage from the epicycle's apparent perigee 10°51′; and the passage of the first station from the apogee 169°9′, and of the second, 190°51′, which we place in the 180° row in the proper columns.

In the case of Venus, we showed that, when the epicycle's centre is 21°9′ in periodic longitude from the apogee, the star makes its stations 14°4′ from the epicycle's apparent perigee. But the passage at the mean distance contains 12° 52′, so that there is a difference of 1°12′. Where the mean distance is 60^{p}, the greatest is 61^{p}15′ and the difference relative to the mean 1^{p}15′. But at the given angular distance from the apogee, the distance is 61^{p}10′ and the difference relative to the mean 1^{p}10′. And again, multiplying 1^{p}15′ by 1°12′ and applying [dividing] the result to [by] 1^{p}10′, we found the difference at the apogee over the mean distance to be 1°17′. And so the angular distance from the epicycle's apparent perigee comes to 14°9′, and the angular distance of the first station from the apogee comes to 165°51′ which we place in the ninth column in the row of 360°; and of the second station to 194°9′, which we place in the tenth column along the same row.

And likewise, when the epicycle is very nearly 20° according to regular longitudinal passage from the eccentric's perigee, the star makes its stations at a distance of 11°44′ from the epicycle's apparent perigee, so that the difference relative to the mean distance comes to 1°8′. Of the linear distances, the least is 58^{p}45′ where the mean is 60^{p}, and the difference between them 1^{p}15′; but that of the given angular distance from the eccentric's perigee 58^{p}50′, and its difference relative to the mean 1^{p}10′. And nultiplying the 1^{p}15′ by the 1°8′ and applying [dividing] the result to [by] 1^{p}10′, we found the difference at the apogee over the mean distance to be 1^{p}13′. Therefore we found the passage from the epicycle's apparent perigee to be 11°39′, and the passage of the first station from the apogee to be 168°21′, and of the second station 191°39′, which we place in the same columns by the number 180°.

In the case of Mercury, we demonstrated that, when the epicycle is 10°17′ in

periodic longitude from the eccentric's apogee, the star makes its stations 32°52′ from the epicycle's apparent perigee. But the passage at the mean distance contains 34°56′, so that there is a difference of 2°4′. Where the mean distance is 60ᵖ, the greatest is 69ᵖ and the difference between them 9ᵖ; but at the given angular distance from the apogee, it is 68ᵖ36′ and the difference relative to the mean 8ᵖ36′. And in the same way as before, multiplying the 9ᵖ by 2°4′ and applying [dividing] the result to [by] 8ᵖ36′, we found the difference at the apogee over the mean distance to be very nearly 2ᵖ10′. And so the angular distance from the epicycle's apparent perigee comes to 32°46′, and the angular distance of the first station from the apogee comes to 147°14′ which we place in the eleventh column beside the number of 360°; and, of the second station, to 212°46′, which we place in the twelfth column along the same row.

And likewise, when the epicycle is 11°22′ in periodic movement from the perigee, then the star makes its stations 35°30′ from the epicycle's apparent perigee so that the difference relative to the mean distance is 0°34′. Of the linear distances, the least is 55ᵖ34′ where the mean is 60ᵖ, and their difference is 4ᵖ26′; but at the given angular distance from the eccentric's perigee is very nearly 55ᵖ42′, and the difference relative to the mean 4ᵖ18′. Again multiplying the 4ᵖ26′ by 0°34′ and applying [dividing] the result to [by] the 4ᵖ18′, we found the difference at the perigee relative to the mean to be 0ᵖ35′. Therefore we found the passage from the epicycle's apparent perigee to be 35°31′, and the perigee of the first station from the apogee to be 144°29′, and, of the second, 215°31′, which we place in the same columns; yet not beside the number 180° in longitude but beside 120° and 240°, since the perigees of Mercury's eccentricity have been demonstrated to be at those places.

Now with these things first set forth, the differences of the intervening passages are established according to the same methods.

For, as an example, let it be required to find the corresponding positions of the apparent anomaly for the first stations when the mean longitudinal passage is 30° from the apogee. In this position, where the mean distance is in every case 60ᵖ, the epicycle's distance is established, by means already formulated, to be 63ᵖ2′ for Saturn, 62ᵖ26′ for Jupiter, 65ᵖ24′ for Mars, 61ᵖ6′ for Venus, and 66ᵖ35′ for Mercury. And so, relative to the mean, the differences in each case are (in the same order, to avoid repetition) 3ᵖ2′, 2ᵖ26′, 5ᵖ24′, 1ᵖ6′, and 6ᵖ35′. But the differences between the mean and apogeal distances (for in every case the given numbers of the distances are greater than the mean) are 3ᵖ25′, 2ᵖ45′, 6ᵖ0′, 1ᵖ15′, and 9ᵖ0′. Now, since the whole differences of the degrees of apparent anomaly at the apogees, relative to the mean distances, come to 1°23′, 1°33′, 5°41′, 1°17′, and 2°10′, in the same order, therefore, multiplying each of them by the difference between the distance at that time and the mean, proper to each star (for example, 1°23′ by 3ᵖ2′), and applying [dividing] the result to [by] the difference of the greatest distance (for instance 3ᵖ25′), we shall have, for each case at the given longitudinal passage, the excess of the degrees of anomaly over those for the mean distance: 1°14′, 1°22′, 5°7′, 1°8′, and 1°35′. But, for the mean distances, the intervals from the epicycle's apparent apogee are 114°8′, 135°38′, 163°9′, 167°8′, and 145°4′; and the intervals for the greatest distances are less than these in all cases except Mercury where it is more. And so, for the other cases, subtracting the differences found for the given distances from intervals at the mean distances, but adding them in the case of Mercury, we shall have, corre-

sponding to 30° of periodic longitude in the columns of the first stations, the degrees of apparent anomaly from the epicycle's apogee: 112°54′ for Saturn, 124° 16′ for Jupiter, 158°2′ for Mars, 166° for Venus, and 146°39′ for Mercury. We fill in the columns of the second stations immediately by placing the differences between the numbers of the first stations and 360° in the same rows in the columns of the second stations: in the case of the given longitude, 247°6′, 235°44′, 201°58′, 194°0′, and 213°21′.

It is easily understood that, even if we do not choose to set out the degrees of anomaly considered with respect to the epicycle's apparent apogee, but, for greater convenience, those not truly determined considered relative to the periodic, we can immediately establish such a table by taking the total addition-subtraction in the Tables of Anomaly corresponding to each number of periodic longitude and subtracting it from the degrees of apparent anomaly found for those up to 180° from the eccentric's apogee and adding it for those beyond 180°. And here is the outlay of the table:

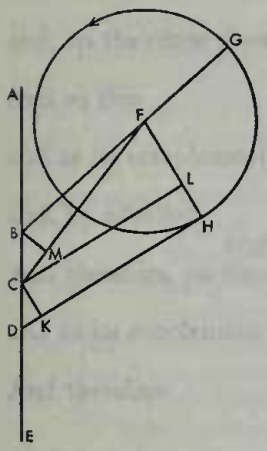
8. Numbers of Anomaly Truly Determined

Common Numbers		SATURN First Station		SATURN Second Station		JUPITER First Station		JUPITER Second Station		MARS First Station		MARS Second Station		VENUS First Station		VENUS Second Station		MERCURY First Station		MERCURY Second Station	
0	360	112	45	247	15	124	5	235	55	157	28	202	32	165	51	194	9	147	14	212	46
6	354	112	45	247	15	124	6	235	54	157	29	202	31	165	52	194	8	147	13	212	47
12	348	112	46	247	14	124	7	235	53	157	34	202	26	165	53	194	7	147	8	212	52
18	342	112	48	247	12	124	9	235	51	157	41	202	19	165	55	194	5	147	1	212	59
24	336	112	51	247	9	124	12	235	48	157	50	202	10	165	57	194	3	146	51	213	9
30	330	112	54	247	6	124	16	235	44	158	2	201	58	166	0	194	0	146	39	213	21
36	324	112	58	247	2	124	21	235	39	158	18	201	42	166	4	193	56	146	25	213	35
42	318	113	3	246	57	124	26	235	34	158	34	201	26	166	9	193	51	146	11	213	49
48	312	113	8	246	52	124	32	235	28	158	55	201	5	166	15	193	45	145	55	214	5
54	306	113	15	246	45	124	39	235	21	159	17	200	43	166	22	193	38	145	39	214	21
60	300	113	22	246	38	124	47	235	13	159	42	200	18	166	29	193	31	145	23	214	37
66	294	113	29	246	31	124	55	235	5	160	10	199	50	166	35	193	25	145	8	214	52
72	286	113	36	246	24	125	3	234	57	160	39	199	21	166	42	193	18	144	58	215	2
78	282	113	44	246	16	125	12	234	48	161	10	198	50	166	50	193	10	144	52	215	8
84	276	113	53	246	7	125	22	234	38	161	44	198	16	166	58	193	2	144	46	215	14
90	270	114	1	245	59	125	32	234	28	162	18	197	42	167	7	193	53	144	40	215	20
96	264	114	10	245	50	125	41	234	19	162	54	197	6	167	14	192	46	144	36	215	24
102	258	114	18	245	42	125	51	234	9	163	31	196	29	167	21	192	39	144	33	215	27
108	252	114	27	245	33	126	0	234	0	164	9	195	51	167	28	192	32	144	30	215	30
114	246	114	35	245	25	126	10	233	50	164	47	195	13	167	35	192	25	144	30	215	30
120	240	114	43	245	17	126	19	233	41	165	25	194	35	167	43	192	17	144	29	215	31
126	234	114	51	245	9	126	28	233	32	166	3	193	57	167	50	192	10	144	29	215	31
132	228	114	58	245	2	126	36	233	24	166	37	193	23	167	56	192	4	144	30	215	30
138	222	115	5	244	55	126	44	233	16	167	8	192	52	168	1	191	59	144	31	215	29
144	216	115	11	244	49	126	51	233	9	167	39	192	21	168	6	191	54	144	33	215	27
150	210	115	16	244	44	126	57	233	3	168	4	191	56	168	10	191	50	144	35	215	25
156	204	115	21	244	39	127	2	232	58	168	28	191	32	168	14	191	46	144	37	215	23
162	198	115	25	244	35	127	6	232	54	168	46	191	14	168	17	191	43	144	38	215	22
168	192	115	27	244	33	127	8	232	52	168	59	191	1	168	19	191	41	144	39	215	21
174	186	115	29	244	31	127	10	232	50	169	8	190	52	168	20	191	40	144	40	215	20
180	180	115	29	244	31	127	11	232	49	169	9	190	51	168	21	191	39	144	40	215	20

9. Demonstration of Venus' and Mercury's Greatest Elongations from the Sun

Now that we have arranged things having to do with the regressions, it would be reasonable next to demonstrate for each dodecatemory the greatest elongations from the sun of Venus and Mercury as established from the given hypotheses. We have the expositions of these with respect to the sun's apparent passage, with the stars at the beginnings of the dodecatemories, and with their apogees in position with respect to the tropic and equinoctial points for our times—that is, with Venus' 25° within the Bull and Mercury's 10° within the Balance. There will be a change in the greatest elongations because of the movement of the apogees and this can be easily corrected by the same methods as before; otherwise it will remain without any appreciable difference for a long time. And that the manner of our methods may be easily understood, it is necessary to show first, as an example, the greatest morning and evening elongations of Venus when it is in the spring equinox and at the beginning of the Ram.

Then let there be through the apogee of eccentricity A the straight line $ABCDE$. And on it let B be supposed the centre of regular movement, C the centre of the eccentric bearing the epicycle, and D the ecliptic's centre. And, with the eccentric's radius CF drawn through, let the epicycle GH be described about F; and let DH be drawn tangent to its morning and western parts. And let BFG and FH be joined; and let CK, CL, and BM be drawn as perpendiculars.



Now, since DA is 25° within the Bull, and DH at the beginning of the Ram, therefore

$$\text{angle } ADH = 55°$$
$$= 110° \text{ to 2 rt.}$$

and its complement

$$\text{angle } DCK = 70° \text{ to 2 rt.}$$

And so also, on the circle about right triangle CDK,

$$\text{arc } CK = 110°,$$

and

$$\text{chord } CK = 98^{p}18'$$

where

$$\text{hypt. } CD = 120^{p}.$$

And therefore

$$CK = HL = 1^{p}1'$$

where

$$CD = 1^{p}15'$$

and

$$\text{epic. rad. } FH = 43^{p}10';$$

and, by subtraction,

$$FL = 42^{p}9'$$

where it is supposed

$$\text{ecc. rad. } CF = 60^{p}.$$

And therefore

$FL = 84^{p}18'$

where

hypt. $CF = 120^{p}$;

and, on the circle about right triangle CFL,

arc $FL = 89°16'$.

And so also

angle $FCL = 89°16'$ to 2 rt.

And

angle $DCK = 70°$ to 2 rt.,

and angle LCK is right. Therefore, by addition,

angle $FCD = 339°16'$ to 2 rt.,

and, by subtraction,

angle $ACF = 20°44'$ to 2 rt.

And so also, on the circle about right triangle BCM,

arc $BM = 20°44'$

and as its supplement

arc $CM = 159°16'$.

And therefore

chord $BM = 21^{p}35'$,

chord $CM = 118^{p}2'$

where

hypt. $BC = 120^{p}$.

And so also, where

$BC = 1^{p}15'$

and

ecc. rad. $CF = 60^{p}$,

there also

$BM = 0^{p}13'$,

$CM = 1^{p}14'$,

and, by subtraction,

$FM = 58^{p}46'$.

And therefore also

hypt. $BF = 58^{p}46'$.

And therefore

$BM = 0^{p}27'$

where

$BF = 120^{p}$;

and, on the circle about right triangle BFM,

arc $BM = 0^{p}26'$.

And so also

angle $BFC = 0°26'$ to 2 rt.

But it was shown that

angle $ACF = 20°44'$ to 2 rt.

And therefore, by addition, as the angle of regular longitudinal passage,

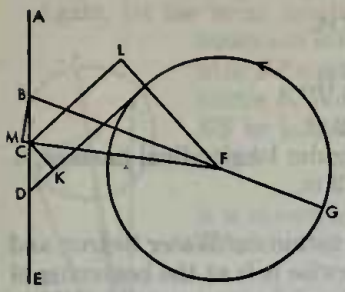
angle $ABF = 21°10'$ to 2 rt.

$= 10°35'$.

The sun's mean passage will therefore be 10°35′ westward from the apogee A and its position, of course, 14°25′ within the Bull; while the true sun will be 15°14′ within the Bull. And so the star, when it is at the beginning of the

Ram, will have a greatest western elongation from the true sun of 45°14′.

Again, let the following drawing be laid out: the tangent drawn to the evening and eastern parts of the epicycle and the star likewise supposed at the beginning of the Ram.



By means of things already demonstrated and with angle ADH remaining the same,

angle $DCK = 70°$ to 2 rt.,

and

chord $CK = LH = 1^p1'$

where

ecc. rad. $CF = 60^p$

and

epic. rad. $FH = 43^p10'$.

And so, by addition,

$FL = 44^p11'$.

And it is clear that

$FL = 88^p22'$

where

hypt. $CF = 120^p$;

and, on the circle about right triangle CFL,

arc $FL = 94°51'$.

And so also

angle $FCL = 94°51'$ to 2 rt.,

and as its complement

angle $FCK = 85°9'$ to 2 rt.,

and, by addition,

angle FCD = angle $BCM = 155°9'$ to 2 rt.

And therefore, on the circle about right triangle BCM,

arc $BM = 155°9'$,

and as its supplement

arc $CM = 24°51'$.

And therefore

chord $BM = 117^p11'$,
chord $CM = 25^p49'$

where

hypt. $BC = 120^p$.

And so also, where

$BC = 1^p15'$,

there also

$BM = 1^p13'$,
$CM = 0^p16'$,

and, by addition,

$FM = 60^p16'$;

and therefore also

hypt. $BF = 60^p17'$.

And therefore

$BM = 2^p25'$

where

$BF = 120^p$;

and, on the circle about right triangle BFM,

arc $BM = 2°19'$.

And so also

angle $BFM = 2°19'$ to 2 rt.

And since it has been shown

angle $DCF = 155°9'$ to 2 rt.,

therefore

angle $BCF = 204°51'$ to 2 rt.

And therefore, by addition, as the angle of the regular longitudinal passage,

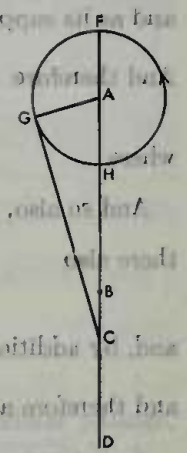
angle $ABF = 207°10'$ to 2 rt.

$= 103°35'$.

The sun's mean passage will therefore be 11°25′ within the Water Bearer, and its true passage 13°38′. And so the star, when likewise it is at the beginning of the Ram, will have a greatest eastern elongation from the true sun of 46°22′.

And for Mercury, let it be supposed as an evening star at the beginning of the Scorpion and as a morning star at the beginning of the Bull; because, with reference to the demonstrations of its heliacal risings on the ecliptic which will follow, it is easier to find the star's greatest elongations from the true sun in these positions. Now, according to Mercury's hypothesis, when the star's apparent passage is given, the mean longitudinal passage is not obtained, because the straight line CF does not always remain the same with and equal to the eccentric's radius as in the hypothesis of the others. But when the regular longitudinal passage is given, the apparent is shown. Therefore, supposing there are two longitudinal positions for each dodecatemory which can carry the star at the beginning of the dodecatemory in question, and calculating the greatest elongations resulting in the passages obtained, in this way we find the greatest elongation for the beginning of the dodecatemory, as will be easily understood through the means set forth. And this will be done first for the greatest eastern elongation at the beginning of the Scorpion.

For let there be the diameter $ABCD$ through the apogee A. And on it let C be supposed the ecliptic's centre and B the centre of the epicycle's regular movement. First let the epicycle's centre be conceived at the apogee itself, so that the sun's mean longitudinal passage is 10° within the Balance, and its true one 8°. And with epicycle FG described about A, let CG be drawn from C tangent to it at the eastern parts; and let the perpendicular AG be joined.



Now, since it has been shown by means already formulated that

epic. rad. $AG = 22\frac{1}{2}^{p}$

where, as the greatest distance,

$AC = 69^{p}$,

therefore

$AG = 39^{p}8'$

where

hypt. $AC = 120^{p}$.

And so, on the circle about right triangle ACG,

arc $ACG = 38°4'$,

and

angle $ACG = 38°4'$ to 2 rt.

$= 19°2'$.

And AC is 10° within the Balance. Therefore the star will be 29°2′ within the Balance when it is at its greatest elongation of 21°2′ from the true sun.

Again, let the mean longitude be supposed 3° from the apogee so that the mean sun is 13° within the Balance, and the true sun 11°4′. And with BE continued through, let epicycle FG be described about centre E. With CG drawn tangent in the same way, let EC and EG be joined. Since, in this position (that is, when it is supposed

angle $ABE = 3°$)

it is shown by means already formulated that, as the angle of difference of eccentricity,

angle $ACE = 2°52′$,

and, as the epicycle's distance at that time,

$CE \fallingdotseq 68^{p}58′$

where

epic. rad. $EG = 22^{p}30′$;

therefore

$EG = 39^{p}9′$

where

hypt. $CE = 120^{p}$.

And so, on the circle about right triangle CEG,

arc $EG = 38°5′$,

and

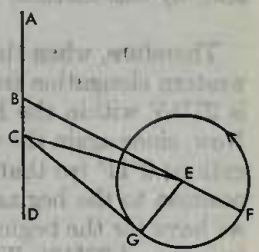
angle $ECG = 38°5′$ to 2 rt.
$\fallingdotseq 19°3′$;

and therefore, by addition,

angle $ACG = 21°25′$.

Therefore, when the star is 1°55′ within the Scorpion, it will have a greatest elongation from the true sun of 20°21′. But it was shown that, when it is 29°2′ within the Balance, it will have a greatest elongation of 21°2′ from the true sun. Since, then, the difference between the positions is 2°53′, and the difference of the greatest elongations 0°11′ so that very nearly 0°4′ correspond to the 0°58′ from the first position to the beginning of the Scorpion, we subtract these from the 21°2′ and thus have for the beginning of the Scorpion the greatest eastern elongation from the true sun of 20°58′.

And next, for the greatest western elongation at the beginning of the Bull, first let the mean longitudinal passage be 39° east of the perigee so that the mean sun is 19° within the Bull, and the true 19°38′. Let a similar drawing be laid out with the epicycle in a position east of the perigee and the tangent drawn to the western parts of the epicycle.



Now since, at this passage (that is, when it is supposed

angle $DBF = 39°$)

it is shown by means already formulated that

angle $DCE = 40°57′$,

and as the distance at that time

$CE = 55^{p}59′$

where

epic. rad. $EG = 22^{p}30'$,

therefore

$EG = 48^{p}14'$

where

hypt. $CE = 120^{p}$;

and, on the circle about right triangle CEG,

arc $EG = 47°24'$.

And so also

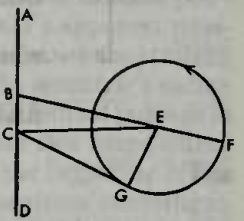
angle $ECG = 47°24'$ to 2 rt.
$= 23°42'$,

and, by subtraction,

angle $GCD = 17°15'$.

And therefore when Mercury is 27°15′ within the Ram, it will have a greatest western elongation from the true sun of 22°23′.

Again, let the mean longitude be supposed 42° from the perigee in the same direction, so that the sun's mean position is 22° within the Bull, and its true 22°31′.



Now since, for this same passage (that is, when it is supposed

angle $DBF = 42°$)

it is shown that

angle $DCE = 44°4'$,

and, as the distance at that time,

$CE = 55^{p}50'$

where

epic. rad. $EG = 22^{p}30'$;

therefore

$EG = 48^{p}19'$

where

hypt. $CE = 120^{p}$;

and, on the circle about right triangle ECG,

arc $EG = 47°30'$.

And so also

angle $ECG = 47°30'$ to 2 rt.
$= 23°45'$,

and, by subtraction,

angle $GCD = 20°19'$.

Therefore, when the Mercury is 0°19′ within the Bull, it will have a greatest western elongation from the true sun of 22°12′. But it was shown that, when it is 27°15′ within the Ram, it will have a similar greatest elongation of 22°23′. Now, since again the difference of the positions is 3°4′, and of the greatest elongations 0°11′ (so that very nearly 0°10′ correspond to the 2°45′ from the first position to the beginning of the Bull), we subtract these from the 22°23′, and we have for the beginning of the Bull the greatest western elongation from the true sun of 22°13′. Which things it was required to find.

For the other dodecatemories, calculating in the same way the greatest western and eastern elongations of both stars, we arranged a table of them in 12 rows and 5 columns. In the first column we arranged the beginnings of the

dodecatemories beginning with the Ram. And in the next four we placed the greatest elongations from the true sun as calculated: the second containing the western elongations of Venus and the third the eastern; and again the fourth the western elongations of Mercury and the fifth the eastern. And here is the table:

10. Greatest Elongations with Respect to the True Sun

Beginnings of the Signs	VENUS				MERCURY			
	Western		*Eastern*		*Western*		*Eastern*	
Ram	45	14	46	22	24	14	19	36
Bull	45	17	45	31	22	13	21	7
Twins	45	34	44	49	20	18	23	41
Crab	45	56	44	25	18	17	26	16
Lion	46	20	44	31	16	35	27	37
Virgin	46	38	44	55	16	8	26	17
Balance	46	45	45	41	17	46	23	31
Scorpion	46	47	46	30	21	32	20	58
Archer	46	30	47	13	26	9	19	28
Goat	46	7	47	35	28	37	19	14
Water Bearer	45	41	47	34	28	17	18	51
Fishes	45	20	47	7	26	24	19	0

BOOK THIRTEEN

1. On the Hypotheses for the Latitudinal Passages of the Five Planets

Two matters are still left for the treatise on the five planets: (1) their latitudinal passage with respect to the ecliptic, and (2) the business of the distances of the heliacal risings and settings. The latitudinal distance of each of them must be first cleared up here, since from this there result noteworthy differences in the risings and settings. Therefore we shall again present first whatever we suppose as common to the obliquities of their circles.

Now, because each planet appears to effect a double latitudinal difference just like the longitudinal anomaly—one due to the eccentric circle relative to parts of the ecliptic, and the other due to the epicycle relative to the sun—in each case we suppose the eccentric inclined to the ecliptic's plane and the epicycle to the eccentric's plane, although (as we said) there is no noteworthy change resulting from this in the longitudinal passage or in the demonstrations of the anomalies for such obliquities as we shall show in what follows. And because, through observations made on each of them, the stars appear in the plane of the ecliptic when the number of truly determined longitude and of truly determined anomaly are each very nearly a quadrant (the one from the eccentric's northern or southern limit, and the other from its own apogee) we suppose the inclinations of the eccentrics about the ecliptic's centre, as in the case of the moon, and relative to the diameters through the northern or southern limits; and we suppose those of the epicycles relative to their diameters directed to the ecliptic's centre and along which the apparent apogees and perigees are observed.

And again, in the case of the three planets Saturn, Jupiter, and Mars, we observed that, when their longitudinal passages are near the eccentric's apogee, then they always appear the farthest north of the ecliptic and farther north at the passages in the epicycle's perigees than at those in their apogees; but when their longitudinal passages are at the eccentric's perigee, on the contrary they appear farther south than the ecliptic. In the case of Saturn and Jupiter, we observed that the eccentric's northernmost limits are near the beginning of the dodecatemory of the Balance, but in the case of Mars near the end of Cancer and almost at its apogee. And so we concluded from these things that the points of the eccentrics at these parts of the ecliptic are inclined towards the north, whereas the points diametrically opposite are equally inclined to the south; and that the perigees of the epicycles are always inclined in the direction of the eccentrics' inclination, while the diameters at right angles to those through their apogees always remain parallel to the plane of the ecliptic.

In the case of Venus and Mercury, we observed that, when their longitudinal passages are at the eccentric's apogees or perigees, the movements at the epicycle's perigees do not differ latitudinally from those at the apogees, but are

alike either north or south of the ecliptic: in the case of Venus always north; and in the case of Mercury, on the contrary, always south. But the passages at the greatest elongations differ most from each other (that is, the western from the eastern) and from those at the epicycle's apogees and perigees (that is, the difference with respect to the eccentric), while the greatest eastern and evening elongation, in the case of Venus, is north at the eccentric's apogee and south at its perigee; and in the case of Mercury, on the contrary, south at the apogee and north at the perigee. But when their passages of truly determined longitude are at the nodes, then the quadrant distances on each epicycle from the apogees or perigees are both in the plane of the ecliptic, and the passages at the perigees differ most from those at the apogees, and, in the case of Venus, make the inclination for the node in the subtractive semicircle southward and for the opposite one northward. But, in the case of Mercury, again the contrary: for the node in the subtractive semicircle northward and for the opposite southward. And so we concluded from this that the eccentrics move their obliquities and make their returns with the periods of the epicycles, being in the same plane with the ecliptic when the epicycles are at the nodes, and, when it is around the apogees and perigees, throwing the epicycle the farthest north in the case of Venus and the farthest south in the case of Mercury. And we concluded that the two epicycles effect differences by inclining their diameters through the apparent apogees the most at the eccentric's nodes, and slanting the diameters perpendicular to these the most at the eccentrics' perigees and apogees (for let us distinguish this inclination by that name), thus contrariwise putting the first diameters in the eccentric's plane at its apogees and perigees, and the second in the ecliptic's plane at the nodes.[1]

[1]These appearances of latitudinal variation can be understood in the heliocentric theory. Kepler, in the *Commentaries on Mars* (Part V, Chapters 67 and 68) and again in the *Epitome of Copernican Astronomy* (Book VI, Parts 2 and 3) gives good accounts of the transformation from the Ptolemaic appearances and theory to his own theory which will be that of Newton in the *Principia*. The latitudinal variations are caused by the inclination of the plane of each planet's orbit to that of the earth with the sun as centre. These inclinations, according to Kepler, are fixed. Newton deduces their fixity except for very slight perturbations in the *Principles*, Book I, Section II, Proposition I.

It is now obvious that for the outer planets there will be greatest variations in latitude at opposite points on the ecliptic and no variation at the quadrants representing the nodes. Furthermore, this variation or inclination will appear greater the nearer the earth and the planet, and therefore when the planet is at the perigee of its epicycle in Ptolemaic theory.

In the case of the lower planets, Venus and Mercury, it is evident that, since its Ptolemaic epicycle represents its Copernican orbit about the sun, there will be two opposite points on the ecliptic where the greatest difference in latitude will be at the greatest elongations and none at the apogee and perigee. And at the quadrants relative to these two points, on the contrary, the greatest difference in latitude will be at the perigee and apogee of the epicycle. The fact that the planet at the apogee and perigee of the epicycle in the case of the first two points has a small latitudinal deviation on the same side of the ecliptic is explained by referring the line of the nodes to the true sun, according to Kepler. This is an argument for the Keplerian dynamical theory of the sun as the centre of the universe. Thus is explained the Ptolemaic inclination of the eccentric which serves to throw the latitude more one side than the other.

2. On the Manner of the Movement of the Obliquities and Slantings According to the Hypotheses

The general conclusion of the hypotheses is, then, that the eccentric circles of the five planets are inclined to the ecliptic's plane about the ecliptic's centre: without movement in the case of the three, Saturn, Jupiter, and Mars, so that diametrically opposite passages of the epicycles are borne in contrary latitudinal directions; but in the case of Venus and Mercury, changing simultaneously with the epicycles towards the same latitude, always towards the north for Venus and always towards the south for Mercury; and that those diameters of the epicycles through their apparent apogees, starting somewhere in the eccentric's plane, are carried aside by little circles lying beside the perigeal limits, say, proportionate in size to the latitudinal deviation, perpendicular to the eccentrics' planes, and having their centres on them. These little circles are revolved regularly and in accord with the longitudinal passages starting from an intersection of the planes with the epicycles (say towards the north) and at the same time carrying the epicycles' planes in the first quadrant's turn to the northernmost limit, of course, then back again to the eccentric's plane; and in the third quadrant's turn to the southernmost limit; and, finally, at the return to the starting point. The beginning and return of this circuit are established—for Saturn, Jupiter, and Mars—from the intersection at the ascending node; in the case of Venus, from the eccentric's perigee; and in the case of Mercury, from the eccentric's apogee.

In the case of the three stars, the diameters perpendicular to these first diameters remain, as we said, always parallel to the ecliptic's plane or have an inappreciable slant to it. But in the case of Mercury and Venus, starting again from some point in the ecliptic's plane, they are carried aside by little circles lying beside, say, their eastern limits, again proportionate in size to the latitudinal deviation, perpendicular to the ecliptic's plane, and having their centres on diameters parallel to the ecliptic's plane. These little circles are revolved at the same speed as the others starting from an intersection of the planes and the epicycles again northwards (let us say), carrying the eastern extremities of these diameters in the same order as before. And again for these stars, the beginning and return of the similar circuit are established, for Venus, from the node in the additive semicircle, and, for Mercury, from the node in the subtractive semicircle.

Now, concerning these little circles by which the oscillations of the epicycles are effected, it is necessary to assume that they are bisected by the planes about which we say the swayings of the obliquities take place; for only in this way can equal latitudinal passages be established on either side of them. Yet they do not have their revolutions with respect to regular movement effected about the proper centre, but about another which has the same eccentricity for the little circle as the star's longitudinal eccentricity for the ecliptic. For the returns on the ecliptic and the little circle are supposed isochronous and the quarterly passages on each apparently agree. But if the little circle's revolutions take place about its proper centre, that which is required can in no way come about, since the passages for the little circle travel each quadrant in equal time, but the epicycle's passages with respect to the ecliptic do not because of the eccentricity assumed in each case. But if they take place

about a centre similar in position to that of the eccentric, and of the quadrants, the returns of the obliquities will traverse corresponding parts of the ecliptic and of the little circle in equal times.

Let no one, seeing the difficulty of our devices, find troublesome such hypotheses. For it is not proper to apply human things to divine things nor to get beliefs concerning such great things from such dissimilar examples. For what is more unlike than those which are always alike with respect to those which never are, and than those which are impeded by anything with those which are not even impeded by themselves? But it is proper to try and fit as far as possible the simpler hypotheses to the movements in the heavens; and if this does not succeed, then any hypotheses possible. Once all the appearances are saved by the consequences of the hypotheses, why should it seem strange that such complications can come about in the movements of heavenly things? For there is no impeding nature in them, but one proper to the yielding and giving way to movements according to the nature of each planet, even if they are contrary, so that they can all penetrate and shine through absolutely all the fluid media; and this free action takes place not only about the particular circles, but also about the spheres themselves and the axes of revolution. We see the complication and sequence in their different movements difficult and hard to come by for the freedom of the movements in the likely stories constructed by us, but in the heavenly thing never anywhere impeded by this mixture. Or rather it is not proper to judge the simplicity of heavenly things by those which seem so with us, when here not even to all of us does the same thing seem likewise simple. For in this way not one of the heavenly occurrences would seem simple to those studying them, not even the unchangeableness of the first motion, since always to be the same is not difficult here with us but impossible. We should instead judge their simplicity from the unchangeableness of the natures in the heavens and their movements. For thus they would all appear simple, more than those things which seem so here with us, since no difficulty or trouble could be conjectured concerning their periods.

3. On the Size of Each of the Obliquities and Slantings

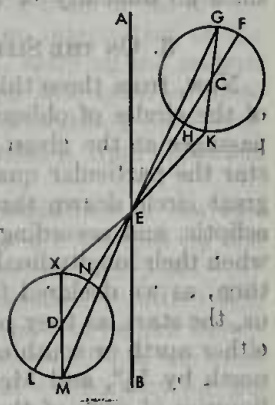
Now, from these things, one could calculate the general position and order of the circles of obliquity. In the case of Venus and Mercury, the apparent passages at the given positions of latitude render easily calculable for each star the particular quantities of the arcs which the obliquities cut off on the great circle drawn through the inclined circle's poles at right angles to the ecliptic, and according to which the latitudinal passages are considered. For when their longitudinal movements are at the eccentrics' apogees and perigees, then, as we declared from nearby observations of the addition resulting for us, the stars, as they pass through the epicycle's perigees and apogees, appear either north or south of the ecliptic by an equal amount, Venus always being north by $\frac{1}{6}°$, and Mercury always south by $\frac{3}{4}°$, so that from this the obliquities of each of the eccentrics are just about those amounts. But at the greatest elongations from the sun they both appear in mean terms about 5° north or south of the opposite greatest elongations. For, when it effects this latitudinal opposition at the eccentric's apogee, Venus appears very little less than 5°, and, when at the eccentric's perigee, very little more than 5°; and Mercury by at most $\frac{1}{2}°$ more or less. And so the slantings of the epicycle

from one side or the other of the eccentric planes subtend, in mean terms, about $2\frac{1}{2}°$ of the circle at right angles to the ecliptic, from which are taken the quantities of the angles resulting from the epicycles' slant to the eccentrics' planes. This will be clear from later demonstrations about them in what follows, not to interrupt at present what we have to say in common about the five planets.

But when the movements of truly determined longitude are at the nodes and mean distances, then, as Venus makes its passage about the epicycle's apogee, it appears 1° north and south of the ecliptic, and, about the epicycle's perigee, very nearly $6\frac{1}{3}°$; so that from this it is concluded the epicycle's obliquity cuts off $2\frac{1}{2}°$ on the circle drawn through its poles in the way we have described. For as a result of the epicyclic anomaly at the mean distances we find that many degrees subtend, for the epicycle's apogee, an angle at the eye of 1°2′, and, for the epicycle's perigee, an angle of 6°22′. And, as Mercury makes its passage about the epicycle's apogee, one would calculate from its heliacal risings that it is very nearly $1\frac{3}{4}°$ north and south of the ecliptic, and, about the epicycle's perigee, very nearly 4°; so that from this the obliquity is established as $6\frac{1}{4}°$. For again, as a result of the epicyclic anomaly at the distances of the greatest obliquities (that is, when the truly determined longitude is a quadrant's distance from the apogee), we find just that many degrees subtend, for the epicycle's apogee, an angle at the eye of 1°46′, and, for the epicycle's perigee, an angle of 4°5′.

In the case of the others—Saturn, Jupiter, and Mars—one could not immediately apprehend the quantities of the obliquities which are both always mixed together, that effected by the eccentric with that effected by the epicycle. But from the latitudinal passages again observed at the perigees and apogees of the eccentrics and epicycles, we separate out each of the two obliquities in this way:

In the plane at right angles to the plane of the ecliptic, let AB be its intersection with the ecliptic's plane, and CD its intersection with the eccentric's plane. And let point E be the ecliptic's centre. About the eccentric's apogee C and perigee D on the intersection of the planes, let there be drawn in the plane of reference the equal circles $FGHK$ and $LMNX$ through the poles of the epicycles. And on these circles let the epicycles' planes be inclined at lines GCK and MDX with equal angles—of course, at C and D. Let also straight lines be drawn from the ecliptic's centre E where the eye is to the epicycles' apogees and perigees: EG and EM to the epogees, and EK and EX to the perigees; for the points K and X embrace, of course, the passages of acronychal opposition, and G and M the synodic passages.



Now, in the case of Mars, we took the latitudinal passages about the acronychal oppositions established at the eccentric's apogee (that is, those about the point K of the epicycle) and those at the eccentric's perigee (that is, those about the point X of the epicycle), because their difference is quite sensible. In the oppositions about the apogee it is $4\frac{1}{3}°$

north of the ecliptic, and in the oppositions about the perigee very nearly 7° south of the ecliptic, so that

$$\text{angle } AEK = 4\tfrac{1}{3}°,$$
$$\text{angle } BEX = 7°.$$

Now, with these things supposed, we find the angle contained by the eccentric's obliquity (that is, angle *AEC*) and that contained by the epicycle's (that is, angle *GCF*) in this way. For since, from what we have demonstrated concerning the anomalies of Mars, it is easily seen that, of those angles subtended by equal arcs about the epicycle's perigeal parts, those about the passages at the eccentric's apogee have to those about the passages at the perigee the ratio which very nearly 5 has to 9[1], and since arcs *HK* and *NX* are equal, therefore

$$\text{angle } CEK : \text{angle } DEX :: 5 : 9.$$

And so, since angles *AEK* and *BEX* are given, and the ratio of angle *CEK* to angle *DEX* is given, and

$$\text{angle } AEC = \text{angle } BED,$$

therefore, if whatever part the difference of the whole quantities is of the difference of the terms of the ratio, we take just that part of the terms of the ratio, and then shall have the quantity in the proper ratio. For this is shown by a short arithmetical lemma.[2] Since the quantities are $4\frac{1}{3}$ and 7 and their difference $2\frac{2}{3}$, and the ratio is 5 to 9 and their difference 4, and $2\frac{2}{3}$ is two-thirds of 4, therefore, taking two-thirds of 5 and 9, we shall have

$$\text{angle } CEK = 3\tfrac{1}{3}°,$$
$$\text{angle } DEX = 6°,$$

and accordingly, by subtraction, as the angles of the eccentric's obliquity,

$$\text{angle } AEC = \text{angle } BED = 1°.$$

And from these we shall have as the arc of the epicycle's obliquity

$$\text{arc } HK = 2\tfrac{1}{4}°,$$

because, according to the Tables of Anomaly, the quantities found for angles *CEK* and *DEX* embrace just that many degrees.

And in the case of Saturn and Jupiter—since we find the passages about the apogeal sections of the eccentrics sensibly no different from those about the perigeal sections just opposite—we calculated what was required, in another way, from the comparison of those about the epicycles' apogees with those about their perigees. As we could easily see from particular observations, in the passages about the heliacal risings and settings, Saturn was at most very nearly 2° north and south, and Jupiter 1°; and in the passages about the acronychal oppositions, Saturn about 3°, Jupiter about 2°. Now, since it is evident from their

[1]For the distances to the star in those two positions would be as $(60 - 39\frac{1}{2} + 6)$ to $(60 - 39\frac{1}{2} - 6)$ or as very nearly 9 to 5. Now, the arcs being small and equal can be considered as perpendiculars to the lines representing the distances from the eye to the star. Then, since for small angles the angles are as their sines, the angles are as 5 to 9 very nearly. The same principle is used for Saturn and Jupiter in the next few pages; but since their epicycles are relatively small and their eccentricities also, it is necessary to make the comparison between the epicycle's apogee and perigee.

[2]For we have

$$(7° - \text{angle } BED) : (4\tfrac{3}{4}° - \text{angle } BED) :: 9 : 5,$$

separando

$$(7° - 4\tfrac{1}{3}°) : (4\tfrac{1}{3}° - \text{angle } BED) :: (9 - 5) : 5$$

or convertendo

$$(7° - 4\tfrac{1}{3}°) : (7° - \text{angle } BED) :: (9 - 5) : 9.$$

anomaly that, of the angles subtended at the eye by equal arcs about the perigeal and apogeal parts of the epicycle, those formed by arcs about the apogees have to those formed by arcs about the perigees, in the case of Saturn, the ratio of 18 to 23, and, in the case of Jupiter, of 29 to 43, and since *FG* and *HK* are equal arcs of the epicycle, therefore, in the case of Saturn,

angle *FEG* : angle *FEK* : : 18 : 23;

in the case of Jupiter

angle *FEG* : angle *FEK* : : 29 : 43.

But angle *GEK* is the difference between the two latitudinal passages for both stars and is a remainder of 1°. Therefore, if the 1° is divided according to these ratios, we shall have, in the case of Saturn,

angle $FEG = 0°26'$;

and, in the case of Jupiter,

angle $FEG = 0°24'$.

And we shall have, in the case of Saturn,

angle $FEK = 0°34'$;

and, in the case of Jupiter,

angle $FEK = 0°36'$.

And so, by subtraction, there will remain, as the angle of the eccentric's obliquity for Saturn,

angle $AEC = 2°26'$;

and, for Jupiter,

angle $AEC = 1°24'$.

For convenience we use, instead of these quantities, $2\frac{1}{2}°$ and $1\frac{1}{2}°$. And immediately it is concluded that, as the arc of the epicycle's obliquity, for Saturn,

arc $HK = 4\frac{1}{2}°$;

and, for Jupiter,

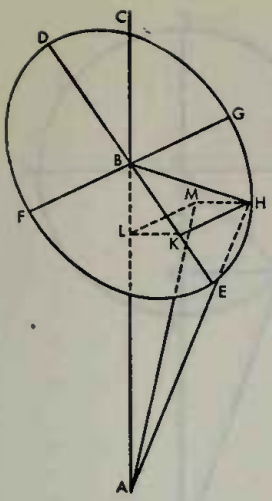
arc $HK = 2\frac{1}{2}°$.

For, again, just this number of degrees in each case embrace very nearly, according to the Tables of Anomaly, the quantities found for angles *FEG* and *FEK*. Which things it was required to find.

4. Construction of Tables for Particular Latitudinal Passages

By these means we have now established the general quantities of the epicycles' and eccentrics' greatest obliquities. But that we may also be able to work out easily at any time the latitudinal passages of the particular distances, we constructed five tables for the five planets, each having as many rows as the Tables of Anomaly and five columns. The first two columns contain the same numbers as in those. The third columns contain the latitudinal deviations from the ecliptic corresponding to sections of the epicycles with the epicycles at their point of greatest obliquity: in the case of Venus and Mercury, at the nodes of the eccentrics, and, for the other three stars, at the northern limits of the eccentrics. In the case of these latter, the fourth columns contain the similar additions about the eccentrics' southern limits. At the same time the greatest deviation of the eccentrics is calculated for these three stars. And, in the case of Venus and Mercury, we worked out the sections, again through one theorem in this way:

In the plane at right angles to the ecliptic's plane, let *ABC* be its intersection



with the ecliptic's plane, and *DBE* its intersection with the epicycle's plane. Let *A* be the ecliptic's centre, *B* the epicycle's, and let *AB* be the distance about the greatest obliquities of the epicycles. With epicycle *DFEG* drawn about *B*, let diameter *FBG* be joined perpendicular to *DE*. And let the epicycle's plane be supposed perpendicular to the plane of reference, so that, of the straight lines drawn perpendicular to *DE*, only *FG* lies in the ecliptic's plane and all the rest are parallel to it.

Given the ratio of *AB* to *BE* and the quantity of the obliquity (that is, angle *ABE*), let it be required to find the stars' latitudinal passages when, for example, they are 45° from *E* the epicycle's perigee. For we wish to demonstrate at the same time, by means of these inclinations, the differences resulting in the longitudinal passages. But these differences must be the greatest about the passages between the perigee *E* and *F* and *G* because they turn out to be the same as those effected without the obliquity.

Then let arc *EH* be taken as 45°. Let *HK* be drawn perpendicular to *BE*, and *KL* and *HM* to the ecliptic's plane. And let *HB*, *LM*, *AM*, and *AH* be joined.

Now, it would be immediately evident that the quadrilateral *LKHM* is a rectangular parallelogram because of *KH*'s being parallel to the ecliptic's plane; and that angle *LAM* embraces the longitudinal addition-subtraction and angle *HAM* the latitudinal passage, while angles *ALM* and *AMH* are right angles because of *AM*'s falling in the ecliptic's plane. It is now necessary to show how great the required passages are for each of these stars, and first for Venus.

Now, since

$$\text{arc } EH = 45°,$$

being at the epicycle's centre,

$$\text{angle } EBH = 45° = 90° \text{ to 2 rt.}$$

And so, on the circle about right triangle *BHK*,

$$\text{arc } BK = \text{arc } HK = 90°.$$

And therefore

$$\text{chord } BK = \text{chord } HK = 84^{p}52'$$

where

$$\text{hypt. } BH = 120^{p}.$$

And so also

$$BK = HK = 30^{p}32'$$

where

$$\text{epic. rad. } BH = 43^{p}10'$$

and, as mean distance,

$$AB = 60^{p};$$

for at this distance occurs the epicycle's greatest obliquity.

Again, since it is supposed that, as the angle of obliquity,

$$\text{angle } ABE = 2°30' = 5° \text{ to 2 rt.},$$

therefore, on the circle about right triangle BLK,

$$\text{arc } KL = 5°,$$

and, as its supplement,

$$\text{arc } BL = 175°.$$

And therefore

$$\text{chord } KL = 5^{p}14',$$
$$\text{chord } BL = 119^{p}53'$$

where

$$\text{hypt. } BK = 120^{p}.$$

And so also, where

$$\text{hypt. } BK = 30^{p}32',$$

and

$$AB = 60^{p},$$

there

$$KL = 1^{p}20',$$
$$BL = 30^{p}30',$$

and, by subtraction,

$$AL = 29^{p}30'.$$

And

$$LM = HK = 30^{p}32'.$$

And so it is concluded that

$$\text{hypt. } AM = 42^{p}27'.$$

And therefore, where

$$\text{hypt. } AM = 120^{p},$$

there

$$LM = 86^{p}19';$$

and, as the angle of longitudinal addition-subtraction for that time,

$$\text{angle } LAM = 92°0' \text{ to 2 rt.}$$
$$= 46°0'.$$

And likewise, since

$$HM = KL = 1^{p}20'$$

where

$$AM = 42^{p}27',$$

and the squares on them added together give the square on AH, therefore in length

$$AH = 42^{p}29'.$$

And

$$HM = 3^{p}46'$$

where

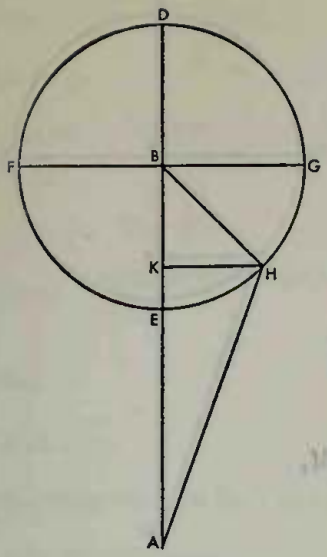
$$\text{hypt. } AH = 120^{p};$$

and, as the angle of latitudinal deviation,

$$\text{angle } HAM = 3°36' \text{ to 2 rt.}$$
$$= 1°48'.$$

And we place this in the third column of Venus' table in the row containing the number 135°.

For measuring the resulting difference of the longitudinal addition-subtrac-



tion, let a similar drawing be laid out having the epicycle not inclined.

Since we demonstrated that

$$BK = HK = 30^{p}32'$$

where

$$AB = 60^{p},$$

so that, by subtraction,

$$AK = 29^{p}28';$$

and since

$$\text{sq. } AK + \text{sq. } HK = \text{sq. } AH,$$

therefore, in length,

$$AH = 42^{p}26'.$$

And therefore

$$HK = 86^{p}21'$$

where

$$\text{hypt. } AH = 120^{p};$$

and, as the angle of longitudinal addition-subtraction,

$$\text{angle } HAK = 92°3' \text{ to 2 rt.}$$
$$\fallingdotseq 46°2'$$

But with the obliquity it was shown to be 46°. Therefore the longitudinal addition-subtraction, because of the epicycle's obliquity, is less by 0°2'. Which it was required to find.

Again, to show the passages of Mercury, let there be laid out a drawing similar to the one before this, with the supposition that

$$\text{arc } EH = 45°;$$

so that again

$$BK = HK = 84^{p}22'$$

where

$$\text{hypt. } BH = 120^{p}.$$

And therefore

$$BK = HK = 15^{p}55'$$

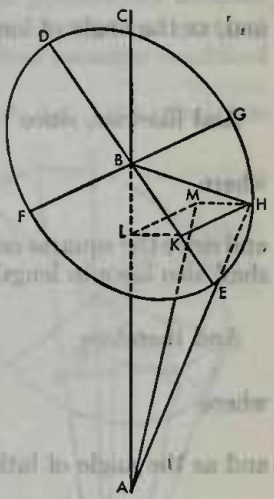
where

$$\text{epic. rad. } BH = 22^{p}30'$$

and, as the distance at the greatest obliquities,

$$AB = 56^{p}40';$$

for we have demonstrated all these things before.



Again, since, as the angle of the epicycle's obliquity, it is supposed

$$\text{angle } ABE = 6°15'$$
$$= 12°30' \text{ to 2 rt.},$$

therefore, on the circle about right triangle BKL,

$$\text{arc } KL = 12°30'$$

and, as its supplement,

$$\text{arc } BL = 167°30'.$$

And therefore

$$\text{chord } KL = 13^{p}4',$$
$$\text{chord } BL = 119^{p}17'$$

where

$$\text{hypt. } BK = 120^{p}.$$

And so also, where it was shown

$$BK = 15^{p}55'$$

and it is supposed

$$AB = 56^{p}40',$$

there also

$$KL = 1^{p}44',$$
$$BL = 15^{p}49'$$

and, by subtraction,

$$AL = 40^{p}51'.$$

And

$$LM = KH = 15^{p}55'.$$

And since

$$\text{sq. } AL + \text{sq. } LM = \text{sq. } AM,$$

we shall have in length

$$AM = 43^{p}50'$$

where

$$LM = 15^{p}55'.$$

And therefore

$$LM = 43^{p}34'$$

where

$$\text{hypt. } AM = 120^{p};$$

and, as the angle of longitudinal addition-subtraction,

$$\text{angle } LAM = 42°34' \text{ to 2 rt.}$$
$$= 21°17'.$$

And likewise, since

$$HM = KL = 1^{p}44'$$

where

$$AM = 43^{p}50',$$

and since the squares on both of them added together give the square on AH, we shall also have in length

$$AH = 43^{p}52'.$$

And therefore

$$HM = 4^{p}44'$$

where

$$\text{hypt. } AH = 120^{p};$$

and as the angle of latitudinal deviation

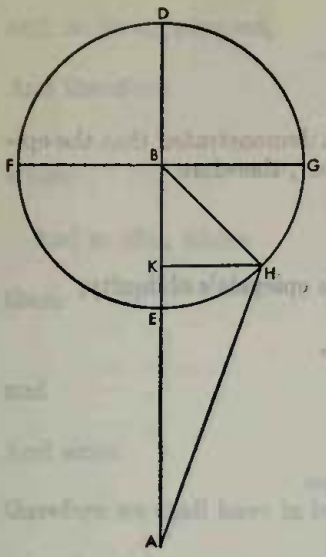
$$\text{angle } HAM = 4°32' \text{ to 2 rt.}$$
$$= 2°16'.$$

And again we shall place this in the third column of Mercury's table in the same row—that is, in the row containing the number of 135°.

Again, for the comparison of the addition-subtraction, let the drawing be laid out without the obliquity.

And since it was shown that, where

$$AB = 56^{p}40',$$



there

$$HK = BK = 15^p55'$$

and, by subtraction, clearly

$$AK = 40^p45',$$

and since

$$\text{sq. } AK + \text{sq. } HK = \text{sq. } AH,$$

therefore we shall have in length

$$AH = 43^p45'$$

where

$$HK = 15^p45'.$$

And therefore

$$HK = 43^p39'$$

where

$$\text{hypt. } AH = 120^p;$$

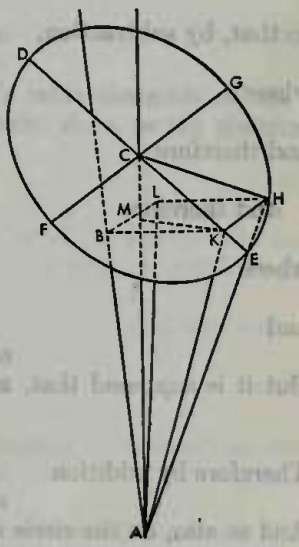
and as the angle of longitudinal addition-subtraction,

$$\text{angle } KAH = 42°40' \text{ to 2 rt.}$$
$$= 21°20'.$$

But it was shown to be 21°17′ with the obliquity. Therefore, in this case, the longitudinal addition-subtraction, because of the epicycle's obliquity, is less by 0°3′. Which was to be found.

Now, for these two stars we worked out the latitudinal passages at the greatest obliquities by taking them when the eccentric is in the same plane with the ecliptic. But, for the other three stars, we used another geometrical theorem since the epicycles' greatest obliquities come about at the eccentrics' greatest obliquities, and it would be helpful to have the latitudinal passages combined from both obliquities calculated out together.

For again, in the plane at right angles to the ecliptic's, let AB be its intersection with the ecliptic's plane, AC its intersection with the eccentric's plane, and DCE its intersection with the epicycle's plane. Let A be supposed the ecliptic's centre, and C the epicycle's. About C let the epicycle $DFEG$ be drawn again so that, of the straight lines drawn perpendicular to DE, the diameter FCG is in the eccentric's plane and parallel to the ecliptic's, and all the rest are parallel to both these planes. Likewise let arc EH be supposed cut off at 45°. And with HK drawn perpendicular from the star's point H, and KB and HL from the points H and K perpendicular to the ecliptic's plane, let BL and AL be joined.



It is required to find the longitudinal addition-subtraction contained by angle BAL, and the latitudinal passage contained by angle HAL.

Then let KM be drawn from K perpendicular to AC, and let CH and AK be joined. And again, because of things already shown, let it be sup-

posed that

$$CK = HK = 84^{p}52'$$

where

$$\text{hypt. } CH = 120^{p}.$$

Then, first in the case of Saturn, since it has been demonstrated that the epicycle's radius is $6^{p}30'$ where the mean distance is 60^{p}, therefore

$$CK = HK = 4^{p}36'$$

where

$$\text{hypt. } CH = 6^{p}30'.$$

And since it is supposed that, as the angle of the epicycle's obliquity,

$$\text{angle } ACE = 4°30'$$
$$= 9° \text{ to 2 rt.},$$

therefore, on the circle about right triangle CKM,

$$\text{arc } KM = 9°,$$

and, as its supplement,

$$\text{arc } CM = 171°.$$

Therefore

$$\text{chord } KM = 9^{p}25',$$
$$\text{chord } CM = 119^{p}38'$$

where

$$\text{hypt. } CK = 120^{p}.$$

And therefore

$$KM = 0°22',$$
$$CM = 4^{p}35'$$

where

$$CK = 4^{p}36'.$$

But, for the greatest obliquity in the apogeal semicircle, from theorems already worked out for the anomalies, it is calculated that, as the distance about the beginning of the Balance,

$$AC = 62^{p}10',$$

so that, by subtraction,

$$AM = 57^{p}35'$$

where

$$KM = 0^{p}22';$$

and therefore

$$\text{hypt. } AK = 57^{p}35'.$$

And therefore

$$KM = 0^{p}46'$$

where

$$\text{hypt. } AK = 120^{p};$$

and

$$\text{angle } KAM = 0°44' \text{ to 2 rt.}$$

But it is supposed that, as the angle of the eccentric's obliquity,

$$\text{angle } BAC = 2°\ 30'$$
$$= 5° \text{ to 2 rt.}$$

Therefore by addition

$$\text{angle } BAK = 5°44' \text{ to 2 rt.}$$

And so also, on the circle about right triangle BAK,

$$\text{arc } BK = 5°44',$$

and, as its supplement,

$$\text{arc } AB = 174°16'.$$

And therefore

$$\text{chord } BK = 6^{p}0',$$
$$\text{chord } AB = 119^{p}51'$$

where

$$\text{hypt. } AK = 120^{p}.$$

And so also, where

$$AK = 57^{p}35',$$

there

$$BK = 2^{p}53',$$
$$AB = 57^{p}31',$$

and

$$BL = HK = 4^{p}36'.$$

And since

$$\text{sq. } AB + \text{sq. } BL = \text{sq. } AL,$$

therefore we shall have in length

$$AL = 57^{p}42'.$$

And likewise, since

$$HL = BK = 2^{p}53',$$

and

$$\text{sq. } AL + \text{sq. } LH = \text{sq. } AH,$$

therefore we shall have in length

$$AH = 57^{p}46'.$$

And so also

$$LH = 5^{p}59'$$

where

$$\text{hypt. } AH = 120^{p};$$

and, as the angle of latitudinal deviation,

$$\text{angle } HAL = 5°44' \text{ to 2 rt.}$$
$$= 2°52'.$$

And we place this in the third column of Saturn's table alongside of 135°.

For the greatest obliquity in the perigeal semicircle, since, as the distance at the beginning of the Ram,

$$AC = 57^{p}40'$$

where it was shown

$$KM = 0^{p}22';$$

and likewise

$$CM = 4^{p}35',$$

and therefore, by subtraction,

$$AM = 53^{p}5';$$

and because it is hardly at all greater than AM,

$$\text{hypt. } AK = 53^{p}5',$$

therefore

$$KM = 0^{p}\ 50'$$

where

$$\text{hypt. } AK = 120^{p};$$

and

angle $KAM = 0°48'$ to 2 rt.

And it is supposed

angle $BAC = 5°$ to 2 rt.

And therefore, by addition,

angle $BAK = 5°48'$ to 2 rt.

And so also, on the circle about right triangle BAK,

arc $BK = 5°48'$,

and, as its supplement,

arc $AB = 174°12'$.

And therefore

chord $BK = 6^p4'$,

chord $AB = 119^p51'$

where

hypt. $AK = 120^p$.

And so also

$BK = 2^p41'$,

$AB = 53^p1'$

where

$AK = 53^p5'$.

And since

sq. AB + sq. BL = sq. AL

and it was shown that of the same parts

$BL = 4^p36'$,

therefore we shall have in length

$AL = 53^p13'$.

And therefore

$BL = 10^p23'$

where

hypt. $AL = 120^p$;

and, as the angle of the longitudinal addition-subtraction,

angle $BAL = 9°56'$ to 2 rt.

$= 4°58'$.

Again, since

$HL = BK = 2^p41'$

where

$AL = 53^p13'$,

and the squares on both added together give the square on AH, therefore we shall have in length

$AH = 53^p17'$.

And therefore

$HL = 6^p3'$

where

hypt. $AH = 120^p$;

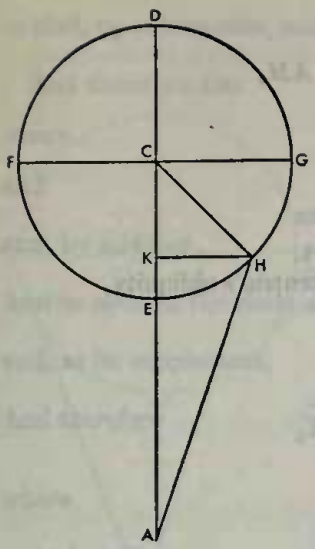
and, as the angle of latitudinal deviation,

angle $HAL = 5°46'$ to 2 rt.

$= 2°53'$.

And we place this in the fourth column of the table beside 135°.

Then, to make the comparison of the longitudinal addition-subtraction for



the perigeal obliquity, again let the figure be drawn as having no obliquity.

And since, where, as the distance at that time,

$$AC = 57^{p}40',$$

there it is supposed

$$CK = HK = 4^{p}36'$$

and, by subtraction,

$$AK = 53^{p}4',$$

and since

$$\text{sq. } AK + \text{sq. } HK = \text{sq. } AH,$$

therefore we shall have in length

$$AH = 53^{p}16'.$$

And so also

$$HK = 10^{p}22'$$

where

$$\text{hypt. } AH = 120^{p};$$

and, as the angle of longitudinal addition-subtraction,

$$\text{angle } HAK = 9°54' \text{ to 2 rt.}$$
$$= 4°57'.$$

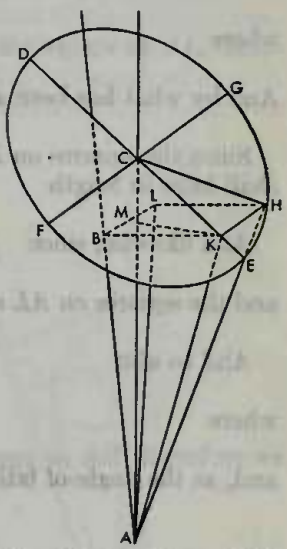
But with the obliquities it was shown to be 4°58′. Therefore the longitudinal addition-subtraction, because of both obliquities, is greater by 0°1′. Which was to be found.

Again, let there be first laid out the drawing with the obliquities, containing the ratios we have already demonstrated for Jupiter, so that

$$CK = HK = 8^{p}8'$$

where

$$\text{epic. rad. } CH = 11^{p}30'.$$



Now since, as the angle of the epicycle's obliquity, it is supposed that

$$\text{angle } ACE = 2°30'$$
$$= 5° \text{ to 2 rt.},$$

therefore, on the circle about right triangle CKM,

$$\text{arc } KM = 5°,$$

and, as its supplement,

$$\text{arc } CM = 175°.$$

And therefore

$$\text{chord } KM = 5^{p}14',$$
$$\text{chord } CM = 119^{p}53'$$

where

$$\text{hypt. } CK = 120^{p}.$$

And so also, where

$$CK = 8^{p}8'$$

and, as the distance about the beginning of the Balance,

$$AC = 62^{p}30',$$

there also

$$KM = 0^{p}21',$$
$$CM = 8^{p}8',$$

and, by subtraction,

$$AM = 54^{p}22';$$

and therefore, since it is hardly at all greater than AM,

$$\text{hypt. } AK = 54^{p}22'.$$

And therefore

$$KM = 0^{p}46'$$

where

$$\text{hypt. } AK = 120^{p};$$

and

$$\text{angle } KAM = 0°44' \text{ to 2 rt.}$$

But it is also supposed that, as the angle of the eccentric's obliquity,

$$\text{angle } BAC = 1°30'$$
$$= 3° \text{ to 2 rt.}$$

And therefore, by addition,

$$\text{angle } BAK = 3°44' \text{ to 2 rt.}$$

And so also, on the circle about right triangle BAK,

$$\text{arc } BK = 3°44',$$

and, as its supplement,

$$\text{arc } AB = 176°16'.$$

And therefore

$$\text{chord } BK = 3^{p}54',$$
$$\text{chord } AB = 119^{p}56'$$

where

$$\text{hypt. } AK = 120^{p}.$$

And so also

$$BK = 1^{p}46',$$
$$AB = 54^{p}20'$$

where

$$AK = 54^{p}22'.$$

And by what has been already demonstrated

$$BL = 8^{p}8'.$$

Since the squares on AB and BL added together give the square on AL, we shall have in length

$$AL = 54^{p}56'.$$

And likewise, since

$$HL = 1^{p}46',$$

and the squares on AL and HL together give the square on AH, we shall have

$$AH = 54^{p}58'.$$

And so also

$$HL = 3^{p}52'$$

where

$$\text{hypt. } AH = 120^{p};$$

and, as the angle of latitudinal deviation,

$$\text{angle } HAL = 3°42' \text{ to 2 rt.}$$
$$= 1°51'.$$

This we place in the third column of Jupiter's table beside 135°.

Likewise, since again as the distance at the beginning of the Ram

$$AC = 57^{p}30',$$
$$CM = 8^{p}8'$$

where it was shown

$$KM = 0^{p}21',$$

so that, by subtraction, and since it is hardly at all greater than AK,

$$AM = AK = 49^{p}22'.$$

And therefore also

$$KM = 0^{p}51'$$

where

$$\text{hypt. } AK = 120^{p};$$

and

$$\text{angle } KAM = 0°49' \text{ to 2 rt.},$$

and, by addition,

$$\text{angle } BAK = 3°49' \text{ to 2 rt.}$$

And so also, on the circle about right triangle AKB,

$$\text{arc } BK = 3°49',$$

and, as its supplement,

$$\text{arc } AB = 176°11'.$$

And therefore

$$\text{chord } BK = 3^{p}59',$$
$$\text{chord } AB = 119^{p}56'$$

where

$$\text{hypt. } AK = 120^{p}.$$

And so also

$$BK = 1^{p}39',$$
$$AB = 49^{p}20'$$

where

$$AK = 49^{p}22'.$$

And therefore, since

$$BL = 8^{p}8',$$

and the squares on AB and BL added together make the square on AL, therefore we shall have in length

$$AL = 50^{p}0'.$$

And so also

$$BL = 19^{p}31'$$

where

$$\text{hypt. } AL = 120^{p};$$

and, as the angle of longitudinal addition-subtraction,

$$\text{angle } BAL = 18°44' \text{ to 2 rt.}$$
$$= 9°22'.$$

Again, since

$$HL = 1^{p}39'$$

where

$$AL = 50^{p},$$

and the squares on them added together give the square on AH, therefore we shall have in length

$$AH = 50^{p}2'.$$

And therefore

$$HL = 3^{p}57'$$

where

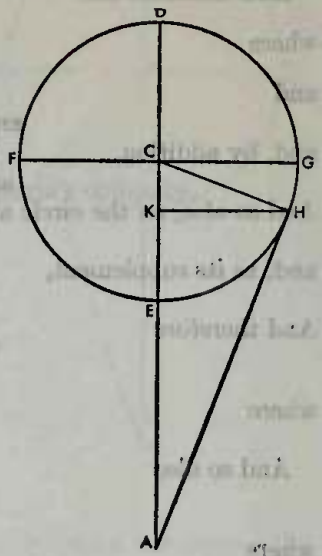
$$\text{hypt. } AH = 120^{p};$$

and as the angle of latitudinal distance

$$\text{angle } HAL = 3°46' \text{ to 2 rt.}$$
$$= 1°53'.$$

This we place in the fourth column of the table beside 135°.

For the comparison of the longitudinal addition-subtractions, let the drawing be laid out without the obliquities.



Since, at the given distance, where

$$HK = CK = 8^{p}8',$$

there by addition

$$AC = 57^{p}30'$$

and, by subtraction,

$$AK = 49^{p}22';$$

and since

$$\text{sq. } AK + \text{sq. } HK = \text{sq. } AH,$$

therefore we shall have in length

$$AH = 50^{p}2'.$$

And so also

$$HK = 19^{p}30'$$

where

$$\text{hypt. } AH = 120^{p};$$

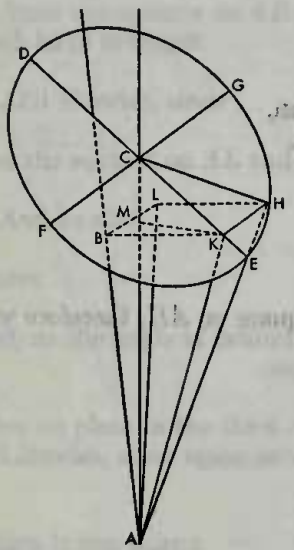
and, as the angle of longitudinal addition-subtraction,

$$\text{angle } HAK = 18°42' \text{ to 2 rt.}$$
$$= 9°21'.$$

But with the obliquities it was shown to be 9°22'. Therefore, again, the longitudinal addition-subtraction, because of both obliquities, is more by 0°1'. Which it was required to find.

Next for the ratios of Mars, first let the drawing of the obliquities be laid out, and again let it be concluded that

$$CK = HK = 27^{p}56'$$



where

$$\text{epic. rad. } CH = 39^{p}30'.$$

Now, since as the angle of the epicycle's obliquity it is supposed that

$$\text{angle } ACE = 2°15'$$
$$= 4°30' \text{ to 2 rt.},$$

therefore, on the circle about right triangle CMK,

$$\text{arc } KM = 4°30'$$

and, as its supplement,

$$\text{arc } CM = 175°30'.$$

And therefore

$$\text{chord } KM = 4^{p}43',$$
$$\text{chord } CM = 119^{p}54'$$

where

$$\text{hypt. } CK = 120^{p}.$$

And so, where

$$CK = 27^{p}56',$$

and as greatest distance

$$AC = 66^{p},$$

there also

$$KM = 1^{p}6',$$
$$CM = 27^{p}54',$$

and, by subtraction,

$$AM = 38^{p}6';$$

and therefore also

$$\text{hypt. } AK = 38^{p}7'.$$

And therefore

$$KM = 3^{p}28'$$

where

$$\text{hypt. } AK = 120^{p};$$

and

$$\text{angle } KAM = 3°19' \text{ to 2 rt.}$$

But it is supposed that, as the angle of the eccentric's obliquity,

$$\text{angle } BAC = 1°$$
$$= 2° \text{ to 2 rt.}$$

And therefore, by addition,

$$\text{angle } BAK = 5°19' \text{ to 2 rt.}$$

And so also, on the circle about right triangle BAK,

$$\text{arc } BK = 5°19'$$

and, as its supplement,

$$\text{arc } AB = 174°41'.$$

And therefore

$$\text{chord } BK = 5^{p}34',$$
$$\text{chord } AB = 119^{p}52'$$

where

$$\text{hypt. } AK = 120^{p}.$$

And so also, where

$$AK = 38^{p}7',$$

there also

$$BK = 1^{p}46',$$

and

$$AB = 38^{p}5'.$$

But

$$BL = 27^{p}56'.$$

And since

$$\text{sq. } AB + \text{sq. } BL = \text{sq. } AL,$$

we shall have in length

$$AL = 47^{p}14'.$$

And since of like parts

$$HL = 1^{p}46'$$

and

$$\text{sq. } AL + \text{sq. } HL = \text{sq. } AH,$$

therefore we shall have in length

$$AH = 47^{p}16'.$$

And so also

$$HL = 4^{p}29'$$

where

$$\text{hypt. } AH = 120^{p};$$

and as the angle of latitudinal distance

$$\text{angle } HAL = 4°18' \text{ to 2 rt.}$$
$$= 2°9'.$$

This we place in the third column of Mars' table beside 135°.

Likewise, for the obliquities at the least distance, since

$$AC = 54^{p},$$
$$CM = 27^{p}54'$$

where it was shown

$$KM = 1^{p}6',$$

so that, by subtraction,

$$AM = 26^{p}6'$$

and it is calculated that

$$\text{hypt. } AK = 26^{p}7';$$

therefore

$$KM = 5^{p}3'$$

where

$$\text{hypt. } AK = 120^{p};$$

and

$$\text{angle } KAM = 4°49' \text{ to 2 rt.}$$

And for that reason, by addition,

$$\text{angle } BAK = 6°49' \text{ to 2 rt.}$$

And so also, on the circle about right triangle ABK,

$$\text{arc } BK = 6°49'$$

and, as its supplement,

$$\text{arc } AB = 173°11'.$$

And therefore

$$\text{chord } BK = 7^{p}8',$$
$$\text{chord } AB = 119^{p}47'$$

where

$$\text{hypt. } AK = 120^{p}.$$

And so also

$$BK = 1^{p}33'$$

and

$$AB = 26^{p}4'$$

where

$$AK = 26^{p}7'.$$

And again, of like parts,

$$BL = 27^{p}56'.$$

And since

$$\text{sq. } AB + \text{sq. } BL = \text{sq. } AL,$$

we shall have in length

$$AL = 38^{p}12'.$$

And so also

$$BL = 87^{p}45'$$

where

hypt. $AL = 120^p$;

and, as the angle of longitudinal addition-subtraction,

angle $BAL = 94°$ to 2 rt.
$= 47°$.

Likewise, since

$HL = 1^p33'$

where

$AL = 38^p12'$,

and since the squares on them added together give the square on AH, therefore we shall have in length

$AH = 38^p14'$.

And so also

$HL = 4^p52'$

where

hypt. $AH = 120^p$;

and as the angle of latitudinal distance

angle $HAL = 4°40'$ to 2 rt.
$= 2°20'$.

This we place in the fourth column of the table beside the 135°.

And now again, for the comparison of the longitudinal addition-subtractions, if we lay out the drawing without the obliquities, then, at the least distance where the difference is necessarily most sensible,

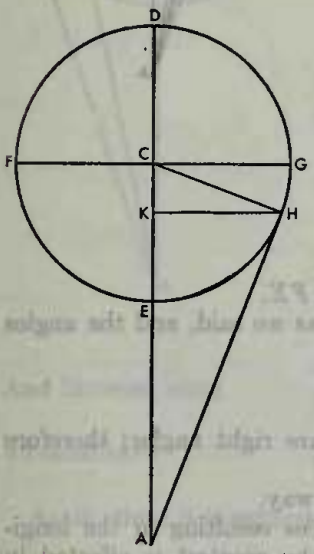
$AC : CK$ or $HK :: 54^p : 27^p56'$;

so that for this reason, by subtraction,

$AK = 26^p4'$

and it is calculated

hypt. $AH = 38^p12'$.



And therefore also, again,

$HK = 87^p45'$

where

hypt. $AH = 120^p$;

and, as the angle of longitudinal addition-subtraction,

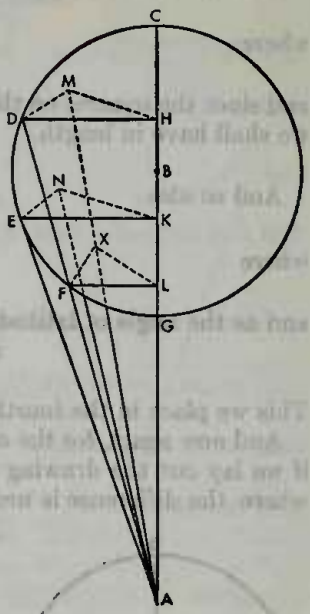
angle $HAK = 94°$ to 2 rt.
$= 47°$.

But from calculations with the obliquities it was shown to be just that much. Therefore, in the case of Mars, the longitudinal addition-subtraction does not differ as a result of the obliquities of the circles. Which was to be found.

The fourth columns of the two tables of Venus and Mercury will contain the latitudinal passages embraced by the greatest slantings of the epicycles which occur at the eccentrics' apogees and perigees. But these passages have been worked out by us independently and separate from the difference due to the eccentrics' obliquities. Otherwise we should need more tables and a difficult calculus, since the evening and morning passages will be unequal and never at

the same parts of the ecliptic; and besides, since the obliquity of the eccentrics varies, the differences in the decreases for the greatest obliquities would not agree with those in the decreases for the greatest slantings. But with the difference separated out, each case can be more easily handled as will be clear from what we are going to add.

Let AB be the intersection of the ecliptic's plane and the epicycle's plane, point A being supposed the ecliptic's centre and B the epicycle's centre. And let epicycle $CDEFG$ be drawn about B slanting to the ecliptic's plane—that is, so that the straight lines drawn in them perpendicular to the intersection CG make equal all the angles resulting at the points CG. Let AE be produced tangent to the epicycle, and AFD cutting it at random. And let DH, EK, and FL be drawn from points D, E, and F perpendicular to CG; and DM, EN, and FX perpendicular to the ecliptic's plane. And let HM, KN, and LX be joined; and also AN and AXM. For AXM is one straight line, since the three points are in two planes—that of the ecliptic and that through AFD at right angles to the ecliptic's.



Now, it is evident that, for the given slanting, angles HAM and KAN contain the longitudinal addition-subtractions of the stars, and angles DAM and EAN the latitudinal addition-subtractions.

It is first necessary to show that the latitudinal passage (angle EAN) formed at the tangent is greater than any others, just like the longitudinal addition-subtraction.

For, since angle EAK is greater than any others,

$$EK : AE > DH : AD$$
$$> FL : AF.$$

But

$$EK : EN :: DH : DM :: FL : FX.$$

For the triangles thus formed are all equiangular as we said, and the angles at M, N, and X are right. And, therefore,

$$EN : AE > DM : AD$$
$$> FX : AF.$$

And, again, the angles DMA, ENA, and FXA are right angles; therefore

$$\text{angle } EAN > \text{angle } DAM$$

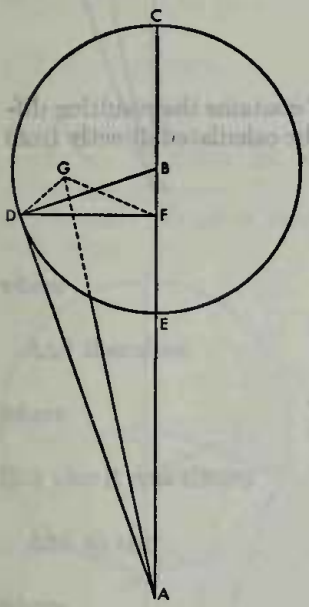
and greater than all the others formed in the same way.

It is immediately evident that, of the differences resulting in the longitudinal addition-subtractions from the slanting, the greatest is effected in the greatest passages at E, since they are contained by the angles subtending the differences of HD, KE, and LF over HM, KN, and LX. Since the same ratio holds for each, it follows, relative to the differences, that the difference between EK and KN has a greater ratio to EA than those of any others to lines similar to AD. It is immediately clear also that, whatever ratio the

greatest longitudinal addition-subtraction has to the greatest latitudinal passage, that ratio the longitudinal addition-subtractions in each case have to the latitudinal passages for all sections of the epicycle, since as EK is to EN so are all lines like FL and DH to those like FX and DM. Which it was required to show.

Now, with these things worked out, let us see first how great an angle is contained, for each of the stars, by the slant of the planes, if we suppose, according to postulates already assumed at the beginning, that about midway between the greatest and least distance each of them is at the most 5° north and south of the opposite passage on the epicycle. For Venus appears to make a deviation at the eccentric's perigee and apogee of hardly more or less than 5°, and Mercury of ½° more and less than 5°.

Now, again, let ABC be the intersection of the ecliptic and the epicycle. With the epicycle drawn about point B aslant the ecliptic's plane as we have described, let AD be joined tangent to the epicycle from the ecliptic's centre A. From D let DF be drawn perpendicular to CBE, and DG perpendicular to the ecliptic's plane. And let BD, FG, and AG be joined.



And let angle DAG, containing half of the given latitudinal deviation, be supposed to be 2½° for either of the stars. And let it be required to find the slant of the two planes—that is, the size of angle DFG.

Now, in the case of Venus, since where

$$\text{epic. rad.} = 43^{\text{p}}10',$$

there

$$\text{greatest dist.} = 61^{\text{p}}15',$$
$$\text{least dist.} = 58^{\text{p}}45'$$

and

$$\text{mean dist.} = 60^{\text{p}},$$

therefore

$$AB : BD :: 60^{\text{p}} : 43^{\text{p}}10'.$$

And since

$$\text{sq. } AB - \text{sq. } BD = \text{sq. } AD,$$

we shall have in length

$$AD = 41^{\text{p}}40'.$$

And likewise, since

$$AB : AD :: BD : DF,$$

we shall have

$$DF = 29^{\text{p}}58'.$$

Again, since it is supposed that

$$\text{angle } DAG = 2°30'$$
$$= 5° \text{ to 2 rt.},$$

therefore, on the circle about right triangle ADG,

$$\text{arc } DG = 5°$$

and

$$\text{chord } DG = 5^{\text{p}}14'$$

where

hypt. $AD = 120^{p}$.

And therefore

$DG = 1^{p}50'$

where

$AD = 41^{p}40'$.

But it was shown

$DF = 29^{p}58'$.

And so also

$DG = 7^{p}20'$

where

hypt. $DF = 120^{p}$;

and, as the angle of slant,

angle $DFG = 7°$ to 2 rt.
$= 3°30'$.



But since the excess of angle DAF over angle GAF contains the resulting difference of longitudinal addition-subtraction, it must be calculated directly from their size. For since it was shown that

hypt. $AD = 31^{p}40'$

and

$DF = 29^{p}58'$

where

$DG = 1^{p}50'$;

and since

sq. AD—sq. DG = sq. AG,
sq. FD—sq. DG = sq. FG,

therefore we shall have in length

$AG = 41^{p}37'$,
$FG = 29^{p}55'$.

And so also

$FG = 86^{p}16'$

where

hypt. $AG = 120^{p}$;

and

angle $FAG = 91°56'$ to 2 rt.
$= 45°58'$.

And likewise, since

$DF = 86^{p}18'$

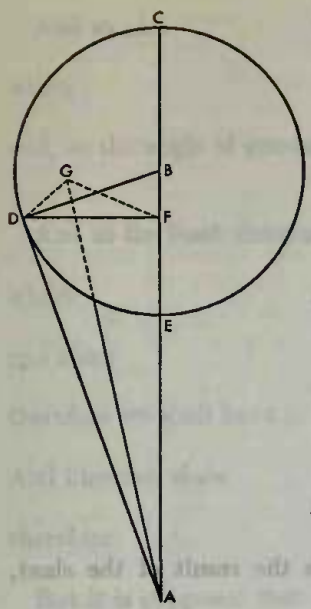
where

hypt. $AD = 120^{p}$;

and we shall have

angle $DAF = 91°58'$ to 2 rt.
$= 45°59'$.

Therefore the longitudinal addition-subtraction, as a result of the slant, is less by $0°1'$.



And, in the case of Mercury, since where
epic. rad. = 22ᵖ30′,
there it was shown
greatest dist. = 69ᵖ,
least dist. = 57ᵖ,
and
mean dist. = 63ᵖ,
therefore
$AB : BD :: 63^p : 22^p30'$.
And since
sq. AB—sq. BD = sq. AD,
we shall have also in length
$AD = 58^p51'$.
And likewise, since
$AB : AD :: BD : DF$,
therefore of the same parts
$DF = 21^p1'$.

Again, since it is supposed that
angle $DAG = 5°$ to 2 rt.,
therefore, on the circle about right triangle ADG,
arc $DG = 5°$;
and
chord $DG = 5^p14'$
where
hypt. $AD = 120^p$.

And therefore
$DG = 2^p34'$
where
$AD = 58^p51'$.
But also it was shown
$DF = 21^p1'$.
And so also
$DG = 14^p40'$
where
hypt. $DF = 120^p$;
and as the angle of slant
angle $DFG = 14°$ to 2 rt.
$= 7°$.

Likewise, for the comparison of the angles of addition-subtraction, since again it was shown
hypt. $AD = 58^p51'$
and
$DF = 21^p1'$
where
$DG = 2^p34'$,
and since
sq. AD − sq. DG = sq. AG,

sq. DF − sq. DG = sq. FG,

therefore we shall have in length

$AG = 58^{p}47'$,

$FG = 20^{p}53'$.

And so also

$FG = 42^{p}38'$

where

hypt. $AG = 120^{p}$;

and

angle $FAG = 41°38'$ to 2 rt.
$= 20°49'$.

And in the same way, since

$DF = 42^{p}50'$

where

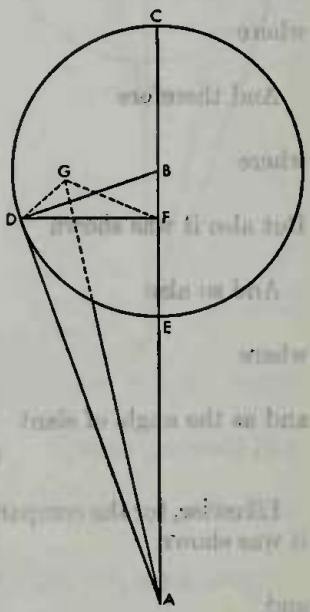
hypt. $AD = 120^{p}$,

therefore we shall have

angle $DAF = 41°50'$ to 2 rt.
$= 20°55'$.

Therefore the longitudinal addition-subtraction, as the result of the slant, is less by 0°6′. Which it was required to find.

Following this, let us see if, when we suppose these to be the sizes of the slants, we find the latitudinal passages at the greatest and least distances consistent with those gotten by observations.



Again, with the same drawing, let Venus' greatest distance be supposed first—that is, let

$AB : BD :: 61^{p}15' : 43^{p}10'$,

so that, since

sq. AB − sq. BD = sq. AD,

therefore it is concluded

$AD = 43^{p}27'$.

But

$AB : AD :: BD : DF$;

and therefore

$DF = 30^{p}37'$.

Again, since it is supposed that, as the angle of slant,

angle $DFG = 7°$ to 2 rt.

and that

$DG = 7^{p}20'$

where

$DF = 120^{p}$,

therefore

$DG = 1^{p}52'$

where

$DF = 30^{p}37'$

and

$AD = 43^{p}27'$.

And so also

$$DG = 5^{p}9'$$

where

$$\text{hypt. } AD = 120^{p};$$

and, as the angle of greatest latitudinal deviation,

$$\text{angle } DAG = 4°54' \text{ to 2 rt.}$$
$$= 2°27'.$$

And at the least distance, since it is supposed that

$$AB = 58^{p}45'$$

where

$$\text{epic. rad. } BD = 43^{p}10',$$

and since

$$\text{sq. } AB - \text{sq. } BD = \text{sq. } AD,$$

therefore we shall have in length

$$AD = 39^{p}51'.$$

And likewise, since

$$AB : AD :: BD : DF,$$

therefore

$$DF = 29^{p}17'.$$

But it is supposed that

$$DF : DG :: 120^{p} : 7^{p}20'.$$

And therefore

$$DG = 1^{p}47'$$

where

$$DF = 29^{p}17'$$

and

$$AD = 39^{p}51'.$$

And so also

$$DG = 5^{p}22'$$

where

$$\text{hypt. } AD = 120^{p};$$

and as the angle of greatest latitudinal deviation

$$\text{angle } DAG = 5°8' \text{ to 2 rt.}$$
$$= 2°34'.$$

Therefore the latitudinal deviation is insensibly less at the apogee, and insensibly more at the perigee, than the deviation of $2\frac{1}{2}°$ assumed in mean terms. For at the greatest distance it is less by 0°3′, and at the least distance more by 0°4′, which obviously cannot be gotten by observations.

Again, let the greatest distance of Mercury be supposed—that is, let

$$AB : BD :: 69^{p} : 22^{p}30',$$

so that by the same means as above it is concluded that of the same parts

$$AD = 65^{p}14'$$

and

$$DF = 21^{p}16'.$$

But here we have, as the supposed angle of slant,

$$\text{angle } DFG = 14° \text{ to 2 rt.},$$

and therefore we have

$$DG = 14^{p}40'$$

where

hypt. $DF = 120^p$.

And therefore

$DG = 2^p36'$

where

$DF = 21^p16'$

and

$AD = 65^p14'$.

And so also

$DG = 4^p47'$

where

hypt. $AD = 120^p$;

and, as the angle of greatest latitudinal deviation,

angle $DAG = 4°34'$ to 2 rt.
$= 2°17'$.

At the least distance it is supposed that

$AB : BD :: 57^p : 22^p30'$.

And since because of the same slant it is supposed that

$DF : DG :: 120^p : 14^p40'$,

therefore

$DG = 2^p32'$

where

$DF = 20^p40'$

and

$AD = 52^p22'$.

And so also

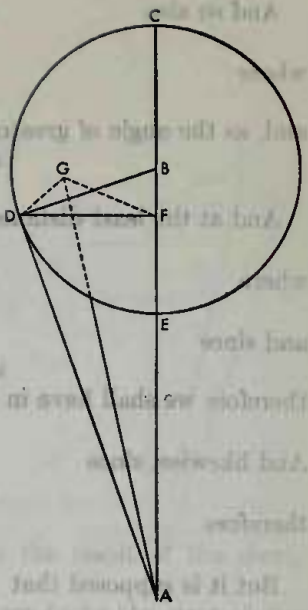
$DG = 5^p48'$

where

hypt. $AD = 120^p$;

and

angle $DAG = 5°32'$ to 2 rt.
$= 2°46'$.



Therefore the latitudinal deviation at the apogee differs at the least by 0°13′ and at the perigee at the most by 0°16′ from the deviation of 2½° assumed in mean terms. In place of these we shall use, for the mean correction in the calculus, the sensible ¼° difference of the observations.

With these things demonstrated—and because, as the greatest longitudinal addition-subtractions are to the greatest latitudinal passages, so are the particular longitudinal addition-subtractions for the other sections of the epicycle to the particular latitudinal passages—the filling in of the latitudinal passages due to slanting, in these fourth columns of Venus' and Mercury's tables, is immediately available to us, although these passages, as we said, are calculated with only the slanting of the epicycles and from the mean addition. Because of greater ease in the calculus which will be given, the difference for Mercury, due to the eccentrics' obliquity at the apogee and perigee, will provide the correction.

Since, according to the given mean terms, the greatest latitudinal passage of both stars on either side of the ecliptic was shown to be 2°30′, due to slant-

ing; since the greatest longitudinal addition-subtraction for Venus is 46° and for Mercury very nearly 22°; and since we have given in their Tables of Anomaly the addition-subtractions corresponding to the particular sections of the epicycles, it follows that whatever part these latter are of the greatest longitudinal addition-subtractions as wholes, we take that part of the 2°30′ for each star, placing the result in the fourth columns of the Tables of Latitude beside the same numbers.

We have the fifth columns for truly determining, by the method of corresponding sixtieths, the latitudinal deviations in the other passages of the eccentrics. For since, as we said, the obliquities and slantings of the epicycles make their return of increase and decrease proportionately with the eccentric return, by the interposition of little circles, and the quantities of all the obliquities and slantings are not very different from that of the moon's oblique circle, and since the particular deviations for obliquities of such amounts are very nearly proportional and we have those of the moon all worked out geometrically, we multiplied each of the entries there by twelve (because the greatest addition there is 5° and here we are considering it as 60°). The results we have placed beside the proper numbers in each of the fifth columns. The tabulated exposition follows:

5. Exposition of the Tables of Latitude

SATURN'S OBLIQUITIES						JUPITER'S OBLIQUITIES						MARS' OBLIQUITIES					
1	2	3		4		5		3		4		5		3		4		5	
Numbers from Apogee		*From Northern Limit*		*From Southern Limit*		*Sixtieths*		*From Northern Limit*		*From Southern Limit*		*Sixtieths*		*From Northern Limit*		*From Southern Limit*		*Sixtieths*	
6	354	2	4	2	2	59	36	1	7	1	5	59	36	0	8	0	4	59	36
12	348	2	5	2	3	58	36	1	8	1	6	58	36	0	9	0	4	58	36
18	342	2	6	2	3	57	0	1	8	1	6	57	0	0	11	0	5	57	0
24	336	2	7	2	4	54	36	1	9	1	7	54	36	0	13	0	6	54	36
30	330	2	8	2	5	52	0	1	10	1	8	52	0	0	14	0	7	52	0
36	324	2	10	2	7	48	24	1	11	1	9	48	24	0	15	0	9	48	24
42	318	2	11	2	8	44	24	1	12	1	10	44	24	0	18	0	12	44	24
48	312	2	12	2	10	40	0	1	13	1	11	40	0	0	21	0	15	40	0
54	306	2	14	2	12	35	12	1	14	1	13	35	12	0	24	0	18	35	12
60	300	2	16	2	15	30	0	1	16	1	16	30	0	0	28	0	22	30	0
66	294	2	18	2	18	24	24	1	18	1	18	24	24	0	32	0	26	24	24
72	288	2	21	2	21	18	24	1	21	1	21	18	24	0	36	0	30	18	24
78	282	2	24	2	24	18	24	1	24	1	24	12	24	0	41	0	36	12	24
84	276	2	27	2	27	6	24	1	27	1	27	6	24	0	46	0	42	6	24
90	270	2	30	2	30	0	0	1	30	1	30	0	0	0	52	0	49	0	0
93	267	2	31	2	31	3	12	1	31	1	31	3	12	0	55	0	52	3	12
96	264	2	33	2	33	6	24	1	33	1	33	6	24	0	59	0	56	6	24
99	261	2	34	2	34	9	24	1	34	1	34	9	24	1	3	1	0	9	24
102	258	2	36	2	36	12	24	1	36	1	36	12	24	1	6	1	4	12	24
105	255	2	37	2	37	15	24	1	37	1	37	15	24	1	10	1	8	15	24
108	252	2	39	2	39	18	24	1	39	1	39	18	24	1	14	1	13	18	24
111	249	2	40	2	40	21	24	1	40	1	40	21	24	1	18	1	18	21	24
114	246	2	42	2	42	24	24	1	42	1	42	24	24	1	23	1	24	24	24
117	243	2	43	2	43	27	12	1	43	1	43	27	12	1	28	1	30	27	12
120	240	2	45	2	45	30	0	1	45	1	45	30	0	1	34	1	37	30	0
123	237	2	46	2	46	32	36	1	46	1	46	32	36	1	41	1	44	32	36
126	234	2	47	2	48	35	12	1	47	1	48	35	12	1	48	1	51	35	12
129	231	2	49	2	49	37	36	1	49	1	49	37	36	1	54	2	0	37	36
132	228	2	50	2	51	40	0	1	50	1	51	40	0	2	1	2	10	40	0
135	225	2	52	2	53	42	12	1	51	1	53	42	12	2	9	2	20	42	12
138	222	2	53	2	54	44	22	1	52	1	54	44	24	2	16	2	32	44	24
141	219	2	54	2	55	46	36	1	53	1	55	46	36	2	25	2	44	46	36
144	216	2	55	2	56	48	24	1	55	1	57	48	24	2	34	2	56	48	24
147	213	2	56	2	57	50	12	1	56	1	59	50	12	2	44	3	12	50	12
150	210	2	57	2	58	52	0	1	58	2	0	52	0	2	54	3	29	52	0
153	207	2	58	2	59	53	12	1	59	2	1	53	12	3	5	3	46	53	12
156	204	2	59	3	0	54	36	2	0	2	3	54	36	3	16	4	9	54	36
159	201	2	59	3	1	56	0	2	1	2	4	56	0	3	27	4	32	56	0
162	198	3	0	3	2	57	0	2	2	2	5	57	0	3	38	4	55	57	0
165	195	3	0	3	2	57	48	2	2	2	6	57	48	3	49	5	24	57	48
168	192	3	1	3	3	58	36	2	3	2	6	58	36	4	0	5	53	58	36
171	189	3	1	3	3	59	12	2	3	2	7	59	12	4	10	6	21	59	12
174	186	3	2	3	4	59	36	2	4	2	7	59	36	4	14	6	36	59	36
177	183	3	2	3	4	59	48	2	4	2	8	59	48	4	18	6	51	59	48
180	180	3	2	3	5	60	0	2	4	2	8	60	0	4	21	7	7	60	0

TABLE 5—*Continued*

VENUS' OBLIQUITIES						MERCURY'S OBLIQUITIES					
1	2	3		4		5		3		4		5	
Numbers from Apogee		*Obliquity*		*Slanting*		*Sixtieths*		*Obliquity*		*Slanting*		*Sixtieths*	
6	354	1	2	0	8	59	36	1	45	0	11	59	36
12	348	1	1	0	16	58	36	1	44	0	22	58	36
18	342	1	0	0	25	57	0	1	43	0	33	57	0
24	336	0	59	0	33	54	36	1	40	0	44	54	36
30	330	0	57	0	41	52	0	1	36	0	55	52	0
36	324	0	55	0	49	48	24	1	30	1	6	48	24
42	318	0	51	0	57	44	24	1	23	1	16	44	24
48	312	0	46	1	5	40	0	1	16	1	26	40	0
54	360	0	41	1	13	35	12	1	8	1	35	35	12
60	300	0	35	1	20	30	0	0	59	1	44	30	0
66	294	0	29	1	28	24	24	0	49	1	52	24	24
72	288	0	23	1	35	18	24	0	38	2	0	18	24
78	282	0	16	1	42	12	24	0	26	2	7	12	24
84	276	0	8	1	50	6	24	0	16	2	14	6	24
90	270	0	0	1	57	0	0	0	0	2	20	0	0
93	267	0	5	2	0	3	12	0	8	2	23	3	12
96	264	0	10	2	3	6	24	0	15	2	25	6	24
99	261	0	15	2	6	9	24	0	23	2	27	9	24
102	258	0	20	2	9	12	24	0	31	2	28	12	24
105	255	0	26	2	12	15	24	0	40	2	29	15	24
108	252	0	32	2	15	18	24	0	48	2	29	18	24
111	249	0	38	2	17	21	24	0	57	2	30	21	24
114	246	0	44	2	20	24	24	1	6	2	30	24	24
117	243	0	50	2	22	27	12	1	16	2	30	27	12
120	240	0	59	2	24	30	0	1	25	2	29	30	0
123	237	1	8	2	26	32	36	1	35	2	28	32	36
126	234	1	18	2	27	35	12	1	45	2	26	35	12
129	231	1	28	2	29	37	36	1	55	2	23	37	36
132	228	1	38	2	30	40	0	2	6	2	20	40	0
135	225	1	48	2	30	42	12	2	16	2	16	42	12
138	222	1	59	2	30	44	24	2	27	2	11	44	24
141	219	2	11	2	29	46	36	2	37	2	6	46	36
144	216	2	23	2	28	48	24	2	47	2	0	48	24
147	213	2	43	2	26	50	12	2	57	1	53	50	12
150	210	3	3	2	22	52	0	3	7	1	46	52	0
153	207	3	23	2	18	53	12	3	17	1	38	53	12
156	204	3	44	2	12	54	36	3	26	1	29	54	36
159	201	4	5	2	4	56	0	3	34	1	20	56	0
162	198	4	26	1	55	57	0	3	42	1	10	57	0
165	195	4	49	1	42	57	48	3	48	0	59	57	48
168	192	5	13	1	27	58	36	3	54	0	48	58	36
171	189	5	36	1	9	59	12	3	58	0	36	59	12
174	186	5	52	0	48	59	36	4	2	0	24	59	36
177	183	6	7	0	25	59	48	4	4	0	12	59	48
180	180	6	22	0	0	60	0	4	5	0	0	60	0

6. Calculus of the Latitudinal Deviation of the Five Planets

With these things so established, we shall work out the latitudinal calculus of the five planets in this way.

In the case of the three stars, Saturn, Jupiter, and Mars, we carry the truly determined longitude to the numbers of the proper table, that of Mars as it is, Jupiter's with a subtraction of 20°, and Saturn's with an addition of 50°. We record the sixtieths in the fifth columns of latitude corresponding to it, and likewise we carry the truly determined number of anomaly to the same numbers. If the truly determined longitude is in the first 15 rows, we multiply the corresponding latitudinal difference in the third column by the given number of sixtieths; but if it is in the rows following, we multiply that in the fourth column. We shall have the star north of the ecliptic by the number of degrees resulting from these operations if we have taken the latitudinal difference from the third column; but south of it by that much if we have taken it from the fourth column.

But in the case of Venus and Mercury, we first carry the truly determined number of anomaly to the numbers of the proper table, and we record separately the corresponding numbers in the third and fourth columns of latitude—those in three of the columns as they are, but those in the fourth column of Mercury with a subtraction of 10° if the truly determined longitude is in the first 15 rows, and with an addition of 10° if below. Then, for Venus, we always add 90° to the truly determined longitude, and for Mercury 270°, subtracting a circle if we can. The result we carry to the same numbers; and whatever number of sixtieths in the fifth column correspond to these numbers, just so many do we take of the numbers recorded from the third column. When the longitude with the given addition is in the first 15 rows, we shall set out the result as south, if the truly determined number of anomaly is in the first 15 rows, but as north if it is in those that follow. And when this number of longitude falls in the rows below the first 15, we shall set out the result as north if the number of anomaly is in the first 15 rows, but as south if it is in those that follow.

Next we carry the truly determined longitude to the same numbers: in the case of Venus, simply as it is, but in the case of Mercury, with an addition of 180°. Whatever number of sixtieths lie beside it in the fifth column, just so many do we take of the numbers recorded from the fourth column. And when, as we said, the longitude we carried over falls in the first 15 rows, we shall set out the result as north if the truly determined number of anomaly is among those up to 180°, but as south if above 180°. When this longitude falls below these 15 rows, we shall set back the result as south if the anomalistic number is among those up to 180°, but as north if above.

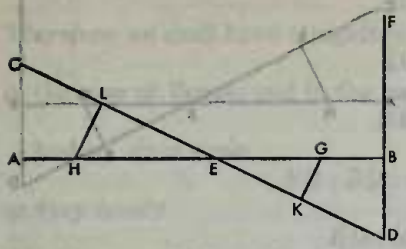
Finally taking, of these sixtieths found from the second carrying over the longitude, the same part as they were of 60, we shall set forth, for Venus, $\frac{1}{6}$ of the result always to the north, and for Mercury, $\frac{3}{4}$ of the result always to the south. Thus, from the mixture of the three settings forth, we get their apparent latitudinal passage with respect to the ecliptic.

7. On the Heliacal Risings and Settings of the Five Planets

Now that we have worked through the latitudinal deviation of the five stars, we have left the filling out of those things which must be considered concerning their heliacal risings and settings. For, just as we described them in detail in the

treatise on the fixed stars, it turns out that their elongations on the ecliptic with respect to the sun are differently unequal in the case of their risings and settings for many causes. The first of these is due to the inequality in their magnitudes; the second to the dissimilarity of the ecliptic's inclinations to the horizon's; and the third to their latitudinal passages.

For if, again, we consider the sections of great circles (*AB* of the horizon and *CD* of the ecliptic); if we suppose the point *E* their intersection either eastern or western, and points *C* and *A* lying on the meridian, and *D* the sun's centre; and if we draw through it and the horizon's pole section *DBF* of a great circle, and suppose the star to rise or set on the horizon *AEB* (clearly at *E* when on the ecliptic; at *G* when north of the ecliptic; and at *H* when south of it), and draw from *G* and *H* the perpendiculars *GK* and *HL*, then again we shall have arc *BD*, as always, equal to the arc of the sun's distance below the earth when the star will first be seen or disappear. For at equal distances below the earth, along the great circle so drawn, the sun's rays give the same light. But since first this distance is established later on to be unequal for the different unequal stars, necessarily, even if all the other things are the same, the ecliptic arcs subtending the right angle subtending the right angle (that is, the distances similar to *ED*) are different, evidently less for the larger stars, and greater for the smaller.



Likewise, if *BD* is the same for the same star, and *BED* the angle of inclination of the ecliptic is unequal either from the differences of the dodecatemories or from those of the latitudes; again the arc *ED* will differ and will be greater when the given angle is decreased, and smaller when it is increased. And likewise even if the inclination is first presupposed the same, but the star not on the ecliptic but north at *G* or south at *H*, then it will no longer appear or disappear first when it is arc *DE* away; but, when it is north of the ecliptic, *DK* will be less, and when south, *DEL* will be greater.

Therefore it is necessary for the examination of particular cases that general sizes of the arcs *BD* be first given, for each of the five planets, from carefully observed risings. Such would be those in summer about the Crab because of the lightness and clearness of the air at that season, and because of the mean value of the ecliptic's inclination to the horizon. Then we find by this examination of the morning observations that, at the beginning of the Crab, Saturn rises generally when it is 14° from the true sun; Jupiter likewise when it is 12¾° away; and the evening Mercury when it is 11½° away.

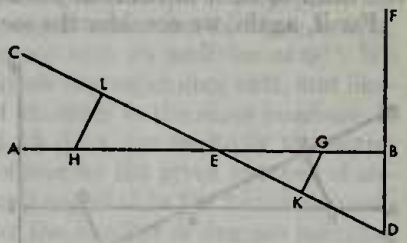
With these things supposed, let the figure of the preceding drawing be laid out since there is no difference in the case of arcs of this size if, for facility in calculation, we take the ratios of the chords (for they are sensibly no different). And let point *E* of intersection of the ecliptic and the horizon be the rising point in the foregoing appearances at the beginning of the Crab for the three morning planets Saturn, Jupiter, and Mars; and the setting point, of course, for the evening planets Venus and Mercury. And let the latitude be taken as the parallel through Phoenicia where the longest day is 14¼ equatorial

hours, since especially at and near this parallel the most and trustworthy observations have been made, those of the Chaldeans on it, and those of Greece and Egypt near it.

Since, then, from the previous demonstration concerning angles [II, 13], when the beginning of the Crab rises in the given latitude, we find

angle $BED = 103°$ to 2 rt.,

and therefore the ratio of the sides about the right angles is very nearly 94 to 75 to the hypotenuse's 120; and since by means of the latitudinal theory, when the three lone stars rise near the beginning of the Crab (that is, are passing near the apogees of the epicycle so as never to be farther from the apogee than a dodecatemory),



we find Saturn and Jupiter practically on the ecliptic, but Mars nearly $\frac{1}{5}°$ north; therefore DE will be the elongation of Saturn and Jupiter from the sun along the ecliptic, but DK that of Mars because of its being farther north by the 12′ of arc KG.

And since

$$KG : KE :: 94 : 75,$$

therefore

$$KE = 0°10'.$$

And it is assumed in the case of Mars

$$DK = 14\tfrac{1}{2}°,$$

so that altogether

$$DE = 14°40'.$$

In the case of Saturn, it is 14°; and of Jupiter, $12\frac{3}{4}°$. And so, since again

$$DE : DB :: 120 : 94,$$

we shall have arc DB of the great circle drawn through the poles of the horizon equal to 11° in the case of Saturn, 10° in the case of Jupiter, and very nearly $11\frac{1}{2}°$ in the case of Mars.

Likewise in the case of Venus and Mercury—since, when the beginning of the Crab sets, it makes the same angle and inclination with the horizon as the foregoing, and it is supposed that about this part the evening Venus rises $5\frac{2}{3}°$ from the true sun and Mercury $11\frac{1}{2}°$—the true sun will be situated at their risings $24\frac{1}{3}°$ within the Twins in the case of Venus, $18\frac{1}{2}°$ in the case of Mercury; the mean sun, 25° in the case of Venus, and very nearly 19° in the case of Mercury. Therefore, the mean longitudinal movement of these stars was that number of degrees. When with this longitude they appear at the beginning of the Crab, Venus is found to be about 14° from the epicycle's apogee, and Mercury about 32°. This is shown by the theorems already expounded on the anomaly. Consequently, in these passages, Venus is found 1° north of the ecliptic and Mercury very nearly $1\frac{2}{3}°$, which of course is KG. And so, since

$$KG : EK :: 94 : 75 :: 1 : \tfrac{3}{4},$$

or very nearly

$$KG : EK :: 1\tfrac{2}{3} : 1\tfrac{1}{3},$$

in the case of Venus we shall have

$$EK = \tfrac{3}{4}°;$$

in the case of Mercury

$$EK = 1\tfrac{1}{3}°.$$

And it is supposed, in the case of Venus,

$$DK = 5\tfrac{2}{3}°$$

where DK is the elongation of either planet from the sun; and in the case of Mercury

$$DK = 11\tfrac{1}{2}°.$$

Therefore we shall have altogether

$$DKE = 6\tfrac{2}{5}°$$

in the case of Venus; and in the case of Mercury

$$DKE = 12\tfrac{5}{6}°.$$

And so, since again

$$ED : BD :: 120 : 94 :: 6\tfrac{2}{5} : 5,$$

or very nearly

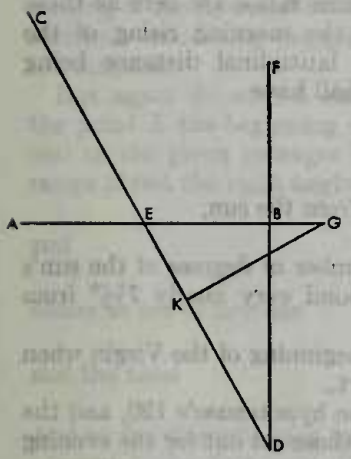
$$ED : BD :: 12\ \tfrac{5}{6} : 10,$$

we shall have the arc BD, the value of the general distance as 5° in the case of Venus, and 10° in the case of Mercury. Which it was required to find.

8. That the Particularities of the Heliacal Risings and Settings of Venus and Mercury Agree with the Hypotheses

We can thus understand that even the extraordinary things concerning the heliacal risings and settings of Venus and Mercury are consequent upon these hypotheses; that is, that in the case of Venus the time from the evening setting to the morning rising about the beginning of the Fishes is about 2 days, but about the beginning of the Virgin 16 days; and that in the case of Mercury the evening risings are eclipsed when it should appear at the beginning of the Scorpion; and the morning risings when it should appear at the beginning of the Bull. And first the case of Venus.

For let a drawing of the apparitions similar to the preceding one be laid out.



Let point E be first supposed near the beginning of the Fishes where Venus, when at the perigee of its epicycle, is north of the ecliptic by very nearly 6⅓°. Let it also be supposed [to represent] the figure of the evening setting where angle BED in the given latitude totals 154° to 2 right angles' 360°, with the greater side of those about the right angle as 117 to the hypotenuse's 120, and the lesser side very nearly 27. Therefore, where the arc DB of the general distance is 5°, there

$$DE = 5°8'.$$

But since the star is 6⅓° north of the ecliptic or arc KG, and

$$117 : 27 :: 6\tfrac{1}{3} : 1\tfrac{1}{2}$$

very nearly, therefore

$$KE = 1\tfrac{1}{2}°,$$

and, by subtraction,

$$KD = 3°38'$$

which is the elongation of the star eastward from the sun, in its evening setting.

Again, in a similar drawing, since in the morning rising

angle $BED = 69°$ to 2 rt.,

therefore the lesser of the sides about the right angle is 68 to the hypotenuse's 120, and the greater is 99. And

$$68° : 120° :: 5° : 8°49',$$
$$68° : 99° :: 6\tfrac{1}{3}° : 9°13';$$

therefore we shall have

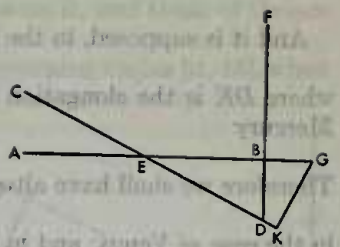
$$DE = 8°49',$$

and the difference due to latitude

$$KE = 9°13';$$

and, by subtraction, eastward of the sun,

$$DK = 0°24'.$$

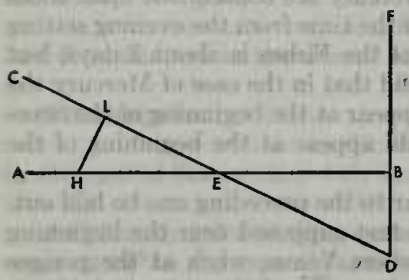


And at the evening setting it was likewise eastward 3°38′. Therefore it has moved, in the time from the evening setting to the morning rising, less than the sun's movement—that is, less than its proper longitudinal passage, because of the epicyclic retrogression of 3°14′. Since, then, the star moves westward by so many degrees, as is easily seen from the Table of Anomaly, when it has moved $1\tfrac{1}{4}°$ at the perigee of the epicycle, and passes through these in mean motion in very nearly 2 days, it is evident that just so much time would be taken for that distance in conformity with the appearances.

Again, in a similar drawing, let the point E be supposed at the beginning of the Virgin where Venus, when at the epicycle's perigee, appears very nearly $6\tfrac{1}{3}°$ south of the ecliptic. And let the evening setting be first proposed when

angle $BED = 69°$ to 2 rt.,

and the lesser of the sides about the right angle is 68 to the hypotenuse's 120, and the greater very nearly 99. Since the same ratios are here as those relative to the morning rising of the Fishes, the latitudinal distance being equal, we shall have



$$\text{arc } ED = 8°49',$$

and, as difference due to latitude,

$$LE = 9°13',$$

and, as the total elongation of the star eastward from the sun,

$$DL = 18°2'.$$

By the Table of Anomaly, as we said, to that number of degrees of the sun's and star's retrogression in mean motion correspond very nearly $7\tfrac{1}{2}°$ from the epicycle's perigee.

Likewise since, at the morning rising near the beginning of the Virgin when

angle $BED = 154°$ to 2 rt.,

the greater side about the right angle is 117 to the hypotenuse's 120, and the lesser side 27, and the same ratios are deduced as those set out for the evening setting in the Fishes, we shall have

$$\text{arc } DE = 5°8',$$

and as difference in latitude

$$EL = 1°30',$$

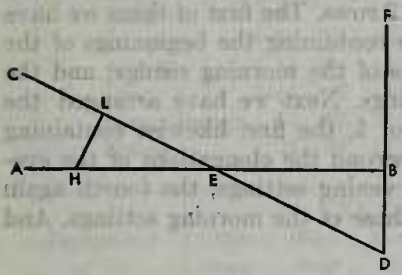
and, as the total elongation of the star westward from the sun,

$$DL = 6°38'.$$

In the same way to these correspond very nearly $2\frac{1}{2}°$ from the epicycle's perigee. Therefore Venus, from the evening setting to the morning rising, will move 10° which it passes through in very nearly 16 days in conformity with the appearances.

These things being proved, it is necessary to consider the incidents of Mercury regarding the eclipsed risings; and first that, at the beginning of the Scorpion, even if it is at its greatest elongation eastward from the sun, the evening rising cannot appear.

For let the drawing of these risings be laid out, E being supposed the point on the ecliptic near the beginning of the Scorpion, where at the setting



angle $BED = 69°$ to 2 rt.,

and the lesser side about the right angle is 68 to the hypotenuse's 120, and the greater 99. Therefore where BD, the general distance, is 10°, there

$$DE = 17°39'.$$

But when the star has that position, it is very nearly 3° south of the ecliptic. And so, since for the given ratios

$$LE = 4°22'$$

where, as arc of latitude,

$$LH = 3°,$$

and the total

$$DEL = 22°.$$

This is the elongation of the star from the true sun necessary for its first appearing. And so, since its greatest elongation from the true sun in the beginning of the Scorpion is only 20°58′ (for this was proved by us in the demonstrations concerning the greatest elongations), it is evident that such risings are eclipsed.

But again if—with a similar drawing of the risings set out—we suppose the point E the beginning of the Bull at the morning rising, then, when the star in the given passages is very nearly $3\frac{1}{6}°$ south of the ecliptic and the ratios about the right angles are the same as before, we shall have

$$DE = 17°39',$$

and

$$LE = 4°37'$$

where as arc of latitude

$$HL = 3°10',$$

and the total

$$DEL = 22°16'.$$

And so the star will have to have an elongation from the true sun of just that amount to be first seen. Since at the most its elongation does not exceed 22°13′ as shown above [Book XII], risings of this kind are eclipsed. And the things proposed have been shown by us to be consonant with the appearances and with the given hypotheses.

9. The Method for the Particular Distances from the Sun of the Risings and Settings

It is immediately evident that, in general (arcs *BD* being supposed for each of the stars and the beginning of the dodecatemories being given at the intersection *E* and therefore angle *BED*), *DE* will also be given and the star's latitudinal passage at this elongation—that is, *KG* or *HL;* and, therefore, *KE* or *EL*, and also the apparent distance *DK* or *DL*. In this way we have calculated for every dodecatemory (not to make the treatise long) the apparent elongations of the risings and settings from the true sun for each of the 5 stars, and for the given latitude only, as being sufficient. Using only the beginnings of the dodecatemories, we have arranged them below, for easy use, in 5 tables for the 5 stars, each containing 12 rows. The first of them we have arranged in columns of 3, the first columns containing the beginnings of the dodecatemories; the second, the elongations of the morning risings; and the third the elongations of the evening settings. Next we have arranged the tables of Venus and Mercury in columns of 5, the first likewise containing the beginnings of the dodecatemories, the second the elongations of the evening risings, the third again those of the evening settings, the fourth again those of the morning risings, and the fifth those of the morning settings. And here is the exposition presented in tables:

10. Exposition of the Tables Containing the Heliacal Risings and Settings of the Five Planets

Beginnings of the Doedcatemories	SATURN				JUPITER				MARS			
	Morning Rising		*Evening Setting*		*Morning Rising*		*Evening Setting*		*Morning Rising*		*Evening Setting*	
Ram	23	1	11	28	20	10	10	19	21	12	11	40
Bull	21	57	11	41	19	6	10	29	20	8	11	48
Twins	17	52	12	26	15	51	11	10	17	21	12	30
Crab	14	2	14	2	12	46	12	46	14	33	14	33
Lion	11	34	15	34	10	40	14	31	12	28	17	19
Virgin	10	53	16	53	10	1	16	12	11	46	20	5
Balance	10	48	17	6	9	57	16	34	11	38	21	1
Scorpion	10	53	16	53	10	1	16	12	11	48	20	19
Archer	11	34	15	34	10	40	14	31	12	34	17	32
Goat	14	2	14	2	12	46	12	46	14	45	14	45
Water Bearer	17	52	12	26	15	51	11	10	17	35	12	36
Fishes	21	57	11	41	19	6	10	29	20	26	11	49

Beginnings of the Dodecatemories	VENUS								MERCURY							
	Evening Rising		*Evening Setting*		*Morning Rising*		*Morning Setting*		*Evening Rising*		*Evening Setting*		*Morning Rising*		*Morning Setting*	
Ram	5	10	4	9	3	0	10	28	9	58	9	43	23	58	23	38
Bull	5	8	4	16	6	16	9	40	10	4	10	15	22	15	22	15
Twins	5	12	5	7	9	15	7	36	10	18	11	47	18	0	16	44
Crab	5	36	8	23	9	50	5	59	12	22	15	34	14	4	12	30
Lion	6	16	13	3	8	2	5	5	13	43	19	59	11	25	10	21
Virgin	7	22	18	2	6	38	4	54	18	1	23	13	10	21	9	59
Balance	7	53	17	43	5	41	4	54	22	49	23	16	9	51	10	0
Scorpion	8	20	13	47	5	28	4	55	20	1	22	1	9	44	10	19
Archer	7	49	8	1	4	39	5	16	18	11	17	25	9	25	11	19
Goat	6	25	4	8	2	43	6	35	13	54	12	10	9	36	14	5
Water Bearer	5	51	3	16	0	30	8	33	11	10	9	50	12	27	17	50
Fishes	5	22	3	38	0	24	10	16	10	11	9	43	19	15	21	46

11. Epilogue

Now that these things have been added, Syrus, and to my mind about all things which ought to be considered in such a treatise have been worked out as much as the time to the present affords for discovery and more accurate revision, and annals suggest as useful for the theory and not just as demonstration, it is therefore fitting and proper that this treatise end here.