Manual of Harmonics

by Nicomachus of Gerasa, the Pythagorean
dictated extempore in the
manner of the ancients

Chapter 1

The book is a manual outlining the doctrine of harmonics

EVEN though the study of intervals and their relations within the harmonic elements is in itself complex and hard to encompass in a single commentary, and even though I, being otherwise in a state of acute mental distraction because of the confusion and haste occasioned by travel, am unable to devote myself to your instruction on these subjects with the appropriate lucidity that demands above all one's leisurely and undivided attention—nevertheless I must exert myself with all zeal, since it is you who bid me, Your Noble Majesty, to set before you the main points in brief, undocumented though they may be, without proper foundation and detailed explication; so that, using this brief note as a manual, you may, with one glance at the chapter headings, recall what is discussed and explained at greater length in each chapter.

And if the gods are willing, as soon as I shall have leisure time and a rest from my journey, I shall compose a longer and more detailed Introduction for you on these same subjects, one that will be thoroughly articulated and crammed, so to speak, with close reasoning, and comprising several volumes. And at my first opportunity, I shall send it to you, wherever I hear that you and your family are residing. In order that it be easier for you to follow, I shall begin at roughly the same point as that at which I started your instruction when I was explaining the subject to you.

Chapter 2

On the two species of the voice, the intervallar and the continuous, and their regions

THE ADHERENTS of the Pythagorean school maintained that there are two species of the human voice subsumed under one genus; they called them the continuous and the intervallar, specifically, deriving their names from the attributes of each. The intervallar, which is the species of melody, stops on every note and renders the change in all the parts perceptible, they assumed to be free from confusion, discrete and graduated by the magnitudes of the intervals lying between each note, forming a progressive series, as it were, and not a blending of the parts of the voice lying adjacent to one another, these being well-defined, readily distinguished and in no instance dissolving into one another. For the melodic species is such that, to trained ears, it renders clear all the notes and the size of the interval in which each note participates. If, on that account, one were to deviate from this species of the voice, he would be said no longer to be singing but to be speaking. The other species is the continuous whereby we converse with one another and read, there being no need for us to make the pitches of the notes explicit and discrete from one another as we string together our discourse right up to the completion of our utterance. For if a person, whether in conversation or in recounting some event or in reading aloud, makes manifest the sizes of the intervals between each note by spacing and changing his voice from one note to another, such a one is said to be speaking or reading no longer, but to be singing. Now since the human voice has two parts, they thought it reasonable that there also be two regions which each of these parts passes through and occupies; that the region of the continuous species is by nature indeterminate in its compass, assuming its own limit from whatever point the speaker begins until he stops, that is, from the first utterance to the final silence, so that it is regulated for the most part by us.

The region of the intervallar species, however, is no longer subject to our control, but to that of nature, and is itself limited by different operations. For its beginning is the first sound that can be perceived by the ear, while its final limit is the utmost sound that can be uttered by the voice. Our awareness and discrimination of the compasses of notes and their changes relative to one another begin at the point at which our sense of hearing receives its very first stimulus, it being possible that fainter sounds not yet perceptible to us are also produced in nature though they still elude our ears. As an example, in the case of weights, there are certain bodies which are almost impossible to express in terms of weight—chaff or bran or other such things; but when such items are added together and the balance-beam of the scale begins to move down, then we say that the first quantum of scientifically determined weight takes place. So too, when the faintness of the voice is intensified little by little to a greater point, we calculate the first degree that is perceptible to our ears to be the beginning of the region of the singing voice. However, it is not our sense of hearing that determines the end of the region, but the human voice. For we define the final limit of the region of this type of voice at that point which it reaches in its melodic progress as it advances musically.

Let it make no difference to us for the present whether we discuss the human voice or that of stringed, wind, and percussion instruments constructed in imitation of our voice; rather, let us pass over for the present the difference between them so that we do not dissipate our explanation at the very outset.

Chapter 3

Among objects of perception, the music of the planets is considered to be the prototype of our music according as we imitate it

IT IS probable that the names of the notes were derived from the seven stars which traverse the heavens and travel around the earth. For they say that all swiftly whirling bodies necessarily produce sounds when something gives way to them and is very easily vibrated; and that these sounds differ from one another in magnitude and in region of the voice either because of the weights of the bodies or their particular speeds, or because of the position in which the motion of each is accomplished, these positions being more subject to fluctuation or, conversely, more resistant. These three differences are clearly observed in the case of the planets, which differ from one another in size and speed and position as they whir continuously and without pause through the ethereal expanse. Hence the word ἀστήρ (star) was formed, inasmuch as each is deprived of στάσις (*status quo*) and is ἀεὶ θέων (constantly running), whereby the words θεός (god) and αἰθήρ (ether) were also formed. However, from the fact that the movement of Kronos is the farthest up from us, the lowest pitched note in the octave was called *hypate*, for *hypaton* means the highest up. On the other hand, from the fact that the course of the Moon is the lowest of all and is situated nearer to the earth, the name *neate* was derived, for *neaton* signifies the lowest. The term *parhypate* was derived from the position of Zeus, being below Kronos on one side, while at the other side, the position of Aphrodite, being above the Moon, occasioned the name, *paraneate*. The term *mese* was derived from the Sun's most central position, this being the fourth from either end, since the *mese* in fact stood at a distance of a fourth in the ancient heptachord. The term *hypermese*, also called *lichanos*, was derived from one of the positions on either side of the Sun, that of Ares, which was assigned the sphere between Zeus and the Sun. On the other side of the Sun between Aphrodite and the Sun, the position occupied by Hermes provided the name *paramese*.

We shall provide more detailed substantiation respecting these identifications, complete with linear and numerical examples, in the commentaries we promised you earlier, Your Most Serene Highness, and we shall explain why we ourselves do not hear this cosmic symphony with its deep complementation of sound and its all-embracing attunement, as tradition describes. But now, because of the pressure of time, we must run over the material that follows.

Chapter 4

The properties in musical notes are regulated by number

WE SAY that sound in general is a percussion of air that is unbroken in its progress to the ear; a note, however, is a pitch without breadth of a melodic voice, and that pitch is a kind of persistence and a sameness of a note that is without intervals in its extent. An interval is a kind of passage from low to high or the reverse. A system is a synthesis of more than one interval.

When a considerable blow or breath falls upon the surrounding air and strikes it in many parts, a loud sound is produced; when a small blow or breath falls upon the surrounding air, a soft sound is produced; when a regular blow falls, a smooth sound is produced; when an irregular blow falls, a harsh sound results; when the blow is applied slowly, a low sound is produced; when quickly, a high sound results.

Wind instruments, such as auloi, trumpets, panpipes and hydraulic organs, necessarily have inverse properties and the same things hold true for the stringed instruments, such as the cithara, lyre, spadix and similar instruments. There are instruments that appear to be intermediate between these in the sense that they share common properties and are subject to the same affects; these are the monochords, commonly called pandouroi but which the Pythagoreans call canons. Classed also among the stringed instruments are the triangular harps; and there are also the transverse auloi along with the photinxes, as the coming discourse will show.

Among stringed instruments, the greater and more powerful the tension, the greater and higher pitched are the notes produced; while the lesser tensions produce more slow moving and lower pitched notes. For when the plectrum displaces the strings, some, when shifted from their proper position, return very quickly and with considerable vibration, striking the surrounding air many times, as if impelled by the very intensity of the tension; on the other hand, others return to their position along the instrument maker's plumbline slowly and without vibration.

Again, in wind instruments the greater bores and greater lengths produce a slow moving and relaxed note. For if the breath, extenuated by a long air-column, issues forth into the surrounding air and strikes and agitates it without much force, the resulting note is low pitched.

We must observe here that the greater and the lesser depend upon the quantity we obtain by our own tightening or slackening of strings, or by our own lengthening or shortening of the parts of air-columns. Wherefore, it is abundantly clear that all these factors are governed by number. For quantity is held to be the property of number and number alone.

Chapter 5

Pythagoras, by adding the eighth string to the seven-stringed lyre, instituted the attunement of the octave

PYTHAGORAS is the first one who—in order that the middle note, when combined itself with both extremes through conjunction, might not produce the consonance of a fourth only, a fourth with hypate at one extreme, and with nete at the other; and in order that we might be able to envisage a more varied scheme, the extremes themselves producing with one another the most satisfying consonance, that is, the octave in a double ratio, which could not result from the two tetrachords—intercalated an eighth note, which he fitted between mese and paramese and separated it from mese by a whole-tone and from paramese by a semi-tone. The result was that the string which was formerly paramese in the heptachord, still being the third string calculating from nete, is called trite and is situated in just this position; the intercalated string, on the other hand, is the fourth string calculating from nete and forms with it the consonance of a fourth, which is the consonance mese formed originally with hypate. The whole-tone between both these two strings, mese and the intercalated string which is named after the former paramese, to whichever tetrachord it is added, whether to the one that is contingent on hypate, in which case it is relatively high in pitch, or to the one contingent on nete, in which case it is relatively deep in pitch, will produce the consonance of a fifth, which is a system of both the tetrachord itself and the added whole tone. Thus the hemiolic ratio [3:2] of the fifth is found to be a system composed of the epitritic ratio [4:3] and of the sesquioctave ratio [9:8]. The whole-tone then is in a sesquioctave ratio.

Chapter 6

How the numerical proportions of the notes were discovered

THE INTERVAL of strings comprising a fourth, that of a fifth, and that formed by the union of both, which is called an octave, as well as that of the whole-tone lying between the two tetrachords, was confirmed by Pythagoras to have this numerical quantity by means of a certain method which he discovered. One day he was deep in thought and seriously considering whether it could be possible to devise some kind of instrumental aid for the ears which would be firm and unerring, such as vision obtains through the compass and the ruler or the *dioptra*; or touch obtains with the balance-beam or the system of measures. While thus engaged, he walked by a smithy and, by divine chance, heard the hammers beating out iron on the anvil and giving off in combination sounds which were most harmonious with one another, except for one combination. He recognized in these sounds the consonance of the octave, the fifth and the fourth. But he perceived that the interval between the fourth and the fifth was dissonant in itself but was otherwise complementary to the greater of these two consonances. Elated, therefore, since it was as if his purpose was being divinely accomplished, he ran into the smithy and found by various experiments that the difference of sound arose from the weight of the hammers, but not from the force of the blows, nor from the shapes of the hammers, nor from the alteration of the iron being forged. After carefully examining the weights of the hammers and their impacts, which were identical, he went home.

He planted a single stake diagonally in the walls in order that no difference might arise from this procedure or, in short, that no variation might be detected from the use of several stakes, each with its own peculiar properties. From this stake he suspended four strings of the same material and made of an equal number of strands, equal in thickness and of equal torsion. He then attached a weight to the bottom of each string, having suspended each by each in succession. When he arranged that the lengths of the strings should be exactly equal, he alternately struck two strings simultaneously and found the aforementioned consonances, a different consonance being produced by a different pair of strings.

He found that the string stretched by the greatest weight produced, when compared with that stretched by the smallest, an octave. The weight on one string was twelve pounds, while that on the other was six pounds. Being therefore in a double ratio [2:1], it produced the octave, the ratio being evidenced by the weights themselves. Again, he found that the string under the greatest tension compared with that next to the string under the least tension (the string stretched by a weight of eight pounds), produced a fifth. Hence he discovered that this string was in a hemiolic ratio [3:2] with the string under the greatest tension, the ratio in which the weights also stood to one another. Then he found that the string stretched by the greatest weight, when compared with that which was next to it in weight, being under a tension of nine pounds, produced a fourth, analogous to the weights. He concluded, therefore, that this string was undoubtedly in an epitritic ratio [4:3] with the string under the greatest tension and that this same string was by nature in a hemiolic ratio with the string under the least tension (for this is the case with the ratio of 9 to 6). In a similar way, the string neighboring on that under the least tension, that is, the string stretched by a weight of eight pounds, compared with that stretched by a weight of six pounds, was in an epitritic ratio, but it was in a hemiolic ratio with the string stretched by a weight of twelve pounds.

Then that interval which is between the fifth and the fourth, that is, the interval by which the fifth is greater than the fourth, was confirmed to be in a sesquioctave ratio [9:8], which is as 9 is to 8. And either way it was proved that the octave is a system consisting of the fifth and the fourth in conjunction, just as the double ratio consists of the hemiolic ratio and the epitritic, as for example, 12, 8, and 6; or conversely, it consists of the fourth and the fifth, just as the double ratio consists of the epitritic ratio and the hemiolic, as for example, 12, 9, and 6, in such order.

And having inured his hand and his hearing to the suspended weights and having established on their basis that ratio of their relations, he ingeniously transferred the bond, which fastened all the strings, from the diagonal stake to the bridge of the instrument, which he called *chordotonon* or string-stretcher, and he transferred the amount of tension on the strings analogous to the weights, to the commensurate turning of the tuning pegs set in the upper part of the instrument. Using this as a standard and as it were an infallible pointer, he extended the test henceforward to various instruments, namely, to the percussion on plates, to auloi and panpipes, to monochords and triangular harps, and the like. And in all of these he found consistent and unchanging, the determination by number.

He called the note partaking of the number 6, hypate, that of the number 8, mese, this number being in an epitritic proportion with the number 6; that of the number 9, he called paramese, which is higher than mese by a whole tone and what is more, stands in a sesquioctave proportion with it; that of 12, he called nete. Filling out the intervening intervals in the diatonic genus with analogous notes, he thus subordinated the octachord to the consonant numbers, the double ratio [2:1], the hemiolic [3:2], the epitritic [4:3], and the difference between them, the sesquioctave [9:8].

Chapter 7

On the division of the octave in the diatonic genus

THUS HE discovered, on the basis of a certain natural necessity, the progression in this diatonic genus from the lowest note to the highest. (For from this procedure he also revealed the structure of the chromatic and enharmonic genera, as it will be possible for me to show you some time later.) This diatonic genus appears, however, to comprehend by nature the degrees and progressions such as follow: semi-tone, a whole-tone, and then a whole-tone. And this is the system of a fourth, consisting of two whole-tones and the semi-tone, so-called. Then, by the addition of another whole-tone, namely, the intercalated whole-tone, the fifth results, being a system of three whole-tones and a semi-tone. Then next in order to this come a semi-tone, a whole-tone, and a whole-tone, being another system of a fourth, that is, another epitritic proportion. So that in the more ancient heptachord all the notes four removed from one another, starting from the lowest, were consonant throughout with each other by a fourth, the semi-tone occupying by transference the first, the middle, and the third place in the tetrachord.

In the Pythagorean octachord, however, whether it be composed of a tetrachord and a pentachord by conjunction, or of two tetrachords separated by a whole-tone from one another by disjunction, the progression by ascent will result in all the notes five removed from one another forming the consonance of a fifth with each other, the semitone as one advances, shifting into the four places, first, second, third, and fourth.

Chapter 8

Explanation of the references to harmonics in the *Timaeus*

IT IS USEFUL, now that we have reached this point, to open up at this opportune moment the passage in the *Psychogony* in which Plato expressed himself as follows: "so that within each interval there are two means, the one superior and inferior to the extremes by the same fraction, the other by the same number. He (the Demiurge) filled up the distance between the hemiolic interval [3:2] and the epitritic [4:3] with the remaining interval of the sesquioctave [9:8]."

For the double interval is as 12 is to 6, but there are two means, 9 and 8. The number 8, however, in the harmonic proportion is midway between 6 and 12, being greater than 6 by one third of 6 (that is, 2), and being less than 12 by one third of 12 (that is, 4). That is why Plato said that the mean, 8, inasmuch as it is of the harmonic proportion, is greater and lesser than the extremes by the same fraction. For the greatest term compared with the smallest is thus in a double proportion; and so it follows that the difference between the greatest term and the middle is 4, compared with the difference between the middle and the smallest, which is 2; for these differences are in a double proportion, 4 to 2. The peculiar property of such a mean is that, when the extremes are added to one another and multiplied by the middle term, a product is yielded which is the double of the product of the extremes; for 8 multiplied by the sum of the extremes, that is, 18, gives 144, which is double the product of the extremes, that is, 72.

The other mean, 9, which is fixed at the paramese degree, is observed to be at the arithmetic mean between the extremes, being less than 12 and greater than 6 by the same number (3). And the peculiar property of this mean is that the sum of the extremes is the double of the middle term itself, and the square of the middle term (which is 81) is greater than the product of the extremes (that is, 72) by the whole square of the differences, that is, by 3 times 3, or 9, for this is the difference.

One can also point out the third mean, more properly called "proportion," in both the middle terms, 9 and 8. For 12 is in the same proportion to 8 as 9 is to 6; for both are in a hemiolic proportion. And the product of the extremes is equal to the product of the middle terms, 12 times 6 being equal to 9 times 8.

Chapter 9

The evidence of Philolaus concerning our statements

THAT the ancients also offered explanations that are in agreement with what has been demonstrated by us—on the one hand, calling the octave *harmonia*, and the fourth *syllaba* (for it is the first "taking together" of consonant notes), and the fifth *dioxeian* (for the fifth, progressing towards the treble, is adjacent to the primitive consonance, the fourth), and, on the other hand, asserting that the system formed of both intervals, the *syllaba* and the *dioxeian*, is the octave (called *harmonia* from the very fact that it is the first consonance "fitted together" from consonant intervals)—Philolaus, the successor of Pythagoras, makes evident by speaking to this effect in the first book of his *On Nature*. Because of haste, we shall content ourselves with a single witness even though many others have often made similar statements on the same subject. The text of Philolaus reads as follows:

> The size of a *harmonia* is a *syllaba* and a *dioxeian*. The *dioxeian* is greater than the *syllaba* by a sesquioctave. For from *hypata* to *mesa* is a *syllaba*, and from *mesa* to *neata*, a *dioxeian*; from *neata* to *trita* is a *syllaba*, and from *trita* to *hypata*, a *dioxeian*. The interval between *trita* and *mesa* is a sesquioctave, the *syllaba* is epitritic, the *dioxeian* is *hamiolic* and the *dia pasan* is a double proportion. Thus a *harmonia* consists of five sesquioctaves and two *dieseis*. A *dioxeian* is three sesquioctaves and a *diesis*, a *syllaba* is two sesquioctaves and a *diesis*.

We must remember that Philolaus means by trite, the note in the heptachord, now called paramese, but called trite before the insertion of the disjunctive whole-tone in the octachord. For this note was distant from paraneate by an incomposite trihemitone, from which interval the intercalated string subtracted a whole-tone, and the remaining semi-tone between trite and paramese was left in the disjunction. Consequently, the ancient trite was distant from the nete by a fourth, which interval the paramese has now assumed in its stead. But there are those who do not understand this and claim that it is impossible for trite to be distant from nete by an interval in the epitritic proportion. Others say not unpersuasively that the intercalated note was not inserted between mese and trite, but between trite and paraneate; but that the ancient trite became paramese in the disjunction; and that Philolaus calls the paramese by the former name, trite, although it was distant from the nete by a fourth.

Chapter 10

On the tuning of the notes by means of numerical proportions

TAKING up our earlier discussion again, let me add the following to it by saying that measurements based on the lengths and thicknesses of strings and on those of the air-columns of auloi are seen to be inverse to measurements that are based on tension, in which case the smaller the term, the lower the pitch, and the greater the term, the higher the pitch. For in the former case, there is an inverse proportion in that the smaller the term, the higher is the pitch, while the greater the term, the lower is the pitch. If, therefore, one takes a long string that is kept under one and the same tension and that lies over a ruler, but is raised far enough above it so as not to touch it, and if one compares the note produced by plucking the entire string with that produced by plucking half the string—the string having been stopped by a bridge or some such contrivance at its very center so that the vibration caused by the plucking of the string may not progress beyond the half-way point—he will find the interval of an octave, the sound of half the string compared with that of the whole string being in a greater proportion, that is, in a duple proportion, a result exactly inverse to the reciprocal data of the length. And if one keeps the vibration down to a third of the string, this part having been measured off exactly, the sound from two thirds of the string will necessarily be in a hemiolic [3:2] relation to that of the sound from the whole string, or inversely proportional to the length of the string. And if one sections off a fourth part of the string, preventing the vibration from reaching any further than that, the sound from three parts of the string will be in an epitritic [4:3] relation to that of the whole string, or inversely proportional to the length of the string. So too in the case of the aulos that has three holes distributed over four equal lengths; if the holes are first stopped by the fingers and if we compared the note of the whole aulos with that produced by the middle hole, the finger being lifted from the hole, a duple proportion would be found and the note produced by the middle hole would form an octave with that of the whole aulos. The same note compared with that produced by the hole below it, namely, the hole lying next to the bottom extremity of the pipe, would stand in a hemiolic [3:2] proportion with it. This latter note, however, compared with that of the whole pipe stands in an epitritic [4:3] proportion with it. But the note produced by the hole next to the mouthpiece compared with that produced by the middle hole, stands in a duple [2:1] relation and, compared with that produced by the whole aulos, in a quadruple [4:1] relation, or inverse to the proportion of the lengths. In the case of syringes (pan-pipes), the lengths of the pipes and the widths of their bores produce a result similar to the thicknesses of the strings; for those made of two strands emit a sound in duple proportion to those made of four strands.

Chapter 11

On the double octave in the diatonic genus

THIS, then, is the diagrammatic structure of the scale in the diatonic genus, consisting of a double octave of quadruple breadth. For this is all that the voice at full capacity encompasses without any insecurity or vacillation, it being difficult for the voice to place itself at either extreme. On the one hand, the voice has a tendency to hoot like a cuckoo in the treble register, while on the other hand, it tends to buzz in the deeper register of the bass notes.

Accordingly, to the old-fashioned lyre, that is, to the heptachord composed of two tetrachords in conjunction—the mese itself delimiting both of the consonant intervals, the lower one contingent on hypate as one ascends, the higher one contingent on nete as one descends—to these they attached two other tetrachords, one at each extremity. Adjoining the original nete the hyperbolaion tetrachord was situated, called "hyperbolaion" because it was composed of a higher and "transcending" vocal register, beginning from the ancient nete, again by conjunction with it. Consequently, the extended tetrachord found its limit after three notes only had been added which were suitably named as follows: trite hyperbolaion, then paranete hyperbolaion, then nete of the same tetrachord.

In order that the tetrachord preceding it [hyperbolaion] and formed by conjunction with the mese, might have its notes so designated as to distinguish it from the hyperbolaion, its notes were given these names: after mese comes trite synemmenon, then paranete synemmenon, then nete synemmenon. And the entire high-pitched range reckoning from the mese itself necessarily completes a heptachord also. To the original hypate they added at the bass the other of the tetrachords mentioned, again by conjunction, the ancient hypate being also comprehended in it since it is the higher of the notes in the tetrachord. In a similar way, for the sake of distlnguishing it from the disposition of the prior tetrachord, this one also acquired more distinctive names. For to each name was added the word, "hypaton," such as: hypate hypaton, parhypate hypaton, diatonos hypaton or lichanos hypaton, for it makes no difference which of the two names we use.

And this entire system from the mese to the hypate of the hypaton tetrachord turns out to be a heptachord composed of two conjunct tetrachords, these using one common note, the ancient hypate. Consequently, from hypate hypaton to nete hyperbolaion there are four conjunct tetrachords. A thirteen-stringed system is found, the seventh string being fixed diatonically from either end. Then, as we said previously, those intending to vary the attunement intercalated the eighth note at a distance of a whole-tone between mese and the ancient trite (or, as some say, between trite and paranete) and clearly emphasized the interval of a fifth. And the mese was found to be no longer truly a middle note. For in the case of strings that are fixed to an equal number of degrees, there cannot be one middle note, but necessarily two, the seventh and eighth strings.

Again, they added one outermost note below the hypate, the lowest of the existing notes, which they called because of this "addition," proslambanomenos, it too being at a distance of a whole-tone below the hypate hypaton as one descends basswards. The purpose of this addition was that the systems on either side of the mese might be octachords, and that the mese might truly be a middle note by being situated among fifteen notes as the eighth string from either end; and that the double octave, the total compass of the scale, might be a doubly duple proportion, that is, a quadruple proportion; and that the order of names, moving upward in succession, might be such as follow:

proslambanomenos
then after an interval of a whole-tone, hypate hypaton
then after a semi-tone, parhypate hypaton
then after a whole-tone, lichanos hypaton, named after
 the finger of the left hand, the finger next to the thumb,
 which is called forefinger and is
 always applied to this string
then after another whole-tone, hypate meson

following in succession after a semi-tone, parhypate meson
and after a whole-tone, lichanos meson, which they
also call "diatonos," naming it after the
diatonic genus itself
then after another whole-tone, mese
then paramesos, after a whole-tone
then trite diezeugmenon, after a semi-tone
then after a whole-tone, paranete diezeugmenon
and after another whole-tone, nete diezeugmenon
succeeding this, after a semi-tone, trite hyperbolaion
then after a whole-tone, paranete hyperbolaion
and following upon all, after a whole-tone, nete hyperbolaion

In reminiscence of the original conjunction in the heptachord, another tetrachord called synemmenon was inserted between the meson tetrachord and the diezeugmenon, complete with its own trite, separated by a semi-tone from the mese; then after a whole-tone, its own paranete, then after another whole-tone, the conjunctive nete of exactly the same pitch and the same sound as the disjunctive paranete. Consequently, there are five tetrachords in all, hypaton, meson, synemmenon, diezeugmenon, hyperbolaion—among which there are two disjunctions and three conjunctions. The disjunctions occur between the synemmenon and the hyperbolaion tetrachords and between the meson and the diezeugmenon tetrachords, each comprising the interval of a whole-tone—and the three conjunctions, one conjoining hypaton with meson, one conjoining meson itself to synemmenon, and the last one conjoining diezeugmenon to hyperbolaion.

The discoveries of all these elements note by note, their causes and developments, how they came about, by whom they were discovered and when, and from what type of origin, I shall explain to you in my extensive treatment, beginning with the tetrachord and ending with the complete *katapyknosis* of the octave, not only in this diatonic genus, but also in the chromatic and the enharmonic, together with the testimonies of ancient witnesses who are the most reliable and who are held in the highest esteem. And in addition we shall set forth the division of the so-called Pythagorean canon up to the twenty-seventh multiple, in rigorous conformity with the intention of this master, not as Eratosthenes or Thrasyllus misrepresented it, but as the Locrian Timaeus rendered it, whom even Plato followed closely.

Chapter 12

On the progression and division of the notes in the three genera

IN ORDER that you may have the progression in the three genera from proslambanomenos up to hyperbolaion nete in an orderly extension, it is suitable that I recapitulate for the sake of clarity, beginning with the statements I made a little earlier.

A note is an indivisible vocal utterance, an auditory monad, as it were. But as the moderns say, it is the incidence of the voice upon a single and simple point of pitch. While, according to some, it is a sound without breadth and without intervals in its region.

An interval is the intervening space between two notes. Relationship, however, is the ratio which measures the distance in each interval; and difference is the excess or deficiency of the notes compared with one another. Those who believe that difference and relationship are the same thing are wrong in their thinking. For consider: 2 to 1 comprises the same difference as 1 does to 2 but not the same relationship. For 2 is twice that of 1, but 1 is half of 2. And further, among all the terms of an arithmetic mean, whether they be three or even more, the difference is the same among all, but the relationship is of one sort or another. But you will be provided with more copious information on this question in my extensive treatment of the subject.

A system is a combination of two or more intervals. But no note among the intervals is consonant with the one immediately following it, but is completely dissonant with it. Among the systems, however, some are consonant, others dissonant. Systems are consonant when the notes comprising them, though they be different in compass, commingle with one another when played together or are somehow sounded simultaneously, in such a way that the sound produced from them is of a oneness like a single voice. Notes are dissonant, however, when the sound emanating from both of them is heard to be disparate in some way and unblended.

Since the first and most elementary consonance is the fourth in a continuous tetrachord in epitritic [4:3] proportion, it is reasonable to find in this tetrachord the variations from one to another of the three genera of melody. For the diatonic, about which we spoke earlier, progresses in this way: semi-tone, then a whole-tone, then a whole-tone—three intervals within four numbers, that is to say, within four notes. For this reason it is called "diatonic," from the fact that it is the only one of the rest of the genera that progresses "by tones." The chromatic progresses in this way: semi-tone, then another semi-tone, then after these, an incomposite trihemitone. And this genus is disposed in such a way that even if it is not composed outright of two whole-tones and a semi-tone, it may be observed, nevertheless, to comprise intervals that are equivalent to two whole-tones and a semi-tone. The enharmonic comprehends by nature such a progression as the following: quarter-tone, which is half of a semi-tone, another quarter-tone, the sum of both being equal to a semi-tone, and the remaining interval of the tetrachord, an entire incomposite ditone. And this is distributed in such a way that it is also equivalent to two whole-tones and a semi-tone; for within these intervals it is impossible for one note to be consonant with another.

It is evident, therefore, that the variations of the genera do not assume their difference within the four notes of the tetrachord, but within the two middle notes only. In the chromatic, therefore, the third note was altered in relation to the diatonic, but the second remained the same as in the diatonic and at the same pitch as the third note in the enharmonic. In the enharmonic the two middle notes were altered in relation to the diatonic in such a way that the enharmonic is opposite to the diatonic, and the chromatic is between these two, for it deviates minimally from the diatonic, by one semi-tone only. From whence we say that men who are changeable have "color."

The extreme notes of the tetrachord, then, are called "fixed notes," for they do not deviate in any of the genera; while the middle notes are "movables," at least in the enharmonic. In the chromatic, however, the second note is both movable and immovable, for relative to the diatonic it does not vary, but relative to the enharmonic it does vary.

The octave, being a system distributed over eight strings, whether one reckons downwards from mese to proslambanomenos or upwards from mese to nete hyperbolaion—amounting to a fourth which consists of two whole-tones and a semi-tone and a fifth which consists of three whole-tones and a semi-tone—does not consist of six whole-tones exactly, as the moderns believe, but of five whole-tones and two semi-tones so-called. If these semi-tones were really halves of whole-tones, what would prevent a whole-tone from being composed of them and the octave from consisting of six whole-tones? We shall offer a clear and very detailed explanation of this problem in our extensive treatment of the subject. Indeed, Philolaus agrees with us in the preceding citation in which he says, "A *harmonia* consists of five sesquioctaves and two *dieseis*," that is, two semi-tones, which would make one whole-tone if they were truly halves of whole-tones.

Accordingly, the names of the three genera amalgamated with one another in the same diagram will be these:*

proslambanomenos	$A_{\mid}$
hypate hypaton	$B_{\mid}$
parhypate hypaton enharmonic	$B_{\mid}+$
parhypate hypaton chromatic and diatonic	C
enharmonic hypaton	C
chromatic hypaton	C♯
diatonos hypaton	D
hypate meson	E
parhypate meson enharmonic	E+
parhypate meson chromatic and diatonic	F
meson enharmonic	F
meson chromatic	F♯
meson diatonic	G
mese	A
trite synemmenon enharmonic	A+
trite synemmenon chromatic and diatonic	B♭

* The sign + indicates the rise of a quarter-tone.

synemmenon enharmonic	$B\flat$
synemmenon chromatic	B
synemmenon diatonic	$C^{\prime}$
nete synemmenon	$D^{\prime}$
paramese	$B^{\prime}$
trite diezeugmenon enharmonic	$B^{\prime}+$
trite diezeugmenon chromatic and diatonic	$C^{\prime}$
enharmonic diezeugmenon	$C^{\prime}$
chromatic diezeugmenon	$C^{\prime}\sharp$
diatonic diezeugmenon	$D^{\prime}$
nete diezeugmenon	$E^{\prime}$
trite hyperbolaion enharmonic	$E^{\prime}+$
trite hyperbolaion chromatic and diatonic	$F^{\prime}$
enharmonic hyperbolaion	$F^{\prime}$
chromatic hyperbolaion	$F^{\prime}\sharp$
diatonic hyperbolaion	$G^{\prime}$
nete hyperbolaion	$A^{\prime}$

Please forgive the haste of such writing as this—for you are aware that you asked me while I was completely unsettled in transit—and, consistent with your most gentle and altogether most intelligent nature, please accept this as first fruits of a sort and friendship's offering. And if the gods be willing, please await my treatise which will be a very copious and complete discourse on the subject discussed here. I shall send it to you directly, as the earliest opportunity presents itself.

Excerpts from Nicomachus

1. They say that Hermes invented the lyre from the tortoise-shell, and providing it with seven strings, handed down the art of lyre-playing to Orpheus. And Orpheus taught Thamyris and Linus. Linus taught Heracles, by whom he was killed. He also taught Amphion, the Theban, who built Thebes with seven gates after the seven strings of the lyre. When Orpheus was killed by the Thracian women, his lyre was thrown into the sea and was cast up in the city of Antissa in Lesbos. Fishermen found it and carried it to Terpander and he took it to Egypt. The story goes that he discovered it and then brought it to perfection and demonstrated it to the priests in Egypt, alleging that he was the first to invent it. Thus it is said that Terpander invented the lyre, and the Achaeans inherited it from Cadmus the son of Agenor. Such are the current reports.

2. There are numbers from which musicians form intervals consisting of whole-tones. In the case of the fourth, which they say is observed to be in the epitritic [4:3] proportion, consisting of two whole-tones and a certain fraction, as, for example, when it begins with parhypate hypaton and ends with parhypate meson; they take as an epitritic number—256 to 192—and, starting from this 192, they take it through the sesquioctave ratios in which the intervals of a whole-tone are computed (the *leimma* being left over), they complete the number 256. Accordingly, then, they raise the interval a whole-tone starting from 192 and arrive at 216 which is the sesquioctave of 192; for it contains 192 and its eighth, 24. Again, starting from 216, they raise it a whole-tone and arrive at 243 which is the sesquioctave of 216; for it contains it and its eighth, 27. To arrive at the completion of the fourth and of the number 256, there remains the number 13 which is neither half of the first whole-tone, calculated in terms of the number 24, nor is it half of the second whole-tone, calculated in terms of the number 27.

In the case of the fifth, which is organized in a hemiolic [3:2] proportion and is composed of three whole-tones and a certain fraction, as, for example, when it begins with parhypate meson and ends with trite diezeugmenon, they deal with the question at issue in the following way. They take as a hemiolic number—768 to 512—and, starting from this 512, they take it through the sesquioctave [9:8] ratios in which the intervals of a whole-tone are computed and, with the left-over *leimma*, they complete the number 768. Accordingly, then, they raise the interval a whole-tone starting from 512 and arrive at 576, which is the sesquioctave of 512; for it contains the number 512 itself and its eighth, 64. Again, starting from 576, they raise the interval a whole-tone and arrive at 648, a sesquioctave of 576, since it contains this number, 576, and its eighth, 72. Again, starting from 648, they raise the interval a whole-tone and arrive at its sesquioctave, 729, which is the sesquioctave of 648 since it contains it and and the number 81, which is an eighth of 648. To complete the hemiolic number, 768, there is left remaining, the number 39. But 39 is neither half of 64, in terms of which the first whole-tone is calculated, nor is it half of 72, in terms of which the second whole-tone is calculated, nor is it half of 81, in terms of which is the third whole-tone calculated. For the whole-tone is not exactly divided into two semi-tones.

And yet it is also true in the case of the fourth consisting of a whole-tone and a leimma and a whole-tone, as, for example, when one starts with proslambanomenos and ends with diatone hypaton, in which case they deal with it in this way. Again, they take as an epitritic number, 288 to 216. And starting with 216, they raise the interval a whole-tone and arrive at 243, which is a sesquioctave of 216 and exceeds 216 by 27. But since we cannot raise the interval a whole-tone starting with 243, for we do not find it to be a complete sesquioctave, we achieve, by lowering it a whole-tone, the sesquioctave's reciprocal of 288. The reciprocal sesquioctave of 288 is 256 because it is contained wholly by 288 plus its eighth, 32. The result of raising the interval starting with 216 through the sesquioctave ratio of a whole-tone, 243 is found; while the result of lowering the interval starting with 288 through the sesquioctave ratio of a whole-tone, 256 is found. But in order that the entire intermediary number be completed, there is left remaining the number 13, which is neither half of 27 nor is it half of 32. Taken in another way, the entire system of a fourth is, in terms of the aforementioned numbers, 72. But in terms of the sesquioctave obtained by raising the interval, the number 27 will be found; while in terms of the sesquioctave obtained by lowering the interval, the number 32 will be found. And if we subtract from 72 the numbers 27 and 32, there is left remaining the number 13, which is neither half of 27 nor of 32.

Again, in the case of the fifth which is constituted of a whole-tone and a *leimma* and two whole-tones, as, for example, when one starts with diatone hypaton and ends with mese, they offer the following arrangement. They take as a hemiolic number, 1296 to 864, and starting with this, the 864, they raise the interval a whole-tone and arrive at 972, which is greater than 864 by 108. But since we cannot raise the interval a whole-tone starting with 972, we find it by lowering the interval. Accordingly, we lower it starting with 1296 and we find the number 1152, which is the reciprocal sesquioctave of 1296 and is exceeded by 1296 by the number 144, which is an eighth of 1152. Again, we lower the interval accordingly starting with 1152 and we find the number 1024, which is the reciprocal sesquioctave of 1152 and is exceeded by 1152 by the number 128, which is an eighth of 1024. Therefore, from 1024 to 972, the point at which the whole-tone is fixed when one raises the interval starting with 864, there is left remaining the number 52. This number is not a half of any of those numbers which were discovered among the whole-tones; it is not half of 108, nor of 128, nor of 144; for it is among these numbers that the whole-tone intervals are found.

3. [Nicomachus says] that in the distribution of the notes of the heptachord the topmost and first string is called hypate. This is indeed because Kronos is the highest and the first reckoning from the sphere of the fixed stars; the lowest is called neate after Selene since it is the most remote sphere of them all, while the middle string is called mese after Helios. Of the strings on either side of nete and hypate, the one next to hypate is called parhypate after Zeus, while the other one next to nete is called paranete not after Hermes, but unconventionally after Aphrodite, unless there is a mistake in the written account; and hypermese is named after Ares,

and trite after Aphrodite. The highest note belongs to Selene, if she assumes the position of nete, while the low note belongs to Kronos, if he assumes the position of hypate.

Those who first began to reckon from the spheres nearest to us say, however, that hypate is the first note and belongs to Selene, since it is the beginning of the notes [of the scale], and that nete, since it is the farthest from us, belongs to Kronos. For the hypate is more related to terrestrial things, because in a multiple substance the power is weaker; so that because of its earthliness it embodies multiplicity and because of its multiplicity it is weak. For the power of all existing things lies in the fact that each is an entity unto itself. It is for this reason that the hypate was allocated to Selene since Selene is herself variable and changeable and has less power because of her greater distance from the sphere of fixed stars; and in this respect there is a certain fixed position of the heavenly bodies relative to her, just as there is among the notes ranging from nete to the hypate which do not admit of another consubstantial kind of distribution extending any further. For this reason hypate is allocated to Selene. For Selene is the first celestial body we encounter since she is nearer to the Earth. Indeed the low note has its origin in the lowest hollow parts of the body near the flanks, while the high note originates in the ears and the brain and the temple, which are the highest parts of the body. Neate is, therefore, the note of Kronos, because he is not susceptible of addition and because he embraces the other [celestial bodies] in his essence and his power. Accordingly then, because of the fact that the one, Selene, is slow and the other, Kronos, is fast in the movement of the cosmos in the same direction, and because of her lagging behind the planets, Selene is identified with the note, hypate. For Kronos is nearest to the sphere of fixed stars, lacking only one thirtieth of a degree, so that in the uniform motion [of the heavens] he lacks two minutes a day from the complete revolution of the universe, that is, one thirtieth of a degree. If we examine Selene in her uniform and daily movement, it is possibile to discover that she falls behind by thirteen degrees and fourteen minutes, so that it is reasonable to consider the one, Kronos, to be the swiftest of all the heavenly bodies and the other, Selene, the slowest.

4. [Nicomachus says] that all those who added other strings to the eight-stringed lyre were induced to do so not by any mathematical logic, but for the gratification of their listeners. Thus, Prophrastus the Pierian added the ninth string, Histiaeus of Colophon the tenth, and Timotheus the Milesian the eleventh, and others added the rest in sequence. Thereafter, the number of strings was brought by them to eighteen. And so Pherecrates, the comic poet, is observed to have censured the practice in his play entitled *Cheiron* for its debauchery of melodies.

In all, then, the strings in the three genera [or even more] amount to a total number of twenty-eight. There are neither more nor less than these owing to the fact that the human voice does not admit of sounds lower than these in the bass; for sounds uttered below these are booming and husky, indistinct intonations that are unarticulated and unmelodious. Sounds uttered above these in the treble screech like the cuckoo, the notes being very like the howling of wolves, and are unintelligible and incapable of participation in a consonance. The number of strings in each genus according as two middle notes are formed in the disjunctions [of tetrachords] so that a tetrachord may be consonant with a pentachord by disjunction—are eighteen. But those notes in the Immutable System which form no more than one middle note, but employ it as the lowest of the higher pitches and the highest of the lower pitches, are limited to fifteen strings for the double octave in the Immutable System. Indeed, Ptolemy agrees with this and says that we must remain fixed with this number, asserting that the tonoi are equal in number to the species of the octave (and to the species) of the fourth and of the fifth; and that it is from these tonoi that every note comprehends its homophone within this number of notes; and that the mese is exactly a middle note and that both extremes are limited, the bass to the proslambanomenos, the treble to the nete hyperbolaion.

5. [Nicomachus says] that since some carried the number of the notes up to twenty-eight, they will be seen not to have fallen outside of the consonance of the universe, but have followed the natural philosophy of Pythagoras and the doctrine of Plato. For the

nature of the divine souls is cut up by the twenty-seventh multiple, the monad, the origin of all things, being taken in addition (just as in the case of musical notes, they call the added note proslambanomenos) and amounts to so great a number; it starts with the seven terms which Plato postulated, and ends with the triple and cubic progressions which he discovered.

6. What is more, the notes of each of the seven spheres produce by nature a single kind of sound; to each of which the elementary speech sounds called vowels are referred. These individually and anything composed of them are not to be spoken aloud by the wise. Wherefore the note has the same power as the monad has in arithmetic and the point has in geometry. These elements are combined with material substances (as, for example, vowels are combined with consonants), just as the soul is combined with the body and *harmonia* is combined with the strings. When the soul is combined with the body, it produces living things; when *harmonia* is combined with the strings, it produces keys and melodies, these combinations being the active and consummating productions of the gods. Wherefore, whenever the initiates pay reverence to such an act, they invoke it symbolically with sigmas and clickings and inarticulate and meaningless sounds. [He says] that those who instituted the heptachord as a natural concord took it from this source; that such a construction comes not from the spheres, but from the sounds themselves that are inserted and interposed in every word, and that these sounds are the only ones among the letters that we call vowels and voiced utterances. But since the distribution of these sounds is simple, it is not adequate to designate anything complex, but there is required for the completion of a whole complex the combination and interconnection of one element with another, just as in the case of the lyre strings. It is in this way that the hebdomad is connected with the tetrad and the monad, as we said in our work on the *Hebdomad*. Therefore, the hebdomad multiplied by the tetrad in a cycle from the first term to the last and back again whether by addition or by subtraction amounts to such a number. For that is also true in the case of the spheres. It is evident how proportionately it is in the case of the

souls containing the spheres and conducting them around in an orderly motion, each one conducting a separate sphere, according to their increment, the sound belonging to each being added; but in rhythm and position each of the others is lacking in number, whence the circuit of sounds is said to be twenty-eight in number according to the convention of the Egyptians.

7. Starting with these facts, Pythagoras discovered that the first division and extension of the soul is limited to this number. For indeed the soul is threefold, getting her substance from the Same, and the Other, and from Being; and like substance, her tripartition is limited to the rational, and the irrational, and the physical (just as melodies are limited to the enharmonic, and the diatonic, and the chromatic). And thence appears the bipartite division in the *Psychogonia* itself which, in the seven notes, gives to the planets the constitution of twenty-eight physical and spiritual notes that is harmonious in essence; and to the sphere of the fixed stars, since it is of the nature of the Same, and as it embraces the planets in motion, it thus gives the cubic increment in the other number, 36, which number is perfect and contains the augment of the right-angled triangle the two sides of which are opposite to the hypotenuse. Perfected with the origin of all things, thence comes the first tetraktys containing the source of the consonances which appears in the numbers, 6, 8, 9, 12, and embraces the ratio of hypate and mese, and nete and paramese, and the sesquioctave. For hypate has reference to the number 6, mese to 8, nete to 12 and paramese to 9. The sesquioctave is first observed in the ratio 8:9. This system when condensed by quarter-tone intervals advances to that number in which the degrees of the zodiac and the circuit of the zodiac is limited.

8. But how, he says, are there said to be seven notes when there are eight spheres? It is because the sphere [of fixed stars] that always moves at the same rate produces a single and unvarying emission of sound; and since it has no other sphere of a speed equal to it or nearly equal, it can have no place in an attunement. [Whereas among the first two planets, such is the case, since they are neither

equal nor unvarying]. But among the planets there are some that are shown to have a great dissimilarity and contrariness to it, having a movement as great as that of the Same is to that of the Other. That is why they let the note emitted by it be unmentioned.

9. The notes which are employed by the professional artists are these twenty-eight:

1. Proslambanomenos
2. Hypate hypaton
3. Parhypate hypaton
4. Hypaton enharmonic
5. Hypaton chromatic
6. Hypaton diatonic
7. Hypate meson
8. Parhypate meson
9. Meson enharmonic
10. Meson chromatic
11. Meson diatonic
12. Mese
13. Trite synemmenon
14. Paranete synemmenon enharmonic
15. Synemmenon chromatic
16. Synemmenon diatonic
17. Nete synemmenon
18. Paramesos
19. Trite diezeugmenon
20. Diezeugmenon enharmonic
21. Diezeugmenon chromatic
22. Diezeugmenon diatonic
23. Nete diezeugmenon
24. Trite hyperbolaion
25. Hyperbolaion enharmonic
26. Hyperbolaion chromatic
27. Hyperbolaion diatonic
28. Nete hyperbolaion

10. Just as some of the tetrachords in the three genera are hypaton, others are meson, others, synemmenon, others, diezeugmenon, and others are hyperbolaion, so it must be supposed that every spherical and divine being assumes something in the universe that has the relation in the attunement and constitution of the cosmos as of a beginning, something else as of a middle, and another as of an end—one thing being a conjunctive coagency, another a disjunctive one. And providence, using all these elements, produced a number that is divine, firm and unshaken and made the universe in melodic accord with itself, binding together every dominant and subordinate essence in such a proportion. Therefore, the first tetraktys, which is also the root of these tetrachords, is the cause in some way of all the divisions according to the genera, bringing about, as I have said, the number thirty-six belonging to the sphere of fixed stars, with the addition of the monad. And Plato, wishing to display this key of all the things that exist, in the thirteenth book of the *Nomoi*, when he had enumerated the last of all the explicative sciences, says, that it also apportioned to humans a consonant and symmetrical function.